The Newman Penrose map and the classical double copy

Gabriel Herczeg[1]
with Gilly Elor[2], Kara Farnsworth[3], and Michael Graesser[4].

[1]Brown Theoretical Physics Center

[2]University of Washington, Seattle

[3]CEICO, Czech Academy of Sciences

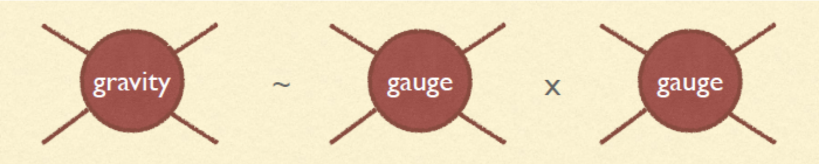
[4]Los Alamos National Laboratory

July 21, 2020

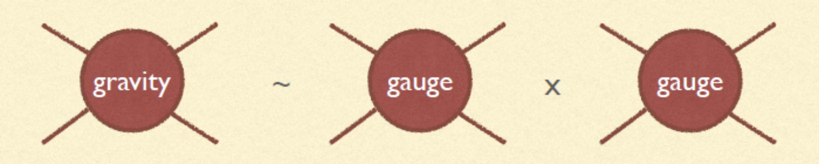
Overview

1. Background
   - BCJ duality and the double copy
   - Classical double copy
   - Examples
   - Self-dual double copy
2. The Newman-Penrose map
   - Newman-Penrose formalism for Kerr-Schild spacetimes
   - The Newman-Penrose map
   - Examples
   - Summary and future work

Double Copy



Double Copy

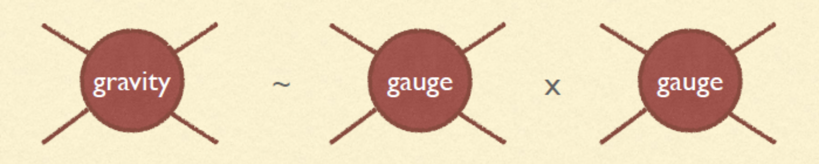


KLT relations

Kawai, Lewellen, Tye, '86:

$$\text{(closed string amplitude)} \sim \text{(open string amplitude)}^2$$

Double Copy



KLT relations

Kawai, Lewellen, Tye, '86:

$$(\text{closed string amplitude}) \sim (\text{open string amplitude})^2$$

BCJ duality

Bern, Carrasco, Johansson, '08:

$$(\text{gravity amplitude}) \sim (\text{gauge amplitude})^2$$

- Proven at tree level, thought to hold in general.

single copy

Tree level:

$$\mathcal{A}_{YM} = \sum_{i \in cubic} \frac{n_i c_i}{D_i}$$

$$c_i \pm c_j \mp c_k = 0$$
$$n_i \pm n_j \mp n_k = 0$$

Color/kinematics duality (BCJ)

single copy

Tree level:

$$\mathcal{A}_{YM} = \sum_{i \in cubic} \frac{n_i c_i}{D_i}$$

$$c_i \pm c_j \mp c_k = 0$$
$$n_i \pm n_j \mp n_k = 0$$

double copy

$$\mathcal{A}_{G} = \sum_{i \in cubic} \frac{n_i \tilde{n}_i}{D_i}$$

single copy

Tree level: $$\mathcal{A}_{YM} = \sum_{i \in cubic} \frac{n_i c_i}{D_i}$$

$$c_i \pm c_j \mp c_k = 0$$
$$n_i \pm n_j \mp n_k = 0$$

double copy

$$\mathcal{A}_G = \sum_{i \in cubic} \frac{n_i \tilde{n}_i}{D_i}$$

zeroth copy

$$\mathcal{A}_S = \sum_{i \in cubic} \frac{c_i \tilde{c}_i}{D_i}$$

Classical double copy

Kerr-Schild metric: $g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$

geodetic

$$k^\mu \nabla_\mu k_\nu = k^\mu \partial_\mu k_\nu = 0$$

null

$$g^{\mu\nu} k_\mu k_\nu = \eta^{\mu\nu} k_\mu k_\nu = 0$$

Ricci truncates after linear order

$$R^\mu{}_\nu = \frac{1}{2}\left(\partial^\mu \partial_\alpha(\phi k^\alpha k_\nu) + \partial_\nu \partial^\alpha(\phi k_\alpha k^\mu) - \partial^2(\phi k^\mu k_\nu)\right)$$

Kerr-Schild double copy

stationary Kerr-Schild: $\partial_0 k_\mu = \partial_0 \phi = 0 \qquad k_0 = 1$

$$R^i{}_0 = \frac{1}{2}\partial_j \left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right] \qquad R^0{}_0 = \frac{1}{2}\partial_i^2 \phi$$

Monteiro, O'Connell, White, '14

Kerr-Schild double copy

stationary Kerr-Schild: $\partial_0 k_\mu = \partial_0 \phi = 0 \qquad k_0 = 1$

$$R^i{}_0 = \frac{1}{2}\partial_j \left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right] \qquad R^0{}_0 = \frac{1}{2}\partial_i^2 \phi$$

Double Copy

$$h_{\mu\nu} = \phi k_\mu k_\nu$$

Monteiro, O'Connell, White, '14

Kerr-Schild double copy

stationary Kerr-Schild: $\partial_0 k_\mu = \partial_0 \phi = 0$ $\quad k_0 = 1$

$$R^i{}_0 = \frac{1}{2}\partial_j \left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right] \qquad R^0{}_0 = \frac{1}{2}\partial_i^2 \phi$$

Double Copy

$$h_{\mu\nu} = \phi k_\mu k_\nu$$

Single Copy

$$A_\mu \equiv \phi k_\mu$$

Monteiro, O'Connell, White, '14

Kerr-Schild double copy

stationary Kerr-Schild: $\partial_0 k_\mu = \partial_0 \phi = 0$ $\quad k_0 = 1$

$$R^i{}_0 = \frac{1}{2}\partial_j \left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right] \qquad R^0{}_0 = \frac{1}{2}\partial_i^2 \phi$$

Double Copy	Single Copy	Zeroth Copy
$h_{\mu\nu} = \phi k_\mu k_\nu$	$A_\mu \equiv \phi k_\mu$	ϕ

Monteiro, O'Connell, White, '14

Kerr-Schild double copy

stationary Kerr-Schild: $\partial_0 k_\mu = \partial_0 \phi = 0 \qquad k_0 = 1$

$$R^i{}_0 = \frac{1}{2}\partial_j \left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right] \qquad R^0{}_0 = \frac{1}{2}\partial_i^2 \phi$$

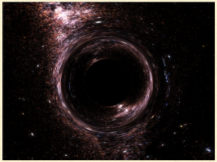
Double Copy	Single Copy	Zeroth Copy
$h_{\mu\nu} = \phi k_\mu k_\nu$	$A_\mu \equiv \phi k_\mu$	ϕ
$R_{\mu\nu} = 0$	$\partial_\mu F^{\mu\nu} = 0$	$\Box \phi = 0$

Monteiro, O'Connell, White, '14

Example: Schwarzschild black hole

Double Copy: $$g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$$

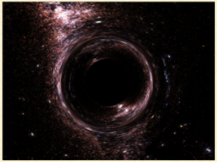
$$\phi = \frac{2GM}{r} \qquad k_\mu = (1, \hat{r})$$



Monteiro, O'Connell, White, '14

Example: Schwarzschild black hole

Double Copy: $g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$



$$\phi = \frac{2GM}{r} \qquad k_\mu = (1, \hat{r})$$

Single Copy:

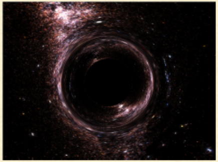
$$\rightarrow A_\mu + \partial_\mu \chi = \left(\frac{2GM}{r}, 0, 0, 0\right)$$

$$\sim \Phi = \frac{kQ}{r}, \qquad \vec{A} = 0$$

Monteiro, O'Connell, White, '14

Example: Schwarzschild black hole

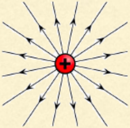
Double Copy: $g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$



$$\phi = \frac{2GM}{r} \qquad k_\mu = (1, \hat{r})$$

Single Copy:

$$\rightarrow A_\mu + \partial_\mu \chi = \left(\frac{2GM}{r}, 0, 0, 0\right)$$



$$\sim \Phi = \frac{kQ}{r}, \qquad \vec{A} = 0$$

Monteiro, O'Connell, White, '14

Other examples

GRAVITY/DOUBLE COPY	GAUGE/SINGLE COPY
Kerr black hole	Rotating disk of charge *(Monteiro, O'Connell, White, '14)*
Plane waves	Plane waves *(Monteiro, O'Connell, White, '14)*
Kerr-Taub-NUT	Dyon *(Luna, Monteiro, O'Connell, White, 15)*
(A)dS backgrounds	(A)dS backgrounds *(Gonzalez, Penco, Trodden, '17)*
BTZ	Constant charge density *(Gonzalez, Penco, Trodden, '17)*
Neutron star	Charged sphere *(Farnsworth, Oliveri, Skordis, soon)*
⋮	⋮

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm\, i \star_0 F\,, \qquad C_{ab} = \pm\, i \star C_{ab}\,,$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm i \star_0 F\,, \qquad C_{ab} = \pm i \star C_{ab}\,,$$

"Fourier transform" of Kerr-Schild

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\hat{k}_\mu \hat{k}_\nu(\phi)$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm\, i \star_0 F\,, \qquad C_{ab} = \pm\, i \star C_{ab}\,,$$

"Fourier transform" of Kerr-Schild

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\hat{k}_\mu \hat{k}_\nu(\phi)$$

- Symmetry of graviton $\implies [\hat{k}_\mu, \hat{k}_\nu] = 0$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm i \star_0 F \,, \qquad C_{ab} = \pm i \star C_{ab} \,,$$

"Fourier transform" of Kerr-Schild

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\hat{k}_\mu \hat{k}_\nu(\phi)$$

- Symmetry of graviton $\implies [\hat{k}_\mu, \hat{k}_\nu] = 0$
- Null condition $\implies \eta^{\mu\nu}\hat{k}_\mu \hat{k}_\nu = 0$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm i \star_0 F\,, \qquad C_{ab} = \pm i \star C_{ab}\,,$$

"Fourier transform" of Kerr-Schild

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\hat{k}_\mu \hat{k}_\nu(\phi)$$

- Symmetry of graviton $\implies [\hat{k}_\mu, \hat{k}_\nu] = 0$
- Null condition $\implies \eta^{\mu\nu}\hat{k}_\mu \hat{k}_\nu = 0$
- Geodesic condition $\implies \hat{k} \cdot \partial = 0.$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Self-dual solutions

$$F = \pm i \star_0 F\,, \qquad C_{ab} = \pm i \star C_{ab}\,,$$

"Fourier transform" of Kerr-Schild

$$g_{\mu\nu} = \eta_{\mu\nu} - 2\hat{k}_\mu \hat{k}_\nu(\phi)$$

- Symmetry of graviton $\implies [\hat{k}_\mu, \hat{k}_\nu] = 0$
- Null condition $\implies \eta^{\mu\nu}\hat{k}_\mu\hat{k}_\nu = 0$
- Geodesic condition $\implies \hat{k} \cdot \partial = 0.$
- Self-dual solutions are *complex* $\implies \phi$ is complex.

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t - z)\,, \quad v = \frac{1}{\sqrt{2}}(t + z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x + iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x - iy)\,.$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2 \phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t - z)\,, \quad v = \frac{1}{\sqrt{2}}(t + z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x + iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x - iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\,\partial_\zeta + d\bar{\zeta}\,\partial_u$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t - z)\,, \quad v = \frac{1}{\sqrt{2}}(t + z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x + iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x - iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\,\partial_\zeta + d\bar{\zeta}\,\partial_u$$

Einstein → Plebański:

$$\phi_{,vu} - \phi_{,\zeta\bar{\zeta}} = (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv}$$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t - z)\,, \quad v = \frac{1}{\sqrt{2}}(t + z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x + iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x - iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\, \partial_\zeta + d\bar{\zeta}\, \partial_u$$

Einstein → Plebański:

$$\phi_{,vu} - \phi_{,\zeta\bar{\zeta}} = (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv}$$

1. $h_{\mu\nu} \sim k_\mu k_\nu \implies (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv} \implies \Box\phi = 0.$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0 \,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t - z)\,, \quad v = \frac{1}{\sqrt{2}}(t + z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x + iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x - iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\,\partial_\zeta + d\bar{\zeta}\,\partial_u$$

Einstein $\to$ Plebański:

$$\phi_{,vu} - \phi_{,\zeta\bar{\zeta}} = (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv}$$

1. $h_{\mu\nu} \sim k_\mu k_\nu \implies (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv} \implies \Box\phi = 0.$
2. Self-dual solutions $\iff \Box\phi = 0.$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t-z)\,, \quad v = \frac{1}{\sqrt{2}}(t+z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x+iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x-iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\,\partial_\zeta + d\bar{\zeta}\,\partial_u$$

Einstein → Plebański:

$$\phi_{,vu} - \phi_{,\zeta\bar{\zeta}} = (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv}$$

1. $h_{\mu\nu} \sim k_\mu k_\nu \implies (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv} \implies \Box\phi = 0.$
2. Self-dual solutions $\iff \Box\phi = 0.$
3. Self-dual single copy: $A_\mu = \hat{k}_\mu \phi.$

Self-dual double copy [Monteiro, O'Connell, White, 2012], [Berman, Chacón, Luna, White, 2019]

Impose vacuum Einstein equations:

$$R_{\mu\nu} = 0 \implies \partial^2\phi + (\hat{k}^\mu \hat{k}^\nu \phi)(\partial_\mu \partial_\nu \phi) = 0\,,$$

Define totally null coordinates:

$$u = \frac{1}{\sqrt{2}}(t-z)\,, \quad v = \frac{1}{\sqrt{2}}(t+z)\,, \quad \zeta = \frac{1}{\sqrt{2}}(x+iy)\,, \quad \bar{\zeta} = \frac{1}{\sqrt{2}}(x-iy)\,.$$

Explicit expression for $\hat{k}_\mu$:

$$\hat{k} = dx^\mu \hat{k}_\mu = dv\,\partial_\zeta + d\bar{\zeta}\,\partial_u$$

Einstein → Plebański:

$$\phi_{,vu} - \phi_{,\zeta\bar{\zeta}} = (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv}$$

1. $h_{\mu\nu} \sim k_\mu k_\nu \implies (\phi_{,u\zeta})^2 - \phi_{,\zeta\zeta}\phi_{,vv} \implies \Box\phi = 0.$
2. Self-dual solutions $\iff \Box\phi = 0.$
3. Self-dual single copy: $A_\mu = \hat{k}_\mu \phi.$
4. Self-dual double copy: $h_{\mu\nu} = \hat{k}_\mu \hat{k}_\nu \phi.$

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Newman-Penrose formalism for Kerr-Schild spacetimes

$$\begin{aligned}
\omega^1 &= dv + \tfrac{1}{2} V \omega^2 , \\
\omega^2 &= du + \bar{\Phi} d\zeta + \Phi d\bar{\zeta} + \Phi\bar{\Phi} dv , \\
\omega^3 &= \Phi dv + d\zeta , \\
\omega^4 &= \bar{\Phi} dv + d\bar{\zeta} ,
\end{aligned}$$

$$g = 2(\omega^1\omega^2 - \omega^3\omega^4) = g_0 + V\omega^2 \otimes \omega^2 , \quad g_0 = 2(du dv - d\zeta d\bar{\zeta}) .$$

1. Φ is a complex function and V is a real function which is determined by Φ for vacuum solutions.

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Newman-Penrose formalism for Kerr-Schild spacetimes

$$
\begin{aligned}
\omega^1 &= dv + \tfrac{1}{2} V \omega^2, \\
\omega^2 &= du + \bar{\Phi} d\zeta + \Phi d\bar{\zeta} + \Phi\bar{\Phi} dv\,, \\
\omega^3 &= \Phi dv + d\zeta\,, \\
\omega^4 &= \bar{\Phi} dv + d\bar{\zeta}\,,
\end{aligned}
$$

$$
g = 2(\omega^1\omega^2 - \omega^3\omega^4) = g_0 + V\omega^2 \otimes \omega^2\,, \quad g_0 = 2(du dv - d\zeta d\bar{\zeta})\,.
$$

1. Φ is a complex function and V is a real function which is determined by Φ for vacuum solutions.
2. When ω^2 is dual to a shear-free, geodesic null vector then Φ obeys:

$$
\Phi_{,v} = \Phi\Phi_{,\zeta}\,, \quad \Phi_{,\bar{\zeta}} = \Phi\Phi_{,u} \implies \Box\Phi = 0.
$$

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

The Map

1. Given a (real) Kerr-Schild spacetime in the Newman-Penrose form, one has a complex harmonic scalar Φ.

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

The Map

1. Given a (real) Kerr-Schild spacetime in the Newman-Penrose form, one has a complex harmonic scalar Φ.
2. We can now obtain a self-dual solution of Maxwell's equations by applying $\hat{k}_\mu$:

$$\boxed{A_\mu = -\frac{Q}{2\pi\epsilon_0}\hat{k}_\mu \Phi.}$$

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

The Map

1. Given a (real) Kerr-Schild spacetime in the Newman-Penrose form, one has a complex harmonic scalar Φ.
2. We can now obtain a self-dual solution of Maxwell's equations by applying $\hat{k}_\mu$:

$$\boxed{A_\mu = -\frac{Q}{2\pi\epsilon_0}\hat{k}_\mu\Phi.}$$

3. Well defined even for non-stationary, non-vacuum spacetimes.

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Examples

1. Schwarzschild $\rightarrow$ Coulomb $+$ i(magnetic monopole)

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Examples

1. Schwarzschild $\rightarrow$ Coulomb $+$ i(magnetic monopole)
2. Kerr $\rightarrow$ rotating electric charge $+$ i(rotating magnetic charge)

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Examples

1. Schwarzschild $\rightarrow$ Coulomb $+$ i(magnetic monopole)
2. Kerr $\rightarrow$ rotating electric charge $+$ i(rotating magnetic charge)

1. The real parts of the gauge fields are the usual classical single copies.

Newman-Penrose map [Elor, Farnsworth, Graesser, Herczeg, 2020]

Examples

1. Schwarzschild $\rightarrow$ Coulomb $+$ i(magnetic monopole)
2. Kerr $\rightarrow$ rotating electric charge $+$ i(rotating magnetic charge)

1. The real parts of the gauge fields are the usual classical single copies.
2. What about non-stationary, non-vacuum solutions?

Photon Rocket spacetime [Kinnersley, 1969]

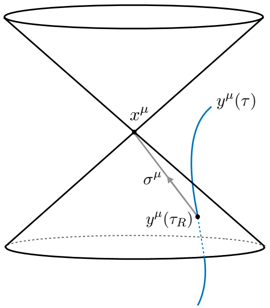


Figure: An arbitrary timelike worldline $y^\mu(\tau)$, parameterized with respect to proper time τ, intersects the past light-cone of an arbitrary point x^μ in Minkowski space at exactly one point $y^\mu(\tau_R)$. This picture defines of $\tau_R(x)$.

Photon Rocket spacetime [Kinnersley, 1969]

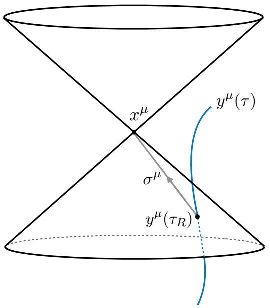
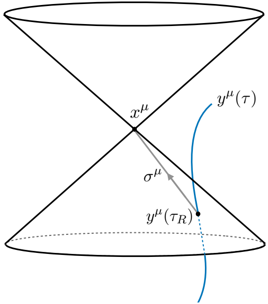


Figure: An arbitrary timelike worldline $y^\mu(\tau)$, parameterized with respect to proper time τ, intersects the past light-cone of an arbitrary point x^μ in Minkowski space at exactly one point $y^\mu(\tau_R)$. This picture defines of $\tau_R(x)$.

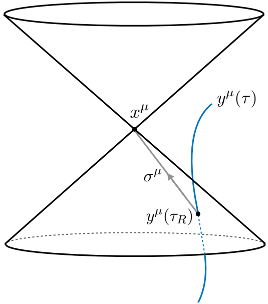
$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2GM(\tau_R)}{r^3}\sigma_\mu\sigma_\nu, \qquad r = \sigma_\mu \frac{dy^\mu}{d\tau_R}.$$

Photon Rocket spacetime [Kinnersley, 1969]



1. Photon Rocket $\rightarrow$ Liénard Wiechert
   $+$ i(arbitrarily accelerating magnetic monopole).

Photon Rocket spacetime [Kinnersley, 1969]



1. Photon Rocket $\rightarrow$ Liénard Wiechert
   $+$ i(arbitrarily accelerating magnetic monopole).
2. Agrees with [Luna, Monteiro, Nicholson, O'Connell, White 2016] without any ad hoc corrections.

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.
4. Can it be extended to self-dual gravity solutions, e.g. Eguchi-Hanson?

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.
4. Can it be extended to self-dual gravity solutions, e.g. Eguchi-Hanson?
5. Can it be extended to double Kerr-Schild solutions, e.g. Taub-Nut?

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.
4. Can it be extended to self-dual gravity solutions, e.g. Eguchi-Hanson?
5. Can it be extended to double Kerr-Schild solutions, e.g. Taub-Nut?
6. Dimensions other than $d = 4$?

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.
4. Can it be extended to self-dual gravity solutions, e.g. Eguchi-Hanson?
5. Can it be extended to double Kerr-Schild solutions, e.g. Taub-Nut?
6. Dimensions other than $d = 4$?
7. Beyond Kerr-Schild?

Summary and future work

1. The Newman-Penrose map extends the classical double copy non-stationary, non-vacuum solutions.
2. It gives the most convincing treatment so far of accelerating black holes.
3. But it does not exhibit the usual double copy structure explicitly.
4. Can it be extended to self-dual gravity solutions, e.g. Eguchi-Hanson?
5. Can it be extended to double Kerr-Schild solutions, e.g. Taub-Nut?
6. Dimensions other than $d = 4$?
7. Beyond Kerr-Schild?

Thank you!

References

G. Elor, K. Farnsworth, M. Graesser, G. Herczeg (2020)
The Newman Penrose Map and the Classical Double Copy
arXiv.org [hep-th] 2006.08630

D. Berman, E. Chacón, A. Luna, C. White (2019)
The Self-dual Classical Double Copy and the Eguchi-Hanson Instanton
JHEP 2019.1 (2019): 107.

A. Luna, R. Monteiro, I. Nicholson, D. O'Connell, and C. D. White, (2016)
The double copy: Bremsstrahlung and accelerating black holes
JHEP vol. 06, p. 023, 2016.

R. Monteiro, D. O'Connell, C. White (2014)
Black Holes and the Double Copy
JHEP 2014.12 (2014): 56

Z. Bern, J. J. M. Carrasco and H. Johansson (2008)
New Relations for Gauge Theory Amplitudes
Physical Review D 78.8 (2008): 085011.

H. Kawai, D. C. Lewellen, and S.-H. Tye (1986)
A Relation Between Tree Amplitudes of Closed and Open Strings
Nuclear Physics B vol. 269, no. 1, pp. 1–23, 1986.

W. Kinnersley (1969)
Field of an Arbitrarily Accelerating Point Mass
Physical Review vol. 186,pp. 1335–1336, 1969.