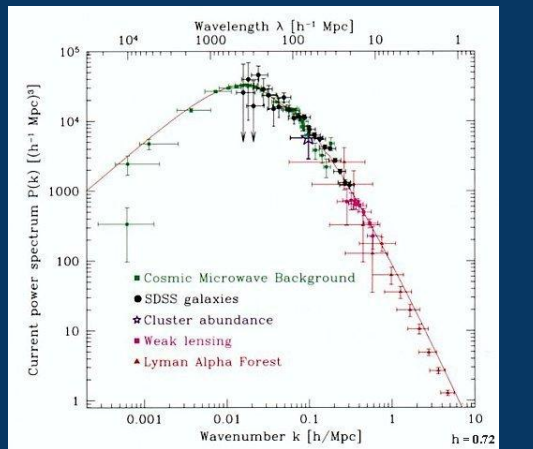
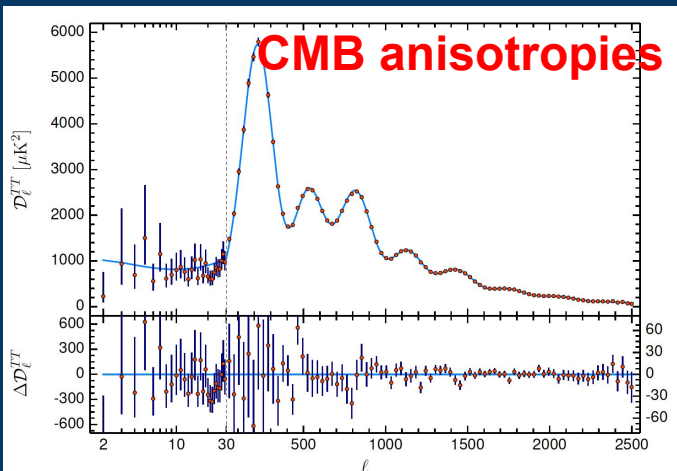


The strong lensing convergence power spectrum as a dark matter probe

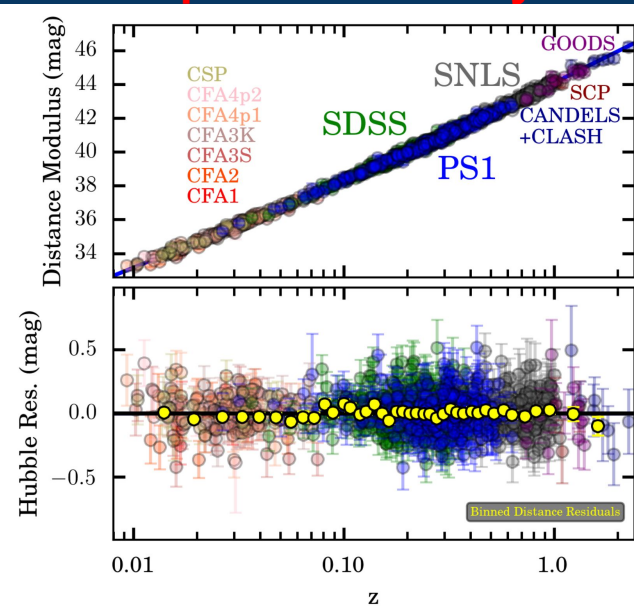
Ana Díaz Rivero
Harvard University

ΛCDM on large scales

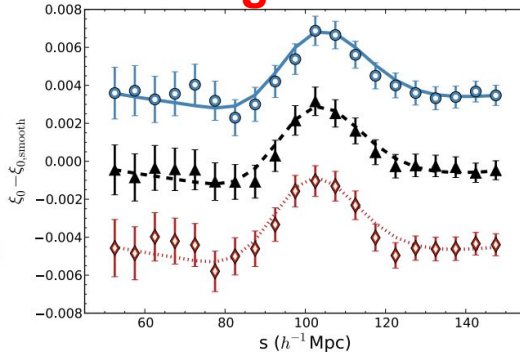
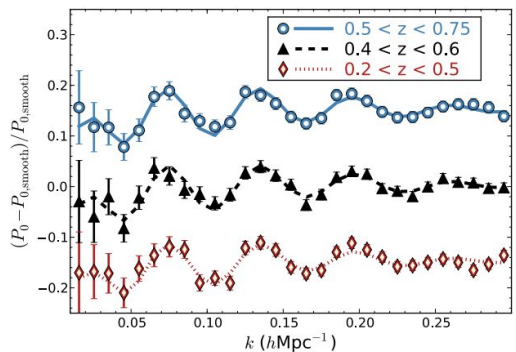
A 6(+) parameter model that is extremely successful on cosmological scales.



Expansion history



Clustering of matter



Planck Collaborat
(2018)

Alam et al. (2016)

Scolnic et al. (2018)

ΛCDM on small scales

Much harder to gauge ΛCDM on small scales (galactic/sub-galactic):

- Deep in the nonlinear regime at low redshifts; require N-body sims for predictions.
- Cannot ignore baryonic physics/astrophysics.
- Stellar formation becomes increasingly inefficient with decreasing halo mass.
- Dark matter models that behave like CDM on large scales can have very different effects on sub-galactic scales.

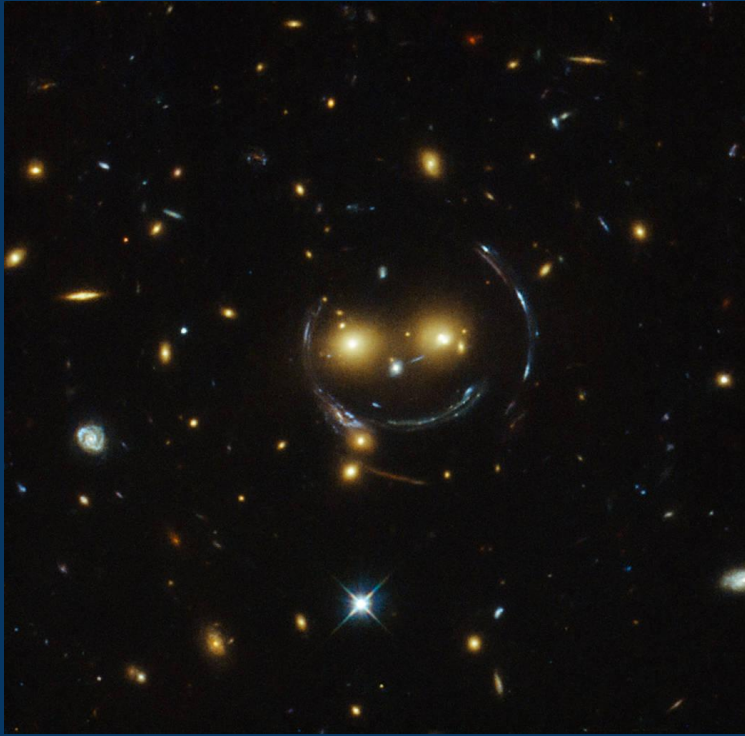
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Let's use the smallest scales to falsify/corroborate the CDM paradigm.

Strong gravitational lensing



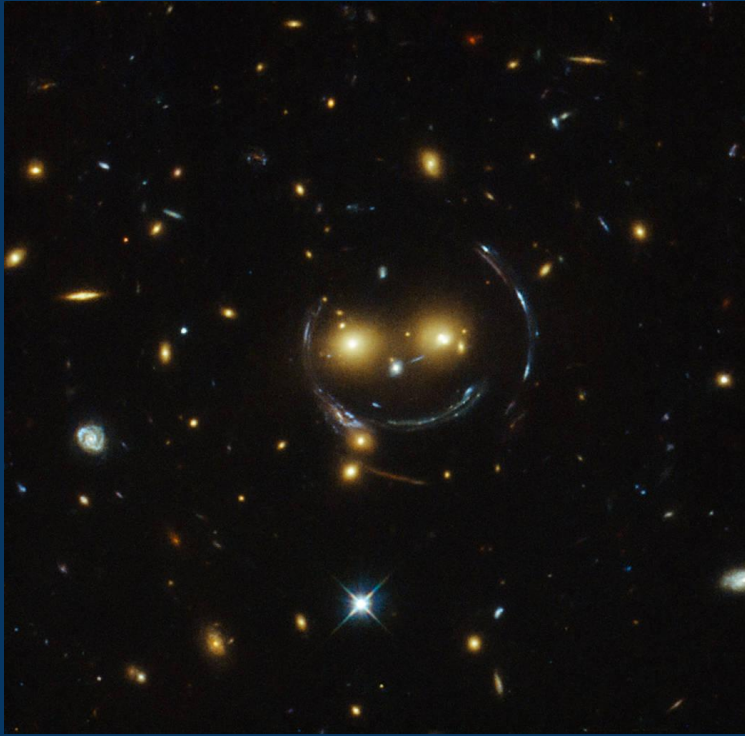
SDSS J1038+4849

$$\mathbf{y}(\mathbf{x}) = \mathbf{x} - \boldsymbol{\alpha}(\mathbf{x}) = \mathbf{x} - \nabla\psi(\mathbf{x})$$

Source plane

Image plane

Strong gravitational lensing as a small-scale probe



SDSS J1038+4849

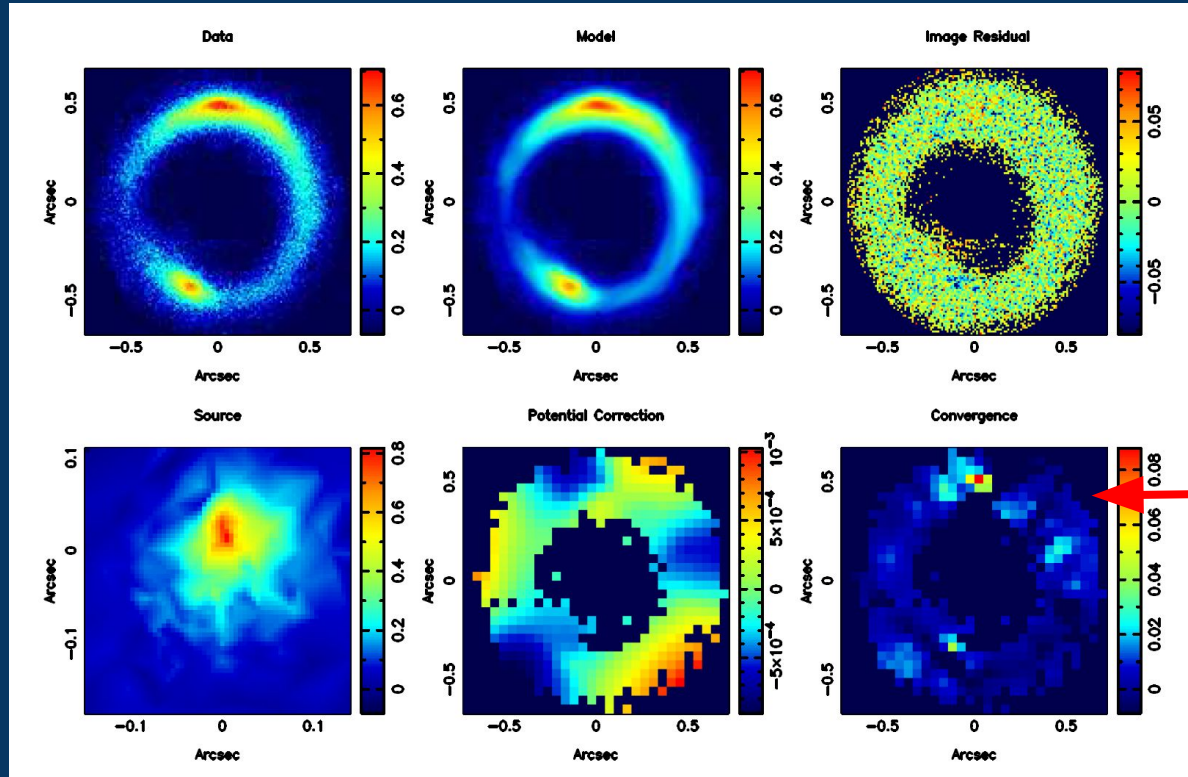
Mao & Schneider (1998): perturbations caused by substructures near lensed quasars can explain anomalous fluxes.

Baryon-independent measurement.

In this talk I focus on **galaxy-galaxy lenses**.

Strong gravitational lensing as a small-scale probe

Dark structures lying close to an image (in projection) can distort it.



Reconstructed
surface mass
density

Strong gravitational lensing as a small-scale probe

Many studies originally assumed that perturbations were caused by **substructure** within the main halo doing the lensing, but it is now clear that the entire line of sight volume has **interloper** halos that can act as **perturbers**.

Strong gravitational lensing as a small-scale probe

Many studies originally assumed that perturbations were caused by **substructure** within the main halo doing the lensing, but it is now clear that the entire line of sight volume has **interloper** halos that can act as **perturbers**.

Understanding whether the subhalos or interlopers are the dominant contribution is crucial when translating detections into DM constraints!

Strong gravitational lensing as a small-scale probe

Direct detection: resolve individual, pretty massive perturbers and infer properties (mass, position). Requires postprocessing and combining many images to convert detections into DM constraints.

Indirect/statistical detection: exploit CDM expectation of a large number of unresolved low-mass structures to statistically detect their collective perturbations on images (marginalizing over individual subhalo properties).

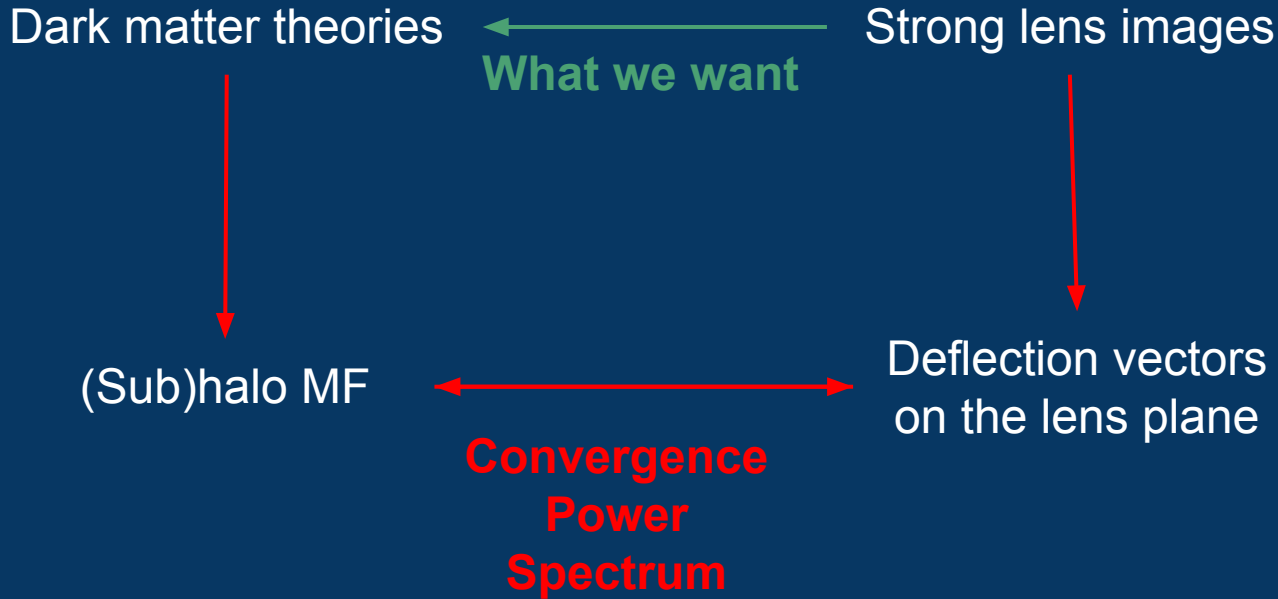
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Convergence power spectrum

Convergence = surface mass density in units of critical density for lensing



Substructure convergence power spectrum

Let us start off considering the limit where all perturbers are **substructure**.

$$\begin{aligned}\kappa &\equiv \Sigma(\mathbf{r})/\Sigma_{\text{crit}} \\ &= \nabla^2\psi(\mathbf{r})\end{aligned}$$

$$\Sigma_{\text{crit}} = \frac{c^2 D_{\text{os}}}{4\pi G D_{\text{ol}} D_{\text{ls}}}$$

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$$\kappa_{\text{sub}}(\mathbf{r}) = \sum_{i=1}^{N_{\text{sub}}} \kappa_i(\mathbf{r} - \mathbf{r}_i, m_i, \mathbf{q}_i)$$

Substructure convergence power spectrum

Let us start off considering the limit where all perturbers are **substructure**.

$$P_{\text{sub}}(\mathbf{k}) = P_{1\text{sh}}(\mathbf{k}) + P_{2\text{sh}}(\mathbf{k})$$

$$P_{1\text{sh}}(k) = \frac{(2\pi)^2 \bar{\kappa}_{\text{sub}}}{\langle m \rangle \Sigma_{\text{crit}}} \int dm d\mathbf{q} m^2 \mathcal{P}_m(m) \mathcal{P}_q(\mathbf{q}|m) \times \left[\int dr r J_0(kr) \hat{\kappa}(r, \mathbf{q}) \right]^2$$

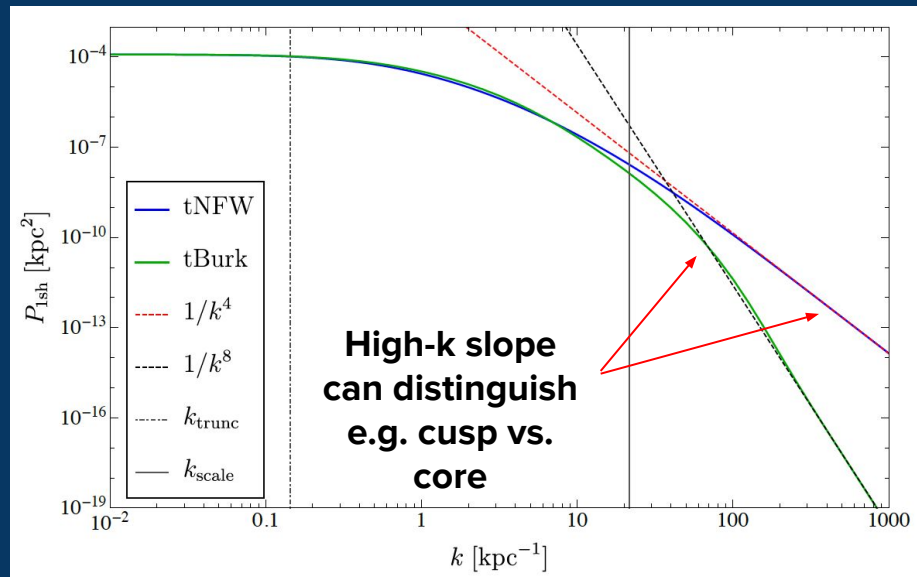
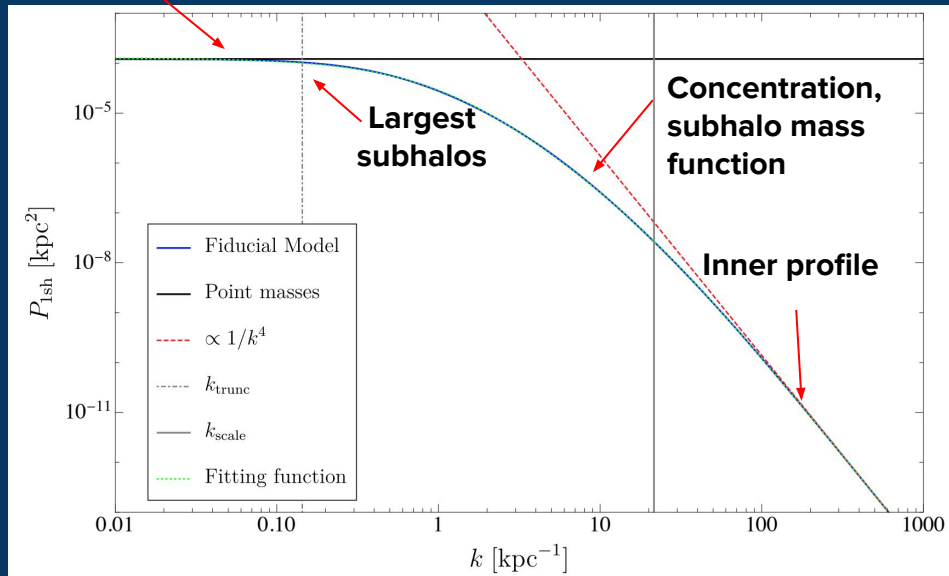
Mass distribution

Other profile properties

Convergence profile
(e.g. projected NFW)

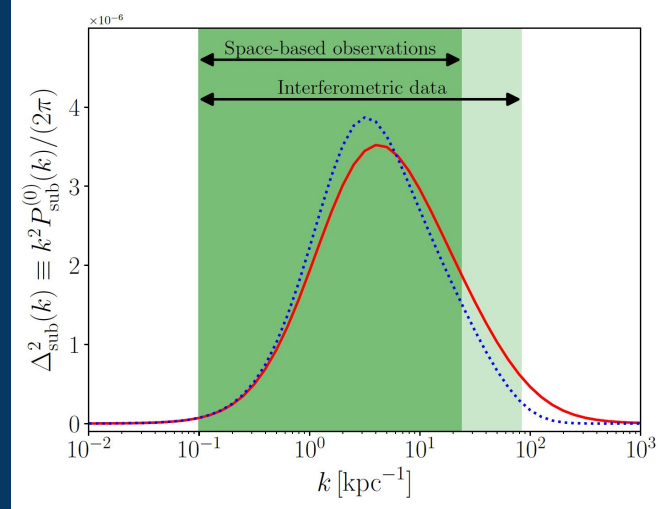
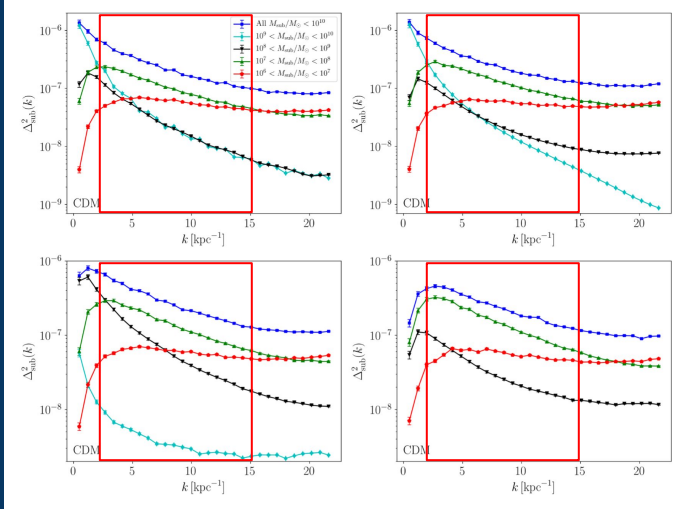
Substructure convergence power spectrum

$$\approx \bar{\kappa}_{\text{sub}} m_{\text{eff}} / \Sigma_{\text{crit}}$$



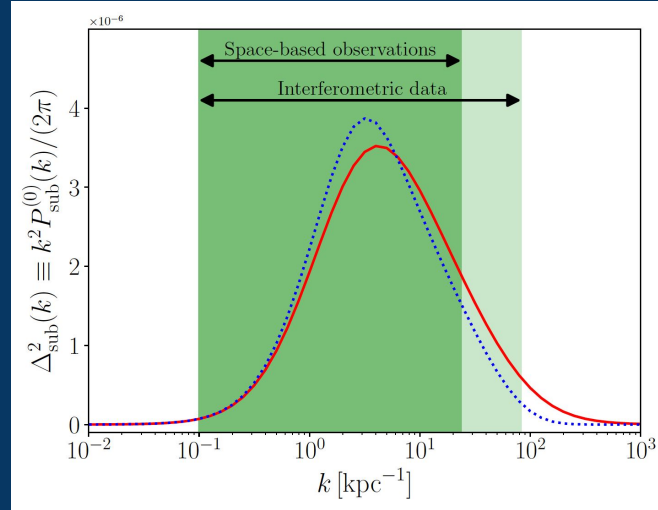
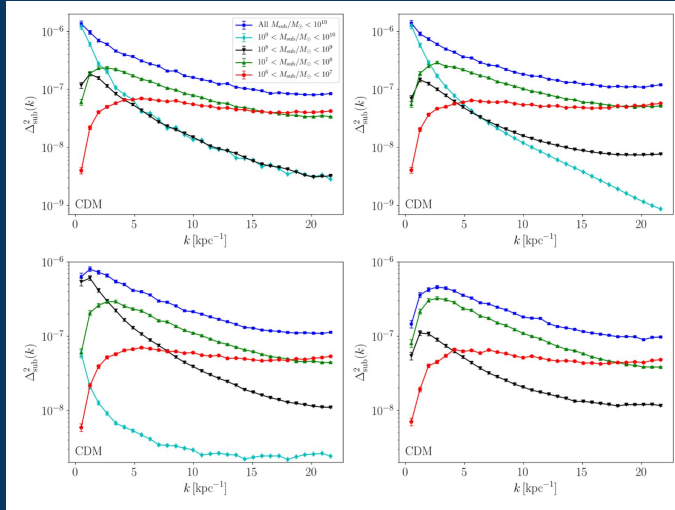
Substructure convergence power spectrum

Sensitivity

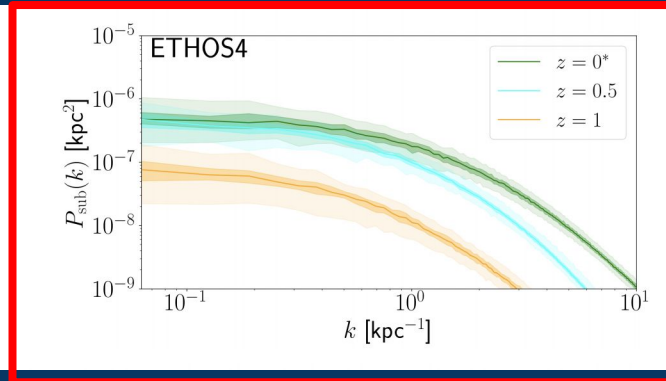
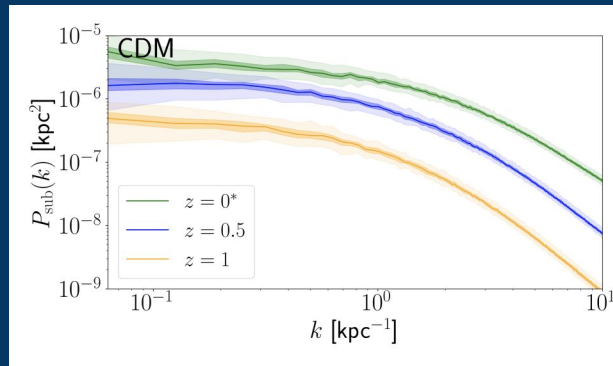


Cyr-Racine+ (2018)

Substructure convergence power spectrum



Cyr-Racine+ (2018)

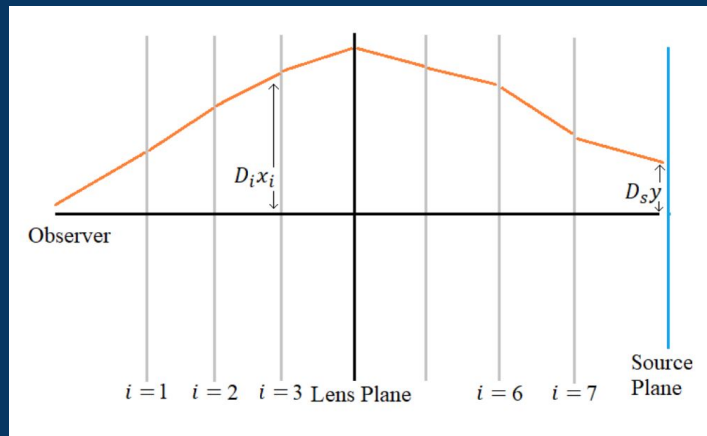
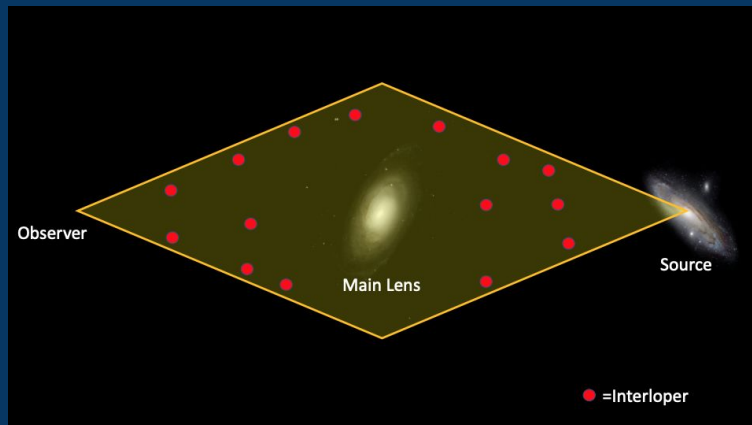


**Exotic DM
senario:
low-mass cutoff
+
self-interactions**

Diaz Rivero+ (2018, 2)

LOS convergence power spectrum

What if we include line-of-sight (LOS) structure?



$$\vec{y} = \vec{x}_1 - \sum_{i=1}^N \vec{\alpha}_i(\vec{x}_i)$$

$$\vec{x}_j = \vec{x}_1 - \sum_{i=1}^{j-1} \beta_{ij} \vec{\alpha}_i(\vec{x}_i), \quad \text{where} \quad \beta_{ij} \equiv \frac{D_{ij} D_s}{D_j D_{is}}$$

$$\kappa_i(\vec{x}) \equiv \frac{\Sigma_i(D_i \vec{x})}{\Sigma_{\text{cr},i}}$$

$$\Sigma_{\text{cr},i} \equiv \frac{c^2 D_s}{4\pi G D_i D_{is}}$$

Recursive multi-lens plane equation

Sengul+ (2020)

LOS convergence power spectrum

Have to define an **effective convergence**, treating interlopers as effective subhalos.

$$\kappa_{i,\text{eff}}(s) = \frac{\Sigma(s; m_{\text{eff},i}, r_{s,\text{eff},i}, \tau_i)}{\Sigma_{\text{cr},l}}$$

$$\begin{aligned} P_{\text{I}}(k) &= \left(\frac{4\pi G}{c^2} \right)^2 D_l^2 \int_0^{\chi_s} d\chi \frac{W_{\text{I}}^2(\chi)}{g^2(\chi)\chi^2} \\ &\times \int dm n(m, \chi) m^2 \\ &\times \int d^2\vec{q} \mathcal{P}(\vec{q} | m, \chi) \left| \tilde{\phi} \left(\frac{D_l r_s}{g(\chi) D_\chi} k; \tau \right) \right|^2 \end{aligned}$$

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Convergence profile
(e.g. projected NFW)

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Mass distribution

$$\times \int dm n(m, \chi) m^2$$

Other profile properties

$$\times \int d^2\vec{q} \mathcal{P}(\vec{q} | m, \chi) \left| \tilde{\phi}\left(\frac{D_l r_s}{g(\chi) D_\chi} k; \tau\right) \right|^2$$

Convergence profile
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Interloper selection
function

$$\kappa_{i,\text{eff}}(s) = \frac{\Sigma(s; m_{\text{eff},i}, r_{s,\text{eff},i}, \tau_i)}{\Sigma_{\text{cr},l}}$$

$$W_{\text{I}} \equiv W_{\text{fg}} + W_{\text{cp}} + W_{\text{bg}} = \frac{f(\chi) D_{\chi s} \chi^2}{D_{\chi} D_s}$$

$$f(\chi) = \begin{cases} 1 - \beta_{\chi l} & \chi \leq \chi_l \\ 1 - \beta_{l\chi} & \chi > \chi_l \end{cases}$$

$$P_{\text{I}}(k) = \left(\frac{4\pi G}{c^2} \right)^2 D_l^2 \int_0^{\chi_s} d\chi \frac{W_{\text{I}}^2(\chi)}{g^2(\chi) \chi^2}$$

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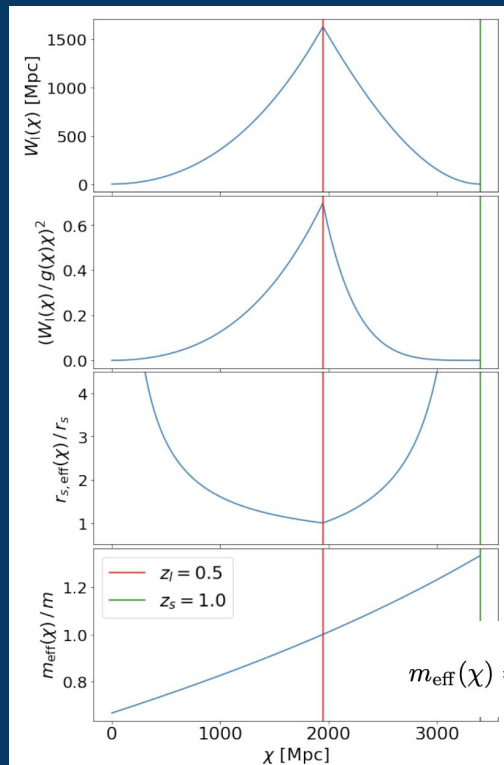
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Interloper
selection
function

Lensing kernel

$$r_{s,\text{eff}}(\chi) = \frac{D_l}{g(\chi) D_\chi} r_s$$

$$m_{\text{eff}}(\chi) = f(\chi) \frac{\Sigma_{\text{cr},l}}{\Sigma_{\text{cr},\chi}} \left(\frac{D_l}{g(\chi) D_\chi}\right)^2 m$$

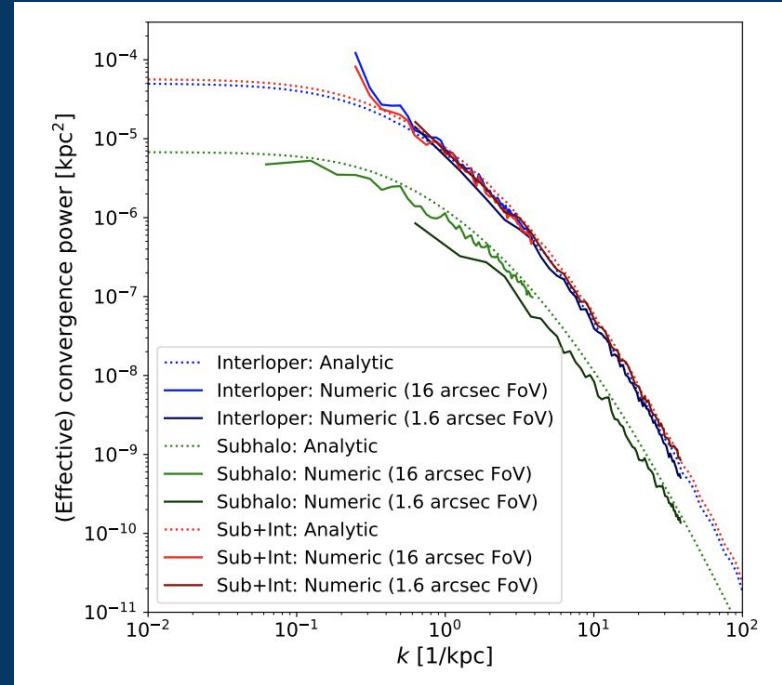
Sengul+ (2020)

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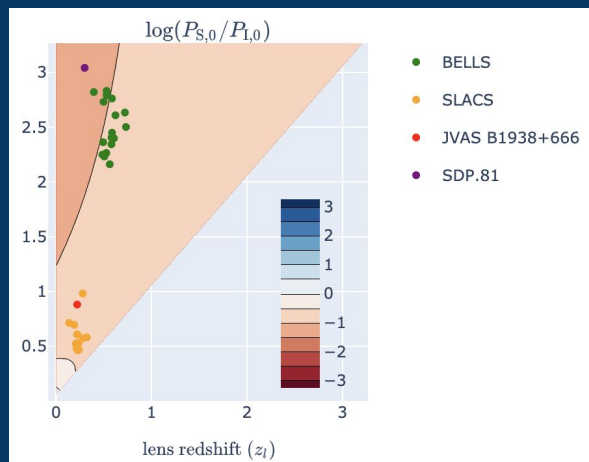
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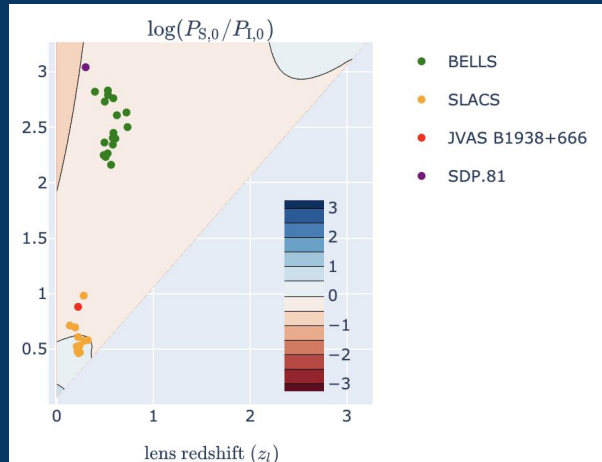


LOS convergence power spectrum

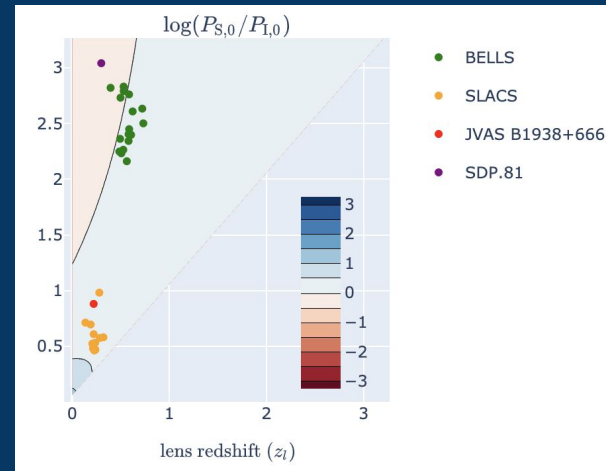
Ratio of substructure to interloper power spectrum
amplitude



$f_{\text{sub}} = 0.4\%$



$f_{\text{sub}} = 2\%$



$f_{\text{sub}} = 4\%$

Fraction of dark matter
halo mass in substructure

f_{sub}



LOS convergence power spectrum

Error due to projection?

$$\kappa_{\text{div}} = \kappa_{\text{eff}} \equiv \frac{1}{2} \nabla \cdot \vec{\alpha}$$

LOS convergence power spectrum

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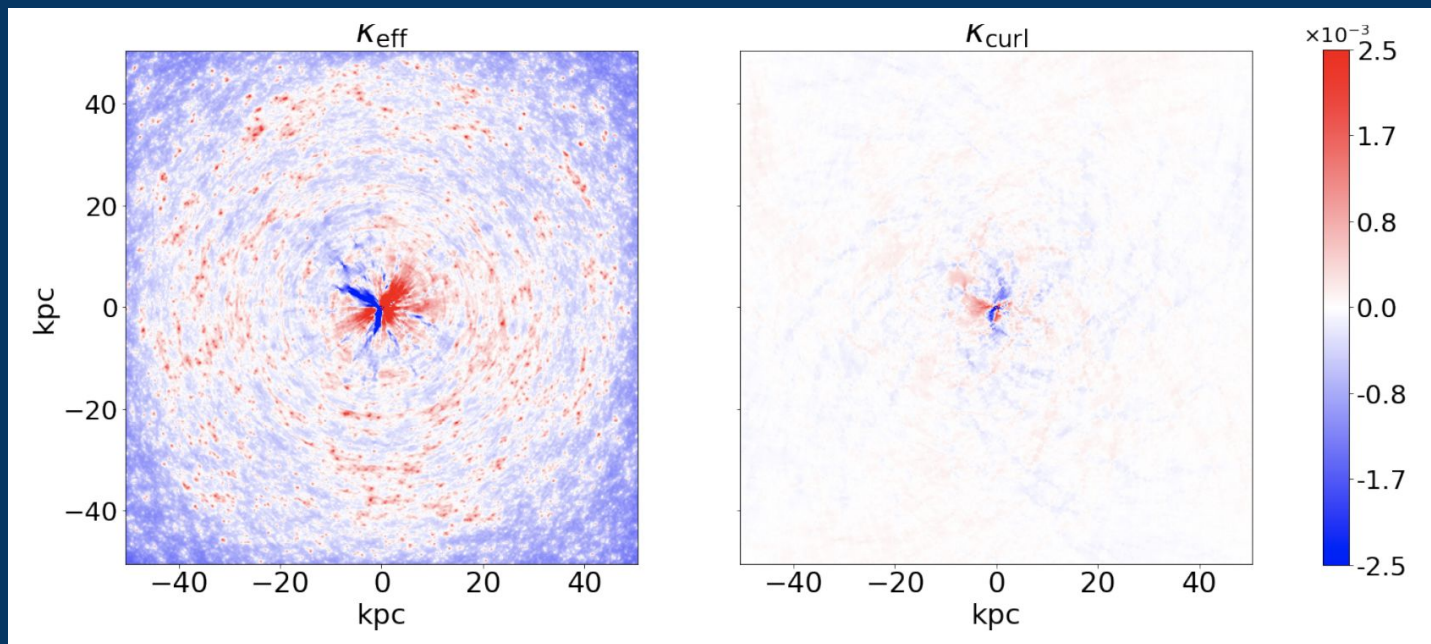
$$\kappa_{\text{curl}} \equiv \frac{1}{2} \nabla \times \vec{\alpha}$$

LOS convergence power spectrum

Error due to projection?

$$\kappa_{\text{div}} = \kappa_{\text{eff}} \equiv \frac{1}{2} \nabla \cdot \vec{\alpha}$$

$$\kappa_{\text{curl}} \equiv \frac{1}{2} \nabla \times \vec{\alpha}$$



Conclusions

GL is the best baryon-independent way we have of probing the low-mass end of the HMF (and only way outside the LG), and consequently probing an untested regime in CDM.

The convergence power spectrum relates length scales to mass scales, bridging the gap between strong lens images and dark matter theories. It is sensitive to lower masses than direct detection methods.

The interloper contribution cannot be ignored: it likely dominates the signal for the SLACS and BELLS galaxy-galaxy lenses.

This is good news! The HMF is a cleaner probe of dark matter than the SMF, which is subject to messy astrophysics.

Questions?

Talk based on:

arXiv: 1707.04590 (**ADR**, F.Y.-Cyr-Racine, C. Dvorkin)

arXiv: 1809.00004 (**ADR**, C. Dvorkin, F.-Y. Cyr-Racine, J. Zavala, M. Vogelsberger)

arXiv: 2020.07383 (A. C. Sengül, A. Tsang, **ADR**, C. Dvorkin, H. Zhu, U. Seljak)