AN EMPIRICAL MEASURE OF ASSET LIQUIDITY*

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ABSTRACT

We develop an operational empirical measure of liquidity consistent with the definition first suggested by Keynes: An asset is more liquid if it is "more certainly realizable at short notice without loss." We define this loss as the difference between the value realizable from optimal sale (with no time constraint) and the value realizable from immediate sale. We estimate the former by treating optimal sale as a search problem: the seller faces a sequence of prices over time and must decide when to accept. Implications of our definition include (1) market efficiency implies perfect liquidity, (2) liquidity is forward-looking and depends on the expected future path of prices rather than current or past prices, (3) liquidity applies to buyers as well as sellers, and (4) any inference about the liquidity of an asset is necessarily model-dependent. We illustrate the use of our measure by applying it to Los Angeles real estate and to investment in Canadian dollars and in pounds sterling.

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I. INTRODUCTION

Our purpose is to develop an operational empirical measure of liquidity. Our measure is consistent with the definition of liquidity first suggested by Keynes: An asset is more liquid if it is "more certainly realizable at short notice without loss."[1] We illustrate the use of our measure by applying it to Los Angeles real estate and to investment in Canadian dollars and in pounds sterling.

To give Keynes's definition empirical content, we must be able to calculate the loss from selling an asset at short notice. We define this loss as the difference between the value realizable from optimal sale (sale with no time constraint) and the value realizable from immediate sale.[2] We estimate the value realizable from optimal sale by formulating optimal sale as a search problem. The seller faces a sequence of prices over time and must decide when to accept. The expected present value of the price that results from the optimal strategy is the value of optimal sale.

In Section II, we set out the search framework and use it to derive some measures of liquidity. In Section III, we use a series of examples to illustrate the behavior of these measures. In particular, we emphasize how liquidity depends on the nature of the stochastic process generating asset prices. In Section IV, we apply our measures to data on real estate and foreign exchange. In Section V, we discuss some possible directions for future research.

Before we begin, we should make clear our position on alternative definitions of liquidity. Since Keynes first introduced the concept of liquidity, other authors have offered a variety of alternative definitions. For example, in some very interesting recent work, Amihud and Mendelson (1986, 1991) identify liquidity with the bid-ask spread. This notion of liquidity differs substantially from that of Keynes. While the bid-ask spread does constitute a loss from realizing an asset, the size of this loss is not uncertain, and it does not depend on the urgency of sale. Despite these differences, however, we have no quarrel with the Amihud-Mendelson definition (or with any other). We have no desire to argue that Keynes's is the only "true" definition of liquidity or even that it is the best. Liquidity has many dimensions, and different definitions may be useful in capturing these different dimensions.

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[1] Keynes (1930), p. 67.

[2] Certainly "short notice" need not imply *immediate* sale. We interpret it thus as an obvious benchmark. Our analysis could easily be modified to accommodate alternative definitions of short notice.

II. The Search Framework

We assume that the seller has an asset he wishes to sell once and for all. He must decide whether to sell it immediately or whether to wait and sell it later. Our seller is a liquidity trader rather than an information trader or speculator: his desire to liquidate now is unrelated to the asset's current price. That is, we consider a problem of optimal liquidation, not one of short-run positioning.[3]

Some simplifying assumptions

In defining the search problem, we make a number of simplifying assumptions:

- There is no explicit search cost: the seller observes a new price each period without cost. The only cost of search is the cost of waiting.
- We represent the cost of waiting with a constant rate of discount, r.
- We ignore any transactions costs of sale (bid-ask spread, etc.).[4]
- The seller is risk-neutral.
- The asset is a pure appreciation asset: there is no cash flow from it until it is sold.
- The market is well enough organized for there to be a unique price each period. That is, the seller has no need to search each period among heterogeneous buyers bidding different prices.[5]

The seller's search problem

The following recursive formula defines the value of the asset under optimal sale:

$$V(I_t) = \max\{p_t, \frac{1}{1+r} \mathrm{E}V(I_{t+1})|I_t\}. \tag{1}$$

where

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[3]The distinction between the two may be illustrated with an example. Suppose an asset's price is $1 this period, $0.90 next period, and $1.10 in every subsequent period–all known with certainty. The solution to the short-run positioning problem is to sell now, buy next period, and sell again the following period (Kohn, 1978). The solution to the liquidation problem, assuming a low enough discount rate, is simply to wait two periods and sell at $1.10.

[4]If these costs are constant over time, they will have little effect on the difference between the value of optimal sale and the value of immediate sale.

[5]Our problem, therefore, is not the conventional one of the search literature–"micro-search." Rather, it is intertemporal or "macro-search." For example, in terms of the housing market, the question we address is not "Should I accept Smith's offer for my house?" but rather "Should I put my house on the market this year?"

I_t = the information set at time t. The information set includes the current price and the distribution over future prices.

$V(\cdot)$ = the value of the asset under optimal sale, as a function of the information set.

p_t = the current price.

$E(\cdot)|I_t$ = the mathematical expectation over possible next-period information, given current-period information.

Equation (1) states that the value of the asset under optimal sale is the maximum of the current price and of the expected present value of continuing to search optimally.

Under very general conditions, the optimal search strategy involves a switchpoint or reservation price (Kohn and Shavell, 1974). Given I_t, there exists a unique reservation price $s(I_t)$, such that the optimal policy is to sell if $p_t > s(I_t)$ and to wait if $p_t < s(I_t)$; if $p_t = s(I_t)$, the seller is indifferent between selling and waiting.

Using (1) and the reservation price property, we have that

$$s(I_t) = \frac{1}{1+r} EV(I_{t+1})|I_t \tag{2}$$

and

$$V(I_t) = \max\{p_t, s(I_t)\} = \begin{cases} p_t \text{ if } p_t > s(I_t) \\ s(I_t) \text{ otherwise} \end{cases} . \tag{3}$$

The level of the reservation price, given the information set, depends on the parameters of the problem. In our case, the only parameter of importance is the discount rate. Kohn and Shavell (1974) show that the reservation price is a decreasing function of the discount rate.[6]

Measures of liquidity

We can use the solution to the optimal sale problem to define liquidity. The loss from immediate sale of the asset, $L(I_t)$, is just the difference between the value of optimal sale and the value of immediate sale (the current price). That is,

$$L(I_t) = \max\{0, s(I_t) - p_t\} \tag{4}$$

We define an index of liquidity, $\lambda(I_t)$, as

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[6]Kohn and Shavell prove the following additional comparative static results: The reservation price is increasing in uncertainty, where uncertainty is defined in terms of a mean-preserving spread. The reservation price is decreasing in search costs, when these are present. It is decreasing in the degree of risk aversion, if the seller is risk-averse.

$$\lambda(I_t) \equiv \frac{V(I_t) - L(I_t)}{V(I_t)} . \tag{5}$$

Since this index depends on the information set I_t, we call it the *conditional liquidity* of the asset.

If there is no loss from immediate sale, the conditional liquidity is unity and the asset is perfectly liquid. If there is some loss from immediate sale, conditional liquidity is less than unity. The greater the percentage loss, the lower is the conditional liquidity.

Because conditional liquidity depends on the information set, it will vary with time. Consequently, an asset may be liquid at some times and illiquid at others. There is an obvious correspondence between high conditional liquidity and the popular notion of a "sellers' market": both indicate a good time to sell.

Conditional liquidity characterizes the liquidity of an asset at any moment. We can construct from it another measure that characterizes the liquidity of an asset on average. The *expected liquidity* of an asset, Λ, is

$$\Lambda \equiv \mathrm{E}\, \lambda(I_t) \tag{6}$$

where the expectation is over all possible values of the information set, I_t.

Conditional liquidity and expected liquidity depend on two things–on information about the behavior of the asset's price over time and on the discount rate used to derive the optimal policy. The dependence on the discount rate presents us with a problem. Since the discount rate is a property of the individual seller, the liquidity of a given asset will differ from seller to seller.

We would like a measure of liquidity that is general rather than individual. To obtain such a measure, we "invert" the problem. That is, we solve for the implicit discount rate that will make a given asset perfectly liquid. We call this rate the *liquidity rate*.

There are four equivalent interpretations of the liquidity rate:

1. It is the lowest discount rate that makes the conditional liquidity of an asset unity.
2. It is the lowest discount rate that makes the loss from immediate sale zero (from (5)).
3. It is the lowest discount rate that makes the reservation price equal to the current price (from (4))
4. It is the lowest discount rate that makes it optimal to sell immediately rather than to wait.

Because the reservation price is decreasing in the discount rate, when an asset is illiquid, its liquidity rate will be high. When an asset is illiquid, only those sellers who are

in a hurry to sell–those with high discount rates–will find it worthwhile to do so immediately.

Like conditional liquidity, the liquidity rate will vary with the information set and so with time.[7]

Buyers' liquidity

We have looked at liquidity from the point of view of sellers–"sellers' liquidity". But there is an obvious analogy between the problem of selling an asset and the problem of purchasing an asset. Corresponding to this second problem, we can define a concept of liquidity that applies to buyers–"buyers' liquidity."

The problem of optimal purchase is similar to that of optimal sale.[8] The value of an asset to a buyer under the optimal policy is

$$V(I_t) = \min\{p_t, \tfrac{1}{1+r}\,\mathrm{E}V(I_{t+1})|I_t\}, \tag{1'}$$

The buyer's reservation price is $s^{\mathrm{B}}(I_t) = \frac{1}{1+r}\,\mathrm{E}V(I_{t+1})|I_t$. The optimal policy is to buy if $p_t < s^{\mathrm{B}}(I_t)$ and to wait if $p_t > s^{\mathrm{B}}(I_t)$.

The buyer's reservation price is distinct from the sellers' reservation price, and we shall discuss the relationship between the two in Section III below. The buyer's reservation price, too, is a decreasing function of the discount rate.

We can define the loss from having to purchase the asset immediately in precisely the same terms as we defined the loss from having to sell immediately (equation (4)). From this, we can define buyers' conditional liquidity and buyers' expected liquidity (equations (5) and (6)). Corresponding to the sellers' liquidity rate, we can define a buyers' liquidity rate.

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[7]As a measure of liquidity of a given asset on average, parallel to its expected liquidity, we could take the expected value of its liquidity rate.

[8]We make the same assumptions about the buyer's problem as we have made about the seller's. In particular, the buyer too is a liquidity trader: he is purchasing the asset once and for all with the intention of holding it indefinitely.

Earlier work on search and liquidity

We are not the first to think about liquidity in terms of search. Most recently, Lippman and McCall (1986) have suggested that the liquidity of an asset be measured as the *expected time to sale* when following the optimal policy.[9]

As Lippman and McCall note, the two measures are not equivalent. An example illustrates the difference. Suppose the current price of an asset is $100, and the value of optimal sale is $100.01. The expected time to sale when following the optimal policy is 100 years. According to Lippman and McCall's definition this asset is highly illiquid. According to our definition, it is almost perfectly liquid. As this example shows, our definition provides a better *economic* measure of illiquidity.

There is, however, a much more basic difference between Lippman and McCall's work and our own. Lippman and McCall assume throughout that asset prices in different periods are independently and identically distributed (i.i.d.). We consider a much broader class of distributions. This is important for two reasons.

First, prices of assets in the real world never follow i.i.d. distributions. Since our goal is an *empirical* measure of liquidity, it is essential that our framework encompass the types of distributions that actually occur.

Second, the liquidity of an asset depends crucially on the nature of the stochastic process generating prices. Consequently, restricting discussion to only one type of process, deprives the discussion of much of its interest.

III. Liquidity and the Price Process

The best way to understand the dependence of liquidity on the nature of the price process is to look at some examples.

Example 1: A random walk

Our first example is a random walk, which in some sense is a benchmark. Intuitively, when an asset's price follows a random walk, there is no benefit to waiting to sell it: it is always optimal to sell the asset immediately. Consequently, an asset whose price follows a random walk is always perfectly liquid.

To verify this formally, assume the price process satisfies:

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[9]Lippman and McCall mention the possibility of defining liquidity, as we have done here, in terms of the difference between the value of optimal sale and the value of current sale. However, they focus primarily on the expected time to sale.

(7) $$Ep_{t+1}|I_t = (1+\rho)p_t \text{ for all } t.$$ [10]

That is, given current information, there is no reason to believe that the asset will rise or fall in value by more or less than the average rate of appreciation, ρ.

Let $V^T(I_t)$ be the value of the asset to a seller when search is restricted to a horizon of no more than T more periods. Then the value of the asset for a horizon of no more than T+1 periods satisfies

(8) $$V^{T+1}(I_{t-1}) = \max\{p_{t-1}, \frac{1}{1+r} EV^T(I_t)|I_{t-1}\}.$$

Now suppose that for some search horizon T, $V^T(I_t) = p_t$ for all possible I_t. Then for a search horizon of T+1,

(9) $$V^{T+1}(I_{t-1}) = \max\{p_{t-1}, \frac{1}{1+r} EV^T(I_t)|I_{t-1}\}$$

$$= \max\{p_{t-1}, \frac{1}{1+r} Ep_t|I_{t-1}\}$$

$$= \max\{p_{t-1}, \frac{1+\rho}{1+r} p_{t-1}\} = p_{t-1}$$

for $r \geq \rho$.

The inductive hypothesis is satisfied for T=0. Since search must be terminated in the current period,

(10) $$V^0(I_t) = p_t.$$

Therefore, for $r \geq \rho$, $V^T(I_t) = p_t$ for all T>0,and it is always optimal to sell immediately. Conditional liquidity is identically unity, as is expected liquidity.

For $r < \rho$, it is always optimal to wait and the asset is never sold. Conditional liquidity is identically zero, as is expected liquidity.

The liquidity rate is identically equal to the rate of appreciation, ρ.

An obvious implication of these results is that *market efficiency implies perfect liquidity*. In an efficient market, prices follow a random walk: consequently, the asset is perfectly liquid. If all asset prices followed a random walk, our measure of liquidity would

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[10]This is a martingale, rather than strictly a random walk, with no restriction on the variance of price changes.

be of little interest. However, there is considerable evidence that many asset prices do not follow a random walk.[11]

Example 2: An i.i.d. process

As we have noted, the i.i.d. process is not of much empirical interest. However, it is of some use in clarifying the meaning of our measures of liquidity.

Assume that the asset appreciates at an average rate of ρ and that the *detrended* price, $\frac{p_t}{(1+\rho)^t}$, is i.i.d. For the standard i.i.d. search problem (Kohn and Shavell, 1974), the reservation price, s, is a constant and satisfies the following equation:

$$s = \frac{1}{r}\int_s^{\infty}(q-s)dF(q) \tag{11}$$

where q denotes draws from the density function $dF(q)$. The reservation price is monotonically decreasing in the discount rate r.

We can translate this into a result for the actual, *non*-detrended, price process. For this process, the reservation price in period t is

$$s_t = s\,(1+\rho)^t, \tag{12}$$

where the detrended reservation price, s, satisfies

$$s = \frac{1+\rho}{r-\rho}\int_s^{\infty}(q-s)dF(q). \tag{13}$$

The solution is defined for all $r > \rho$. The value of s is monotonically decreasing in r.

For the corresponding problem of optimal purchase, there is a detrended reservation price for buyers, s^B, analogous to that for sellers. It satisfies

$$s^B = \frac{1+\rho}{\rho-r}\int_0^{s^B}(s^B-q)dF(q), \tag{14}$$

which is defined for all $r < \rho$. The value of s^B is monotonically decreasing in r.

For any realization of the price, p_t, we can calculate the seller's liquidity rate, $r(p_t,)$ that just makes it optimal to sell at that price. We can also calculate the buyer's liquidity rate $r^B(p_t,)$ that just makes it optimal to buy. Figure 1 plots r and r^B as functions of the realized price for a numerical example. Notice that r is bounded below by ρ and that r^B is bounded above by ρ. The asset is liquid from the point of view of a seller when the price is high and

[11] See for example Bekaert and Hodrick (1992): "[t]here is now considerable evidence that excess returns on a variety of assets are predictable".

when r, consequently, is low. It is liquid from the point of view of a buyer when the price is low and when r^B, consequently, is high.[12]

Example 3: A Markov chain

A process that is particularly useful is the Markov chain. It is useful because (1) it can approximate any other process, and (2) it is easy to calculate reservation prices for a Markov chain. We shall use the Markov chain in this way in our empirical applications.

In a Markov chain there is a finite number of states, K. Each state k is described by the pair $\{p_k, I_k\}$, where p_k is the price associated with the state k and I_k is an information set including other information associated with the state.[13] The probability of moving from state i to state j is π_{ij}, the i, jth element of a transition matrix Π.

There is a reservation price, s_k, associated with each state k. The value of the asset in state k under optimal sale is

$$V_k = \max\{p_k, s_k\} \tag{15}$$

where

$$s_k = \frac{1}{1+r} \sum_{i=1}^{K} \pi_{ij} V_i. \tag{16}$$

Consequently, the K reservation prices satisfy the following set of nonlinear equations:

$$\begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} = \frac{1}{1+r} \Pi . \begin{bmatrix} \max\{p_1, s_1\} \\ \vdots \\ \max\{p_K, s_K\} \end{bmatrix} \tag{17}$$

We can calculate the conditional liquidity in state k, λ_k, according to equations (4) and (5). We can then calculate the expected liquidity using the ergodic probabilities–the long-run probabilities of being in each of the states:

$$\Lambda = \sum_{k=1}^{K} \alpha_k \lambda_k,$$

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[12]The buyer's reservation price is decreasing in the discount rate. Therefore a high value of the discount rate is needed to make the reservation price equal a low value of the actual price.

[13]The information set could, for example, include a distribution function over prices, so that each state has associated with it a particular distribution. See Kohn (1978) or Lippmann and McCall (1976).

where α_k is the ergodic probability of state k.[14]

Example 4: The Engel-Hamilton model: a mean-averting process

The process we use in our empirical applications is the Engel-Hamilton process (Engel and Hamilton, 1990). This is a segmented Markov trend process. It takes the log-difference of the asset price to be drawn each period from one of two Normal distributions, referred to as "regimes." Each regime has a distinct mean and variance. The mean of each regime is the average rate of appreciation for that regime.[15] Transition between the two regimes is governed by an unobservable two-state Markov chain. The model is defined by six parameters–the two means, μ_1 and μ_2; the two variances, σ_1^2 and σ_2^2; and the two transition probabilities p_{11} and p_{22}.

This framework encompasses a broad variety of processes. For example, if the transition probabilities satisfy $p_{11} = 1 - p_{22}$, the model reduces to a random walk. Or, if the transition probabilities are small and the means are of opposite sign, the process exhibits "long swings"–periods of sustained appreciation followed by periods of sustained depreciation.[16]

The Engel-Hamilton model has been used successfully in a wide variety of finance and finance-related applications: foreign exchange (Engel and Hamilton 1990, Engel 1992); interest rate spreads (Lahiri and Wang 1994), inflation (Evans and Lewis 1992); and the equity premium puzzle (Abel 1994).

To illustrate the behavior of the liquidity of an asset whose price follows an Engel-Hamilton process, we generated an artificial "time series" of asset prices for such a process. A plot of the series is shown in Figure 2. We chose parameter values that would generate a "long swings" process.[17]

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[14]Not every Markov chain has ergodic probabilities (is ergodic). Whether it does or not depends on the properties of the transition matrix, Π. If ergodic probabilities exist they can be calculated from the transition matrix.

[15]In the basic version of the process, which we adopt, draws from each regime are independent. More complicated versions allow serial correlation of the draws.

[16]Such a process is consistent with the behavior of asset prices described by Fama and French (1988), Poterba and Summers (1988), and others–positive autocorrelations at short horizons, negative autocorrelations at longer horizons, and near zero autocorrelation at very long horizons.

[17]We set the average rates of appreciation at 3% and -1% respectively and the variances for both regimes at 5. We set the transition probabilities at $p_{11} = 0.94$ and $p_{22} = 0.86$.

From this series we calculated the seller's liquidity rate and the buyer's liquidity rate at each time period. We did this by approximating the Engel-Hamilton process with a Markov chain. Each state is characterized by a pair of numbers–the price and the probability that the process is in the first regime. From this Markov chain we calculate the loss from immediate sale, L_k, for each state, k. We then calculate the discount rate for each state that would make L_k zero–the liquidity rate associated with that state. The seller's rate and the buyer's liquidity rate are shown in Figure 2 along with the price series.

Buyer's liquidity is roughly, but not exactly, inverse to seller's liquidity. When the asset is liquid from the point of view of a seller (low seller's liquidity rate) it is generally illiquid from the point of view of a buyer (low buyer's liquidity rate).

Notice that a high seller's liquidity rate, indicating illiquidity, is associated with periods of rising prices. This is because the underlying process is "mean-averting"–the "mean" in this case being the average rate of appreciation of the asset.[18] When the process is in the appreciation regime, the expected rate of appreciation is above average and will likely continue to be so for several periods. When the process is in the depreciation regime, the expected rate of appreciation is below average and is likely to continue so for several periods.

Liquidity is defined in terms of the expected loss from selling immediately. It is therefore forward-looking and depends on the expected future path of prices. With a mean-averting process, rising prices make future increases more likely. Selling immediately means foregoing this anticipated appreciation. Moreover, as the price continues to rise, the probability of being in the appreciation regime increases and the expected loss of immediate sale increases with it.

It should be clear, therefore, that *any inference about the liquidity of an asset is model-dependent.*

Example 5: An AR(1) model: a mean-reverting process

To illustrate the model dependence of measures of liquidity, we reinterpret the data generated by the Engel-Hamilton process of Example 3 as if it were generated by a different, mean-reverting process. We detrend the data and fit to it an AR(1) model.[19]

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[18]The average rate of appreciation is $\omega\mu_1+(1-\omega)\mu_2$, where $\omega = (1-p_{11})/[(1-p_{11})+(1-p_{22})]$ is the ergodic probability of being in the first regime.

[19]The model is estimated from the log-detrended data. The trend is estimated to be 1.4% per "quarter" or 5.7% per year, and the intercept, lag-coefficient, and error variance are estimated to be 0.00, 0.94, and 7.77E-04 respectively.

Figure 3 shows the underlying price series, the estimated trend, and the seller's and buyer's liquidity rates implied by the AR(1) model.[20]

These results, which assume a mean-reverting model, are very different from those we obtained when we assumed the (correct) mean-averting Engel-Hamilton model. The most striking difference is in *when* the asset is illiquid. Now it is illiquid, not when prices are rising, but when prices are below trend (illiquid from the point of view of a seller, as indicated by a high seller's liquidity rate). With a mean-reverting process, when prices are below trend they are likely to increase by more than the trend rate. Consequently, immediate sale involves a sacrifice, and the asset is illiquid.[21]

IV. APPLICATIONS

We now apply our measure of liquidity to two assets–residential real estate and foreign exchange. Residential real estate is an asset that is generally considered to be relatively illiquid. Foreign exchange is one that is considered to be relatively liquid.

In each case, we begin by estimating a model of the asset's prices. As we have seen, the measure of liquidity will depend quite strongly on the model chosen. Research is only now beginning to explore predictable patterns in asset prices, and it therefore provides us with relatively little guidance. Because of its simplicity, and because it has been used quite widely, we rely on the Engel-Hamilton model. As models of asset prices improve, so will the resulting measures of liquidity.[22]

Application 1: Los Angeles real estate

Figure 4 plots our price series for Los Angeles real estate. The data consist of 91 quarterly observations for an index of home prices in the Los Angeles area for the period

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[20]We do this as before by approximating our process, here an AR(1) process, with a Markov chain. We then calculate reservation prices and liquidity rates. Details of our procedure are available on request.

[21]There are other differences between the results generated by the two models. Results based on the mean-averting process show sellers' liquidity rate fluctuating over roughly the same range, but much more smoothly. The buyers' liquidity rate fluctuates over a much smaller range, and it moves more closeley with the sellers' liquidity rate. Because the average rates of appreciation are different in the two cases (it is estimated from a misspecified model in the mean reversion case), the boundary rates which separate the buyers' and sellers' liquidity rates are different as well.

[22]For simplicity, we restrict ourselves to univariate models. There is some evidence that additional variables may help in predicting future asset prices (see, for example, Fama and French, 1988).

1971:I through 1993:III deflated by the GDP deflator.[23] Notice the two major cycles of the past 30 years. There was a rapid appreciation from the mid-70s until about 1980, when a combination of extremely high nominal interest rates and recession brought about a mild real decline in prices. Another period of rapid appreciation began in 1986 and ended in 1990, to be followed by a sharp depreciation.

Table 1 shows the Engel-Hamilton model fitted to this data.[24] Notice the high average rate of appreciation in the high growth regime–over 12% per year in real terms. The average rate of depreciation in the low growth regime is much smaller–about 2.8% per year in real terms. The process is in the high growth regime about 30% of the time (this is its ergodic probability). The average rate of appreciation is 1.98% per year in real terms. Both regimes are very persistent: the probability of remaining for another quarter in the high growth regime is 92%; the probability of remaining in the low growth regime is 97%.

Figure 4 plots the seller's liquidity rate implied by this model.[25] The rate varies between about 2% and 14% on an annualized basis, and is highly autocorrelated.[26] Liquidity displays the pattern we would expect from a "long swings" mean-averting model. Real estate is relatively liquid in the early 1970s and in the early 1980s when real prices are flat. It becomes illiquid during the two large run-ups in prices in the late 1970s and late 1980s, and becomes liquid again as prices fall in the early 1990s. There are occasional spikes of illiquidity when the inferred probability of being in the appreciation regime temporarily jumps, as it did in mid-1983 with the small appreciation that occurred then.

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[23] The data were provided to us by Karl Case. These numbers represent average market transactions prices. They are therefore the result of a process of search by many individuals over unobservable non-transactions prices. As we noted before, we abstract from this problem of "micro-search" to focus on the problem of "macro-search" (when to put real estate on the market rather than whether to accept a specific offer). We treat residential real estate as a pure appreciation asset, because we lack date on current net income. If the latter does not fluctuate much, it may not affect liquidity a great deal.

[24] We estimated the model using a maximum likelihood procedure. We used a randomized grid search, which is robust against local maxima, to find a starting value and then used Hamilton's (1990) EM algorithm.

[25] We use the same procedure to calculate the liquidity rate as described in footnote 20 above. For clarity, we plot only the seller's liquidity rate. As we saw in the numerical examples, buyer's and seller's liquidity move largely in parallel.

[26] Recall that the average rate of appreciation determines a floor beneath the seller's liquidity rate.

Application 2: Foreign exchange

In our second application, we look at the liquidity of investments in foreign exchange. Since our model is quarterly, we assume that investors hold *securities* denominated in a foreign currency rather than cash. Consequently, for each currency, we construct a series giving the dollar value of a fund reinvested quarterly in Treasury bills denominated in that currency.[27] Each series consists of 80 observations, running from 1973:IV through 1993:III.

Figures 5 and 6 plot such series for the pound sterling and for the Canadian dollar respectively.[28] The dollar value of the pound fund reflects the long swings of the dollar-pound exchange rate over this period. The pound appreciated against the dollar in the late 1970s (by 110% between mid-1976 and the end of 1980), depreciated in the strong-dollar period of the early 1980s (by 26% by 1985), and appreciated again after 1985 (by some 250% between then and 1992). The U.S.-Canada exchange rate exhibited much milder swings over this period. Consequently, the U.S. dollar value of the Canadian dollar fund grows fairly steadily, with a decline relative to trend in the decade 1976-1986 and a steep appreciation relative to trend from 1986-1991.

Table 1 shows the Engel-Hamilton model fitted to this data. The parameter estimates for the two currencies are very different. The difference between the two regimes is more extreme for the pound fund—an average rate of appreciation of 24% per year in the high growth regime versus 3% depreciation in the low growth regime. The Canadian dollar fund appreciates on average in both regimes, by 13% and 3.6% respectively. The Canadian dollar fund is in its high appreciation regime 85% of the time, versus 44% for the pound fund.[29] Consequently, the average rate of appreciation is higher for the Canadian dollar fund–11.75% versus 8%.

Figures 5 and 6 plot the sellers' liquidity rates for the two currencies. The most striking difference is in the variability of liquidity. The liquidity rate varies much more for the pound than for the Canadian dollar–between 8% and 25% for the pound, between 11.75% and +3.50% for the Canadian dollar. The standard deviation for the pound liquidity rate is nearly 20 times that for the Canadian dollar (it is even higher than that for Los Angeles real estate). Autocorrelation of the liquidity rate is similar for the two currencies (0.63 and 0.67 respectively), but lower than it is for real estate (0.87).

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[27]We use averages of daily interest and exchange rates (bid/ask averages) in the third month of each quarter.

[28]Note that these series are in nominal terms while the real estate series is in real terms.

[29]The high appreciation regime is also particularly persistent for the Canadian dollar fund.

The liquidity of the pound fund exhibits the same pattern, typical of the "long swings" model, that we observed for Los Angeles real estate. It is relatively liquid during its periods of depreciation and relatively illiquid during its periods of appreciation. In contrast, the Canadian dollar fund does not seem to exhibit significant swings, and its liquidity remains relatively constant.

V. FURTHER RESEARCH

We are currently exploring a number of possible applications and extensions of our measure of liquidity:

1. We are calculating liquidity measures for Treasury securities at different maturities. We wish to see whether differences and variation in these measures explain liquidity premia in the term structure of interest rates.

2. We are examining whether variations in liquidity measures of currencies explain changes in the interest rate premium of one currency over another.

3. We intend to experiment with alternative models of price processes (mean-reverting as well as mean-averting). How well the liquidity measures derived from different models perform in various applications may provide some evidence as to which of the alternative models is the "true" one.

3. The risk of illiquidity is a financial attribute that has many parallels with the risk associated with fluctuations in an asset's return. Can the risk of illiquidity be diversified away and are investors paid to bear it? Is it possible to define an asset's *liquidity beta* with respect to the liquidity of the market portfolio. Does liquidity beta help to explain asset returns?

4. Our measure of the value of an asset under optimal sale holds some promise as a basis of marking assets to market. For assets that are illiquid and that need not be sold immediately, the value of optimal sale seems a much more reasonable measure of their value than the current market price.[30]

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[30]We are also working on some technical issues in the computation of liquidity liquidity rates. The main one is how to solve the search problem (and compute reservation prices) while explicitly allowing for a nonstationary (integrated) price process. We are also trying to find efficient ways to calculate standard errors for our estimated liquidity rates.

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Table 1: Parameter Estimates for the Engel-Hamilton Model

Parameter	LA Real Estate	Pound Sterling	Canadian Dollar
μ_1	3.162 (0.333)	5.478 (1.028)	3.167 (0.304)
μ_2	-0.707 (0.312)	-0.785 (2.003)	0.900 (0.979)
σ_1^2	2.691 (0.730)	16.967 (6.267)	2.954 (0.601)
σ_2^2	4.579 (0.955)	30.570 (11.441)	3.477 (1.423)
p_{11}	0.922 (0.049)	0.803 (0.125)	0.969 (0.029)
p_{22}	0.965 (0.026)	0.847 (0.136)	0.830 (0.197)
ω*	0.845	0.438	0.310
trend rate of growth (annualized)	11.75%	8.06%	1.98%
log-likelihood	-162.26	-214.72	-199.25

Standard Errors, computed from numerical evaluation of the inverse hessian matrix, in parentheses. Real estate prices are in real terms while the currency funds represent nominal returns. * the ergodic probability of being in the high-growth regime.

Figure 1: Liquidity Rates for Sellers and Buyers

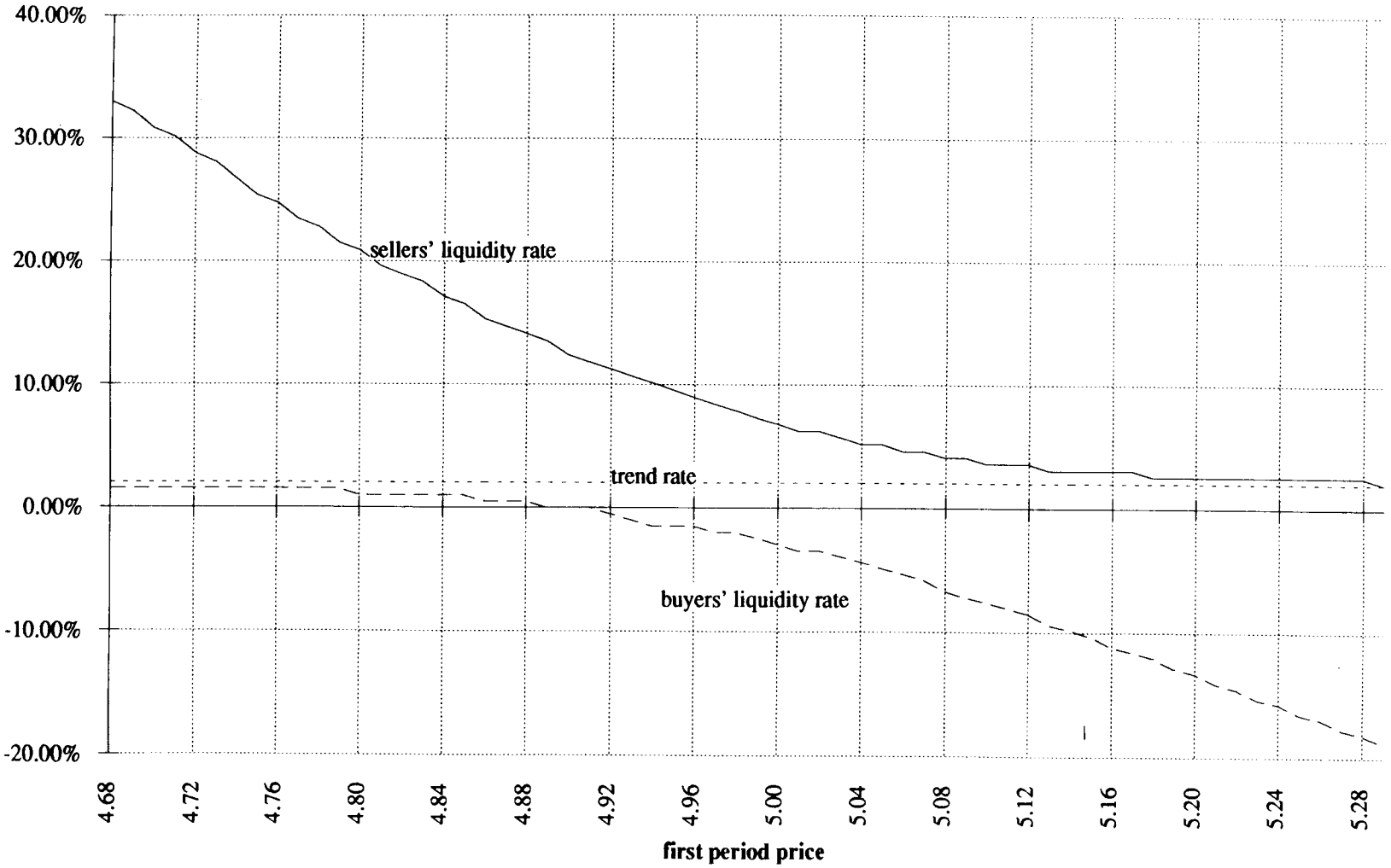

Figure 2: A series generated by the Engel-Hamilton model and the implied liquidity rates

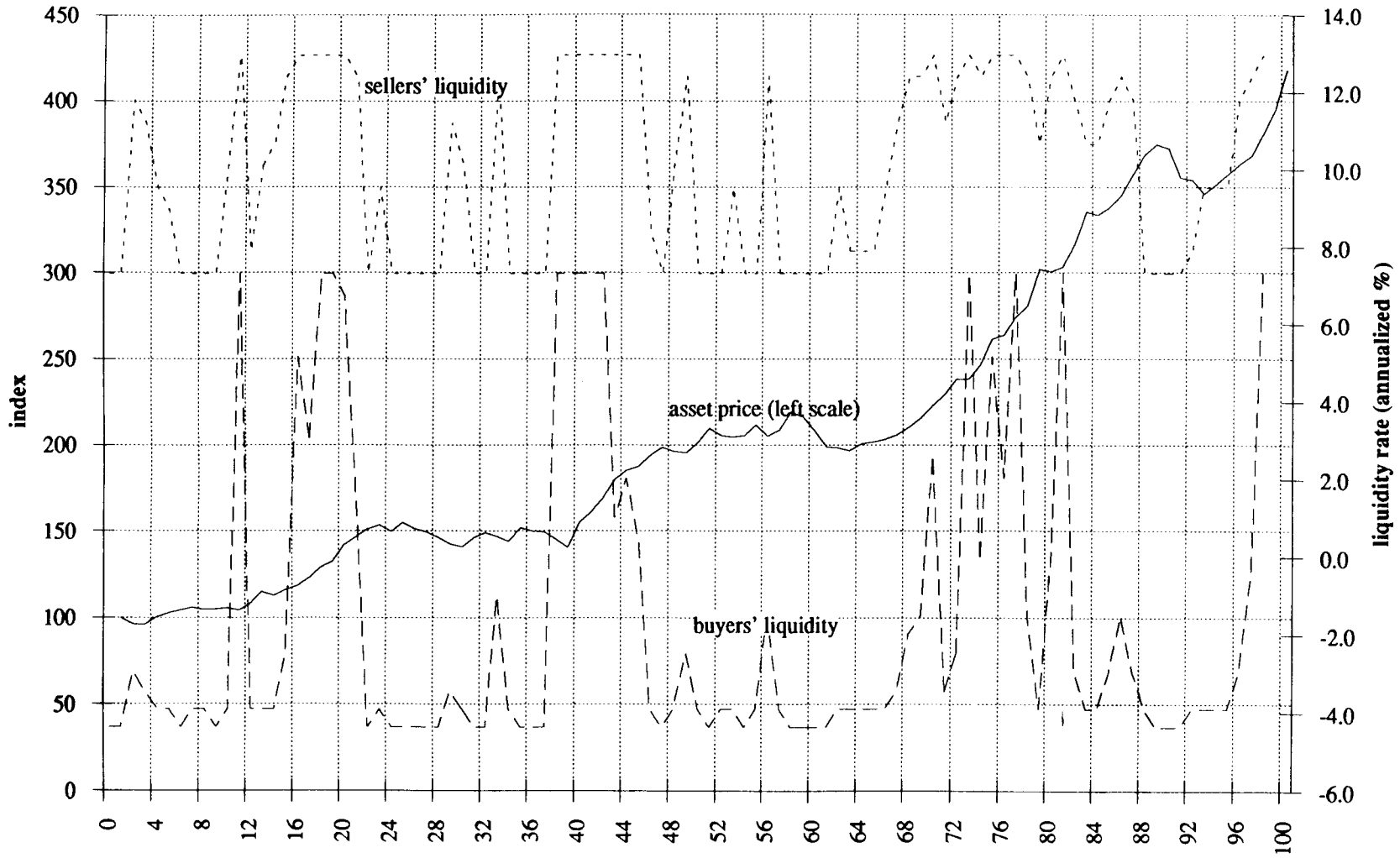

Figure 3: Liquidity rates implied by an AR(1) model

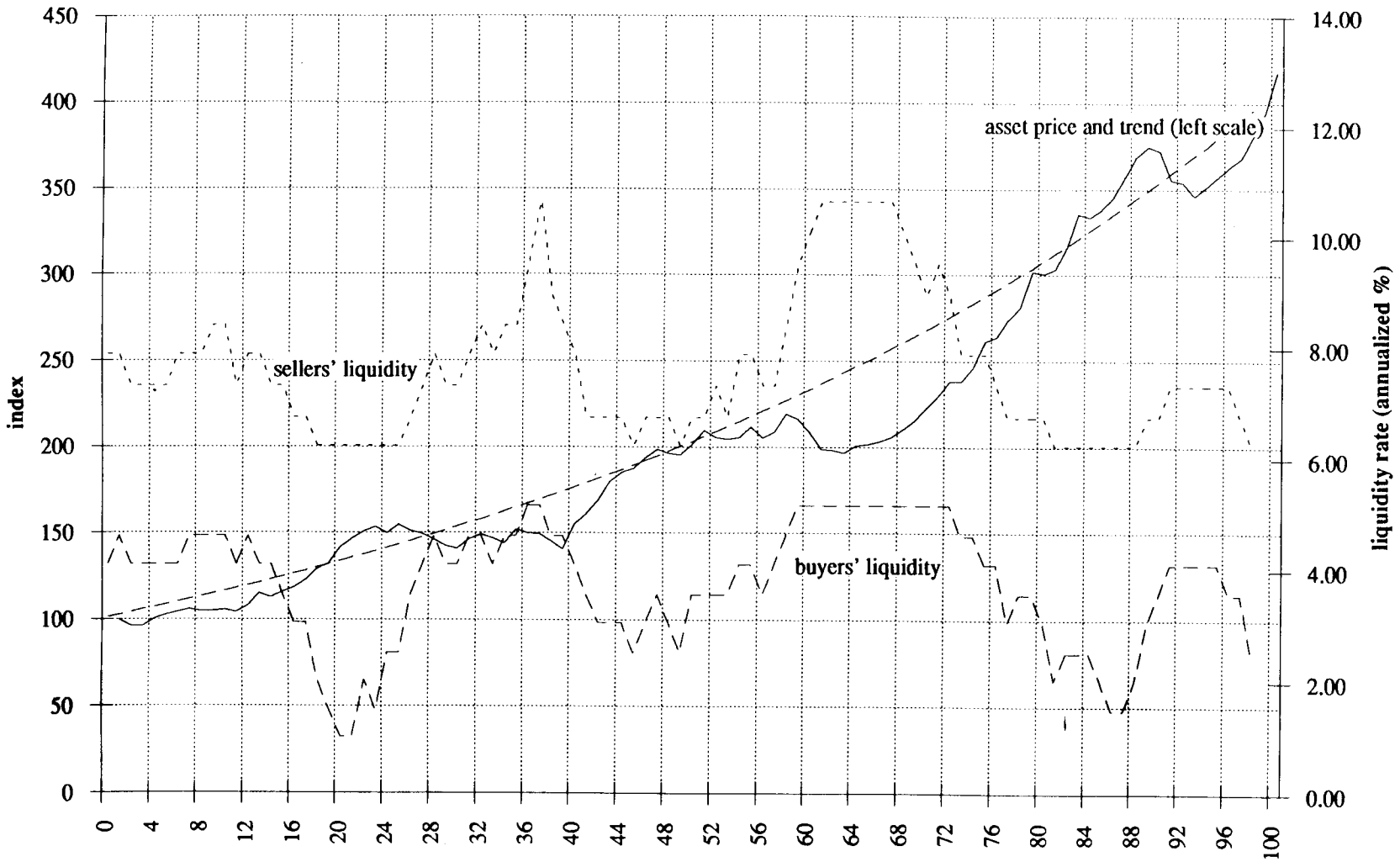

Figure 4: Los Angeles Residential Real Estate

price (solid, left scale) and sellers' liquidity rate (dashed, right scale)

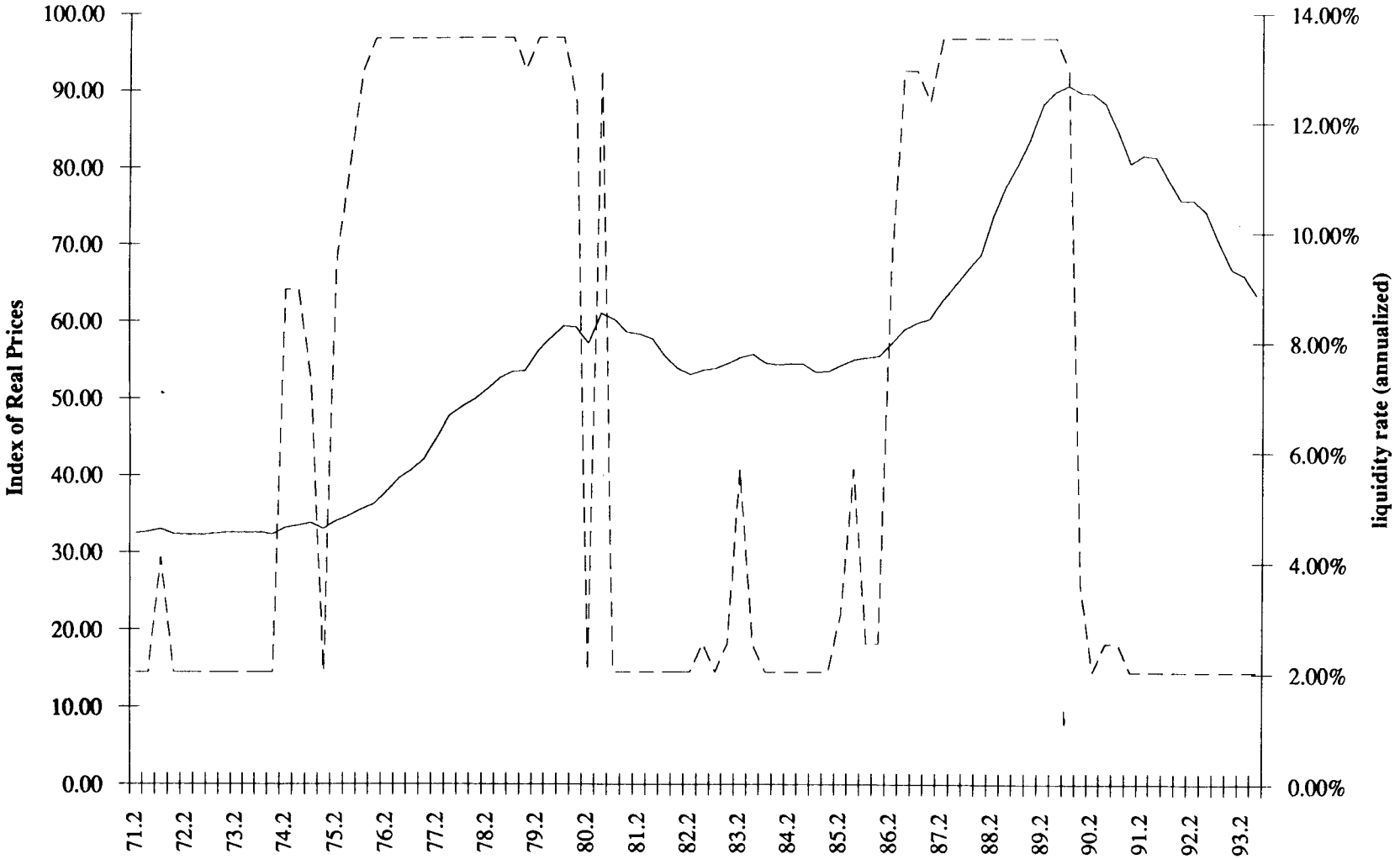

Figure 5: The Pound Sterling

price (solid, left scale) and sellers' liquidity rate (dashed, right scale)

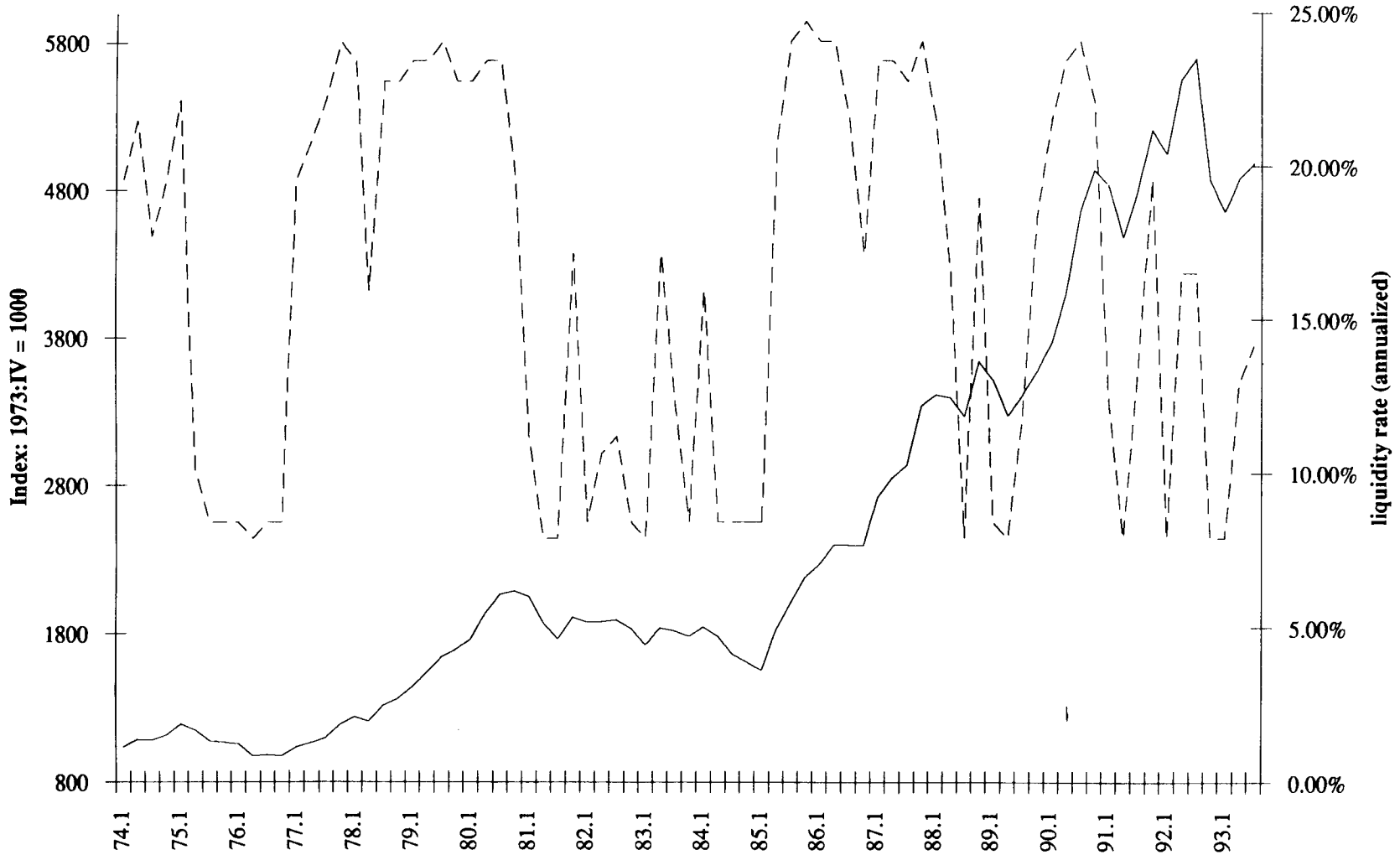

Figure 6: The Canadian Dollar

price (solid, left scale) and sellers' liquidity rate (dashed, right scale)