AN EXPLANATION FOR EARLY UNIVERSE'S STABILITY, AND THE DOMINANCE OF STANDARD MODEL

based on HERTZBERG, MJ 1807.05233; 1904.04262; 1910.04664; and 1911.04648

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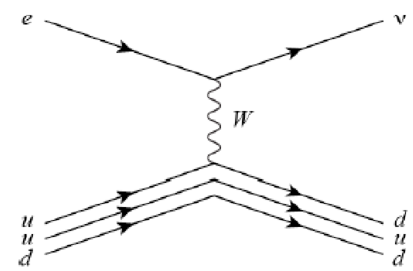
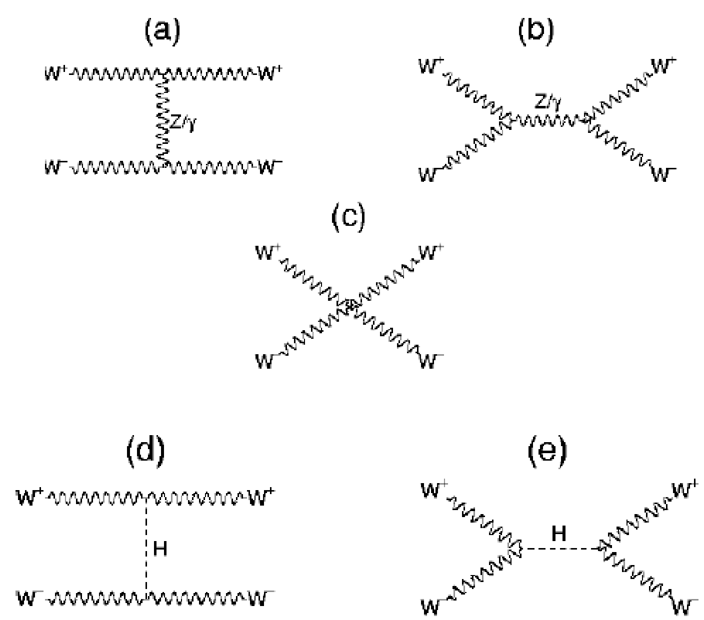
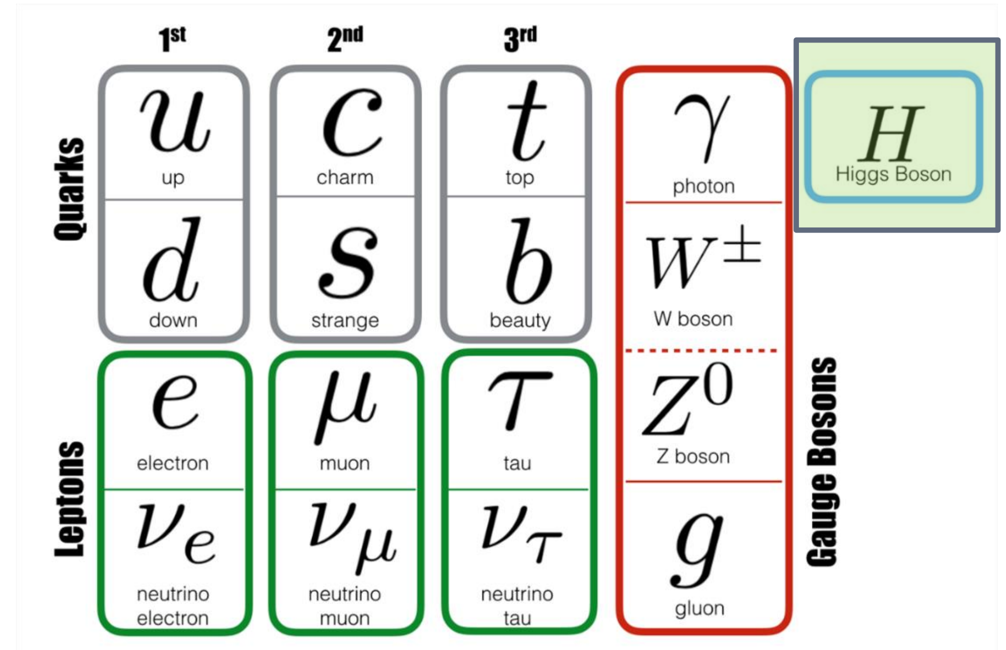
UNITARITY AT TREE LEVEL

W/Z-bosons unitarize fermion scattering

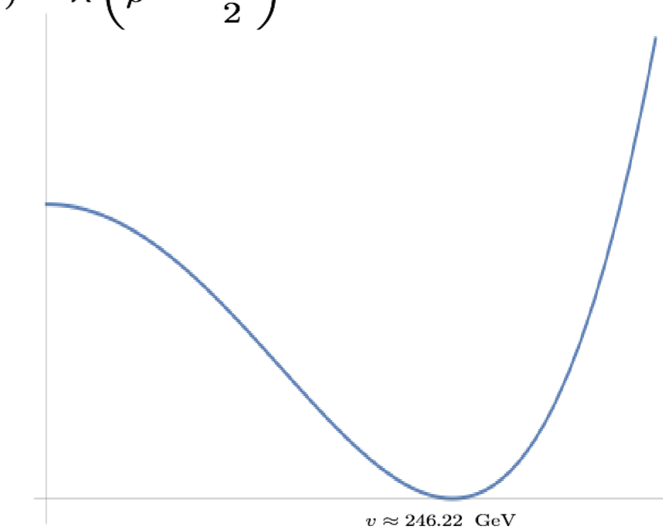




The Higgs unitarizes WW scattering



$$V(\rho) = \lambda \left(\rho^2 - \frac{v^2}{2}\right)^2$$



$$\frac{\lambda}{4} h^4$$

$$\rho = \frac{v+h}{\sqrt{2}}$$

and so on…

$$\text{QED + Fermi Theory} \xrightarrow{\text{Unitarity}} \text{Electro-Weak Theory}$$

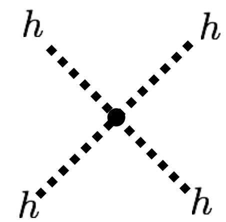
(all the way up to Planckian scales)

(Minimal bottom-up construction)

ANALYZING HIGGS BEYOND TREE LEVEL

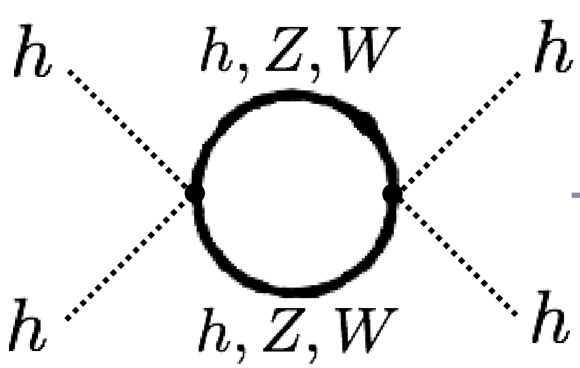
- Since Higgs couples to all massive SM particles. Loop corrections due to all of them

$$\frac{\lambda}{4}h^4 \rightarrow$$



$+$

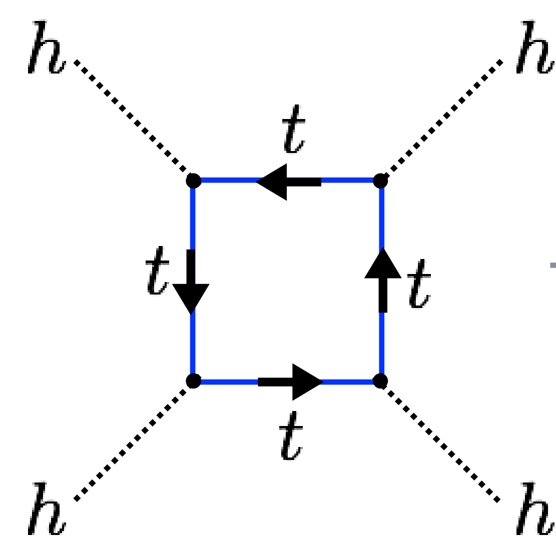
Bosons in the loop



$+ \quad \ldots$

Fermions in the loop

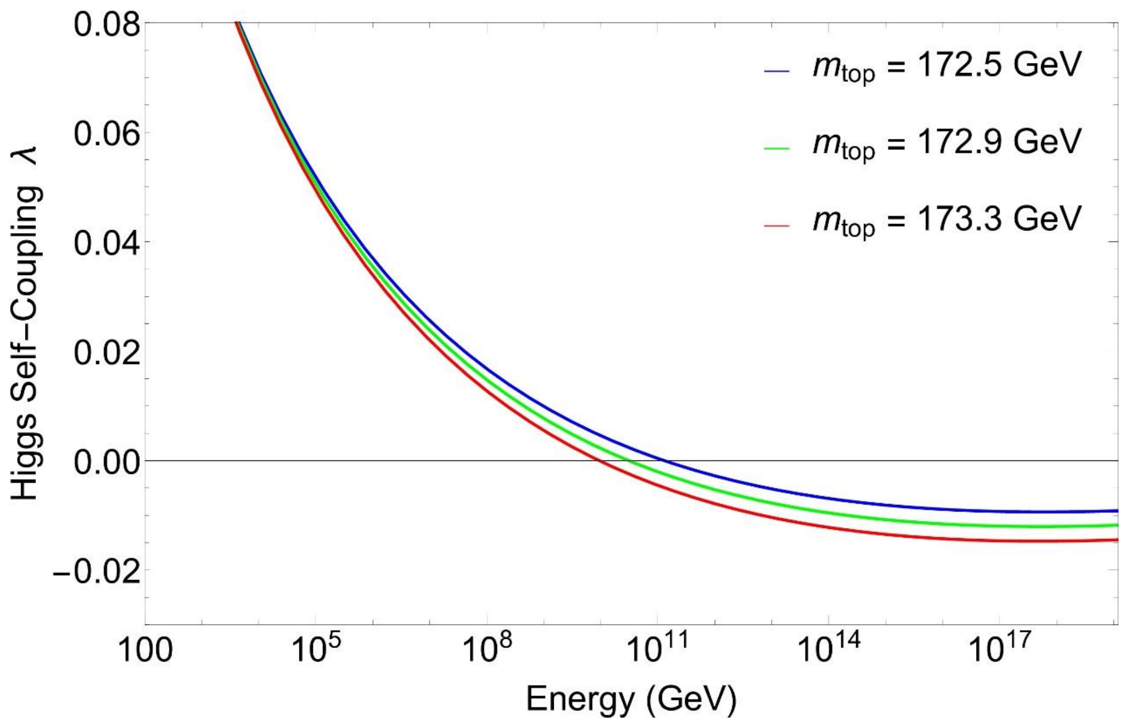
$+$

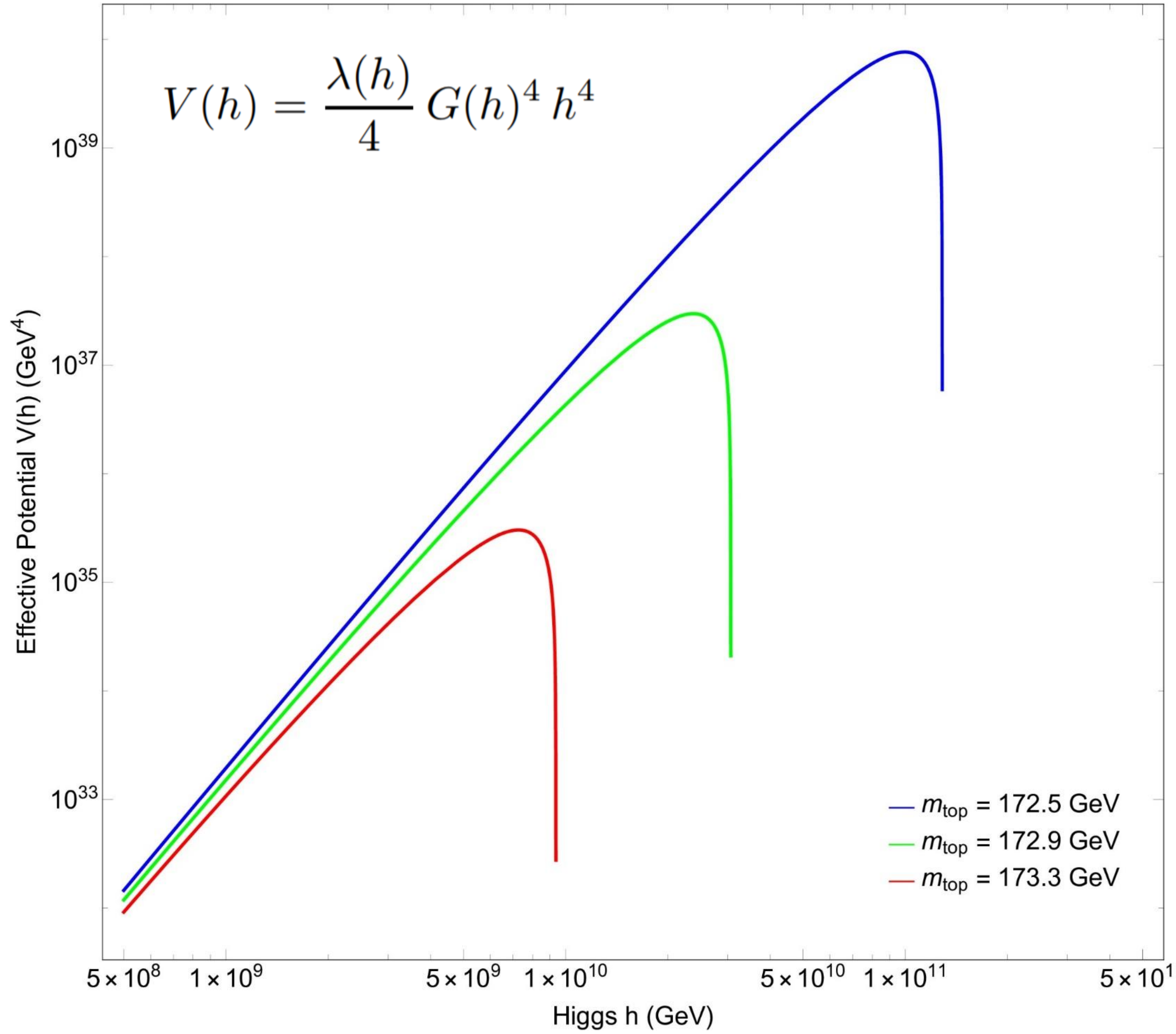


$+ \quad \ldots$

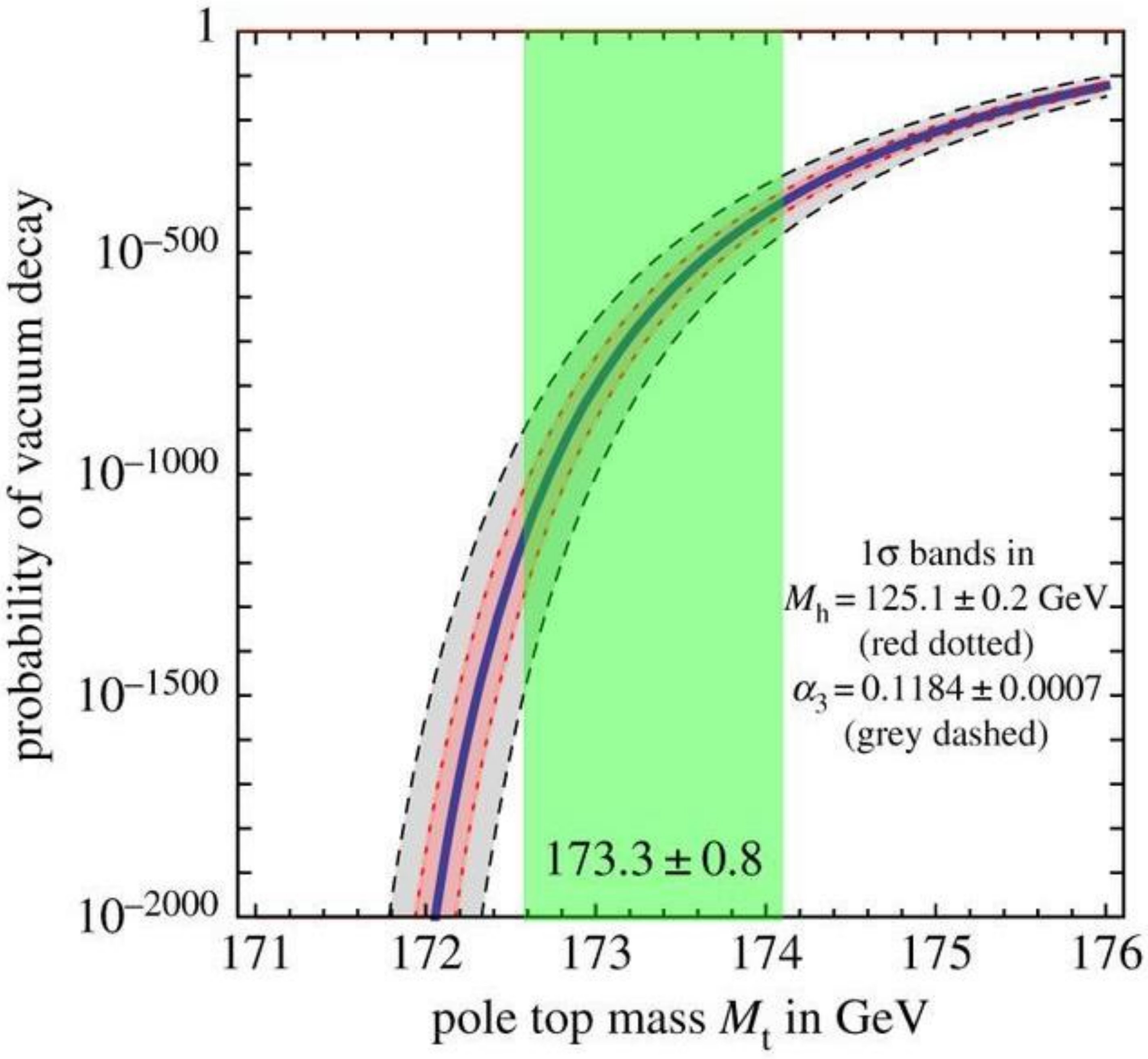
$$\beta_\lambda^{(1)} \equiv \frac{d\lambda}{d\log(E/M_Z)} = \frac{1}{16\pi^2}\left(24\lambda^2 + \frac{3}{2}(2g^4 + \frac{3}{2}(g^2 + g'^2)^2)\right) - \frac{1}{16\pi^2}(6y_t^4)$$

A catastrophe
(two loop result below)





PROBABILITY OF DECAY



(Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia; Espinosa)

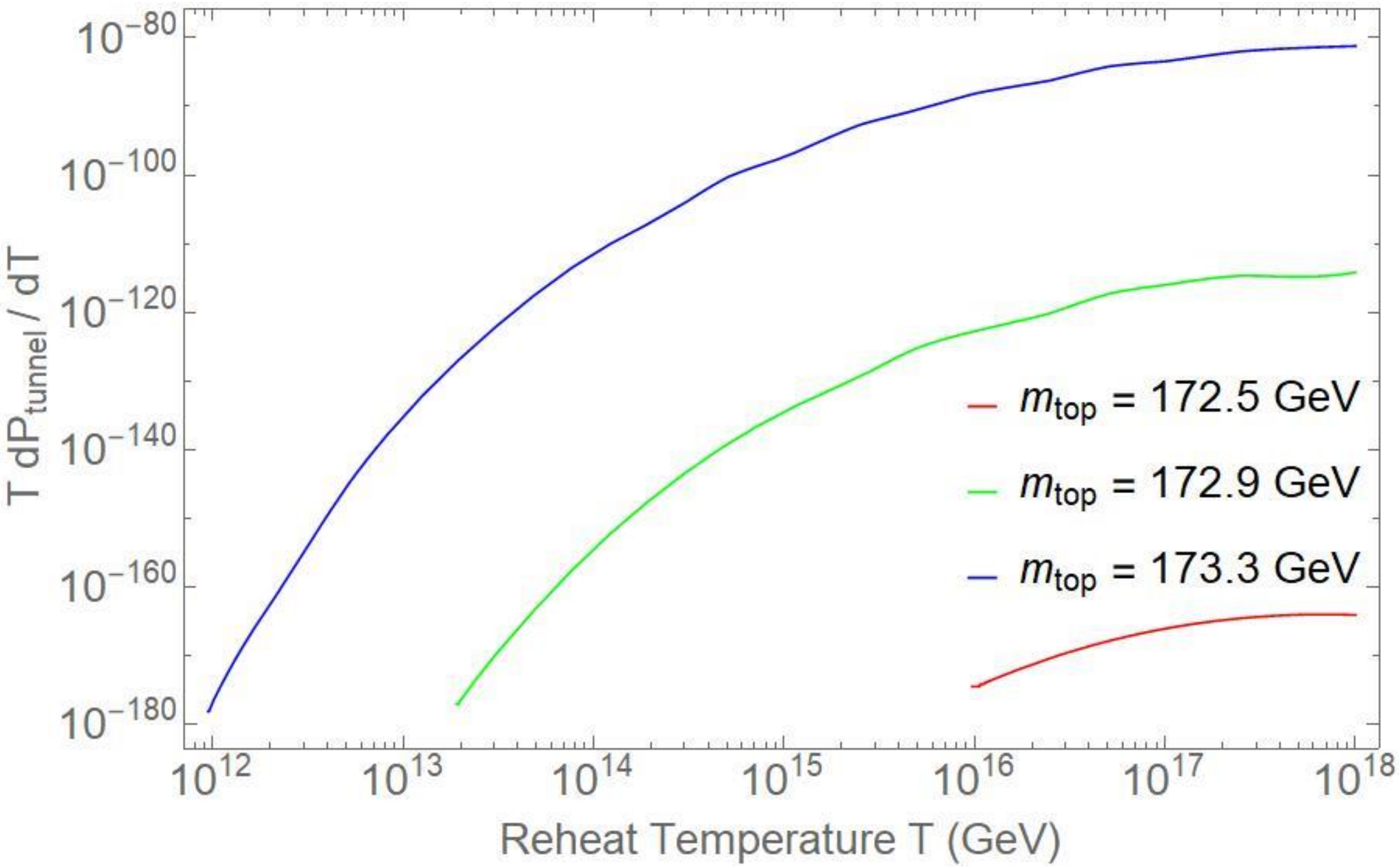
But very early epochs in
the history of the Universe

FINITE TEMPERATURE EFFECTS

- Early Universe was at high temperatures. Temperature corrected effective potential; + Higgs could be fluctuating a lot!

- Assuming adiabatic expansion, calculate the total probability of tunneling: "What is the probability that there would have been 1 bubble nucleation anywhere in the observable Universe, till today"

$$P_{tunnel} \sim \int_{T_0}^{T} dT \, \frac{V_0 T_0^3}{H} \left(\frac{S_B}{T}\right)^{3/2} e^{-\frac{S_B}{T}}$$



Standard Model is good even for Planckian temperatures

(MJ, Hertzberg 1910.04664)
also **(Espinosa, Giudice, Isidori, Miro, Riotto, Strumia, …)**

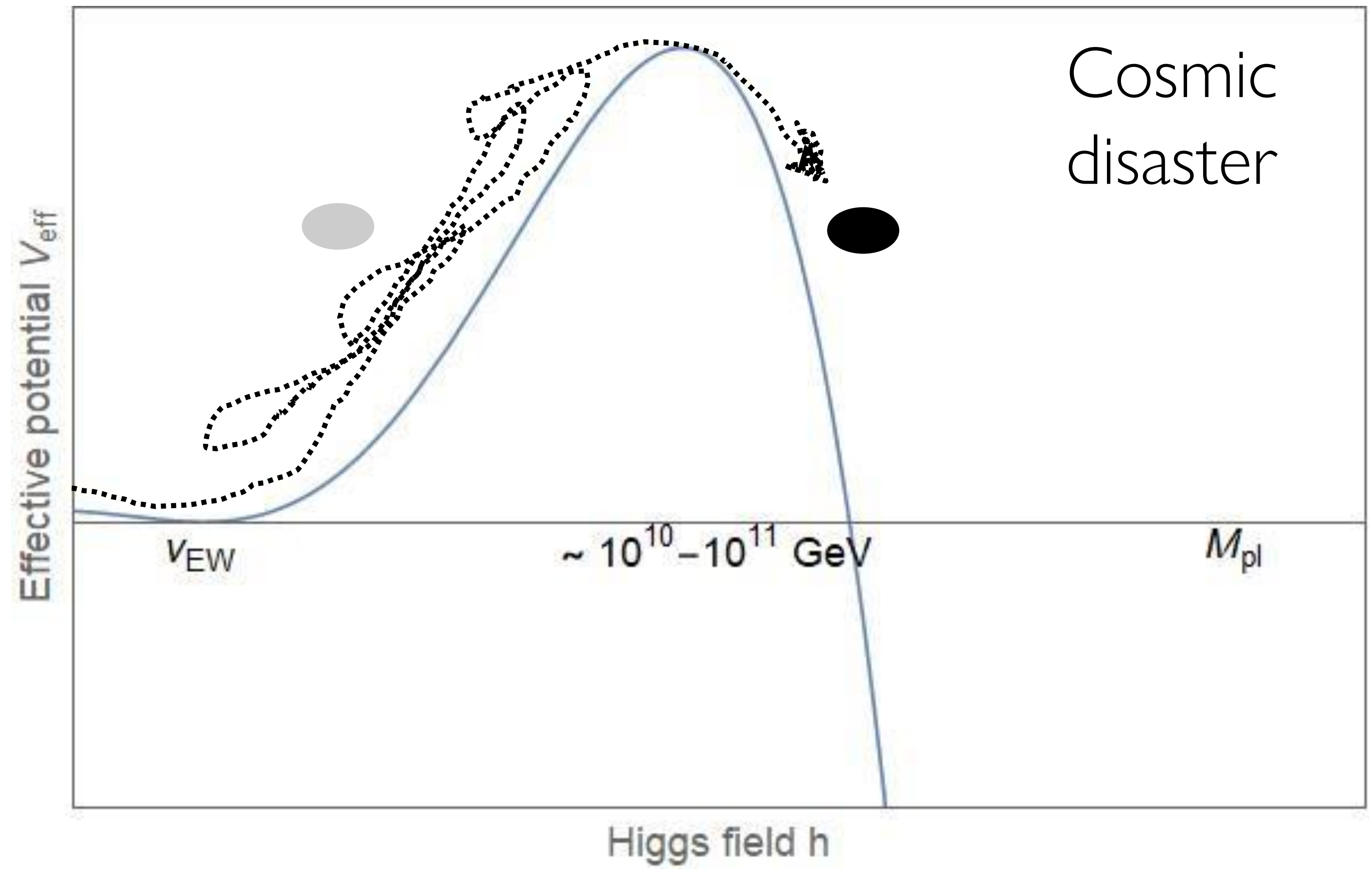
LAUNCHING HIGGS INTO INFLATIONARY UNIVERSE

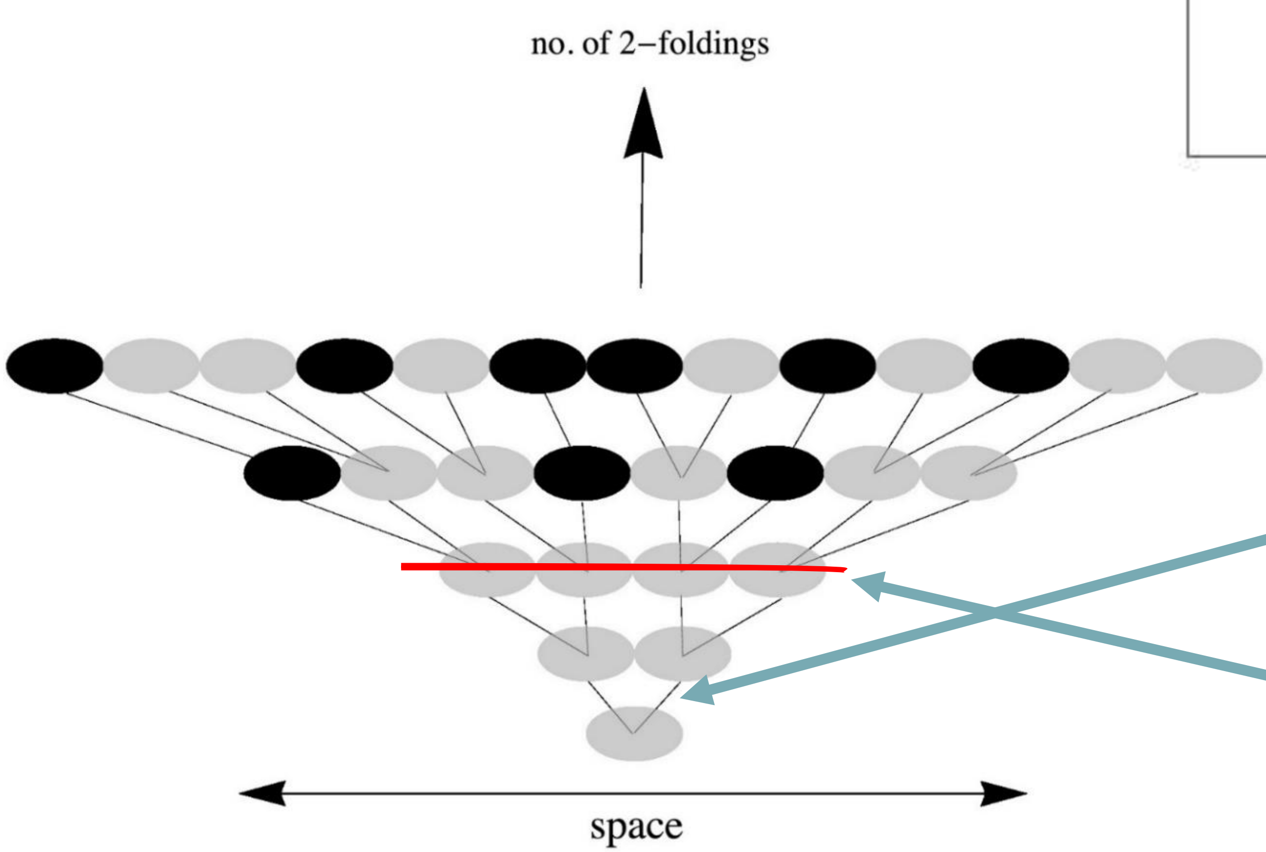
(East, Espinosa, Giudice, Isidori, Kearney, Kohri, Matsui, Miro, Morgante, Riotto, Senatore, Shakya, Strumia, Tetradis, Yoo, Zurek,…)

De-Sitter Fluctuations

$$\Delta h = \frac{H}{2\pi} \quad \text{(kick)}$$

per Hubble patch
per unit time
(a random walk behavior)





Usual approach in the literature

- Initial delta at zero,
- Gaussian pdf,
- Evolve till some N=60

LAUNCHING HIGGS INTO (ETERNAL) INFLATIONARY UNIVERSE

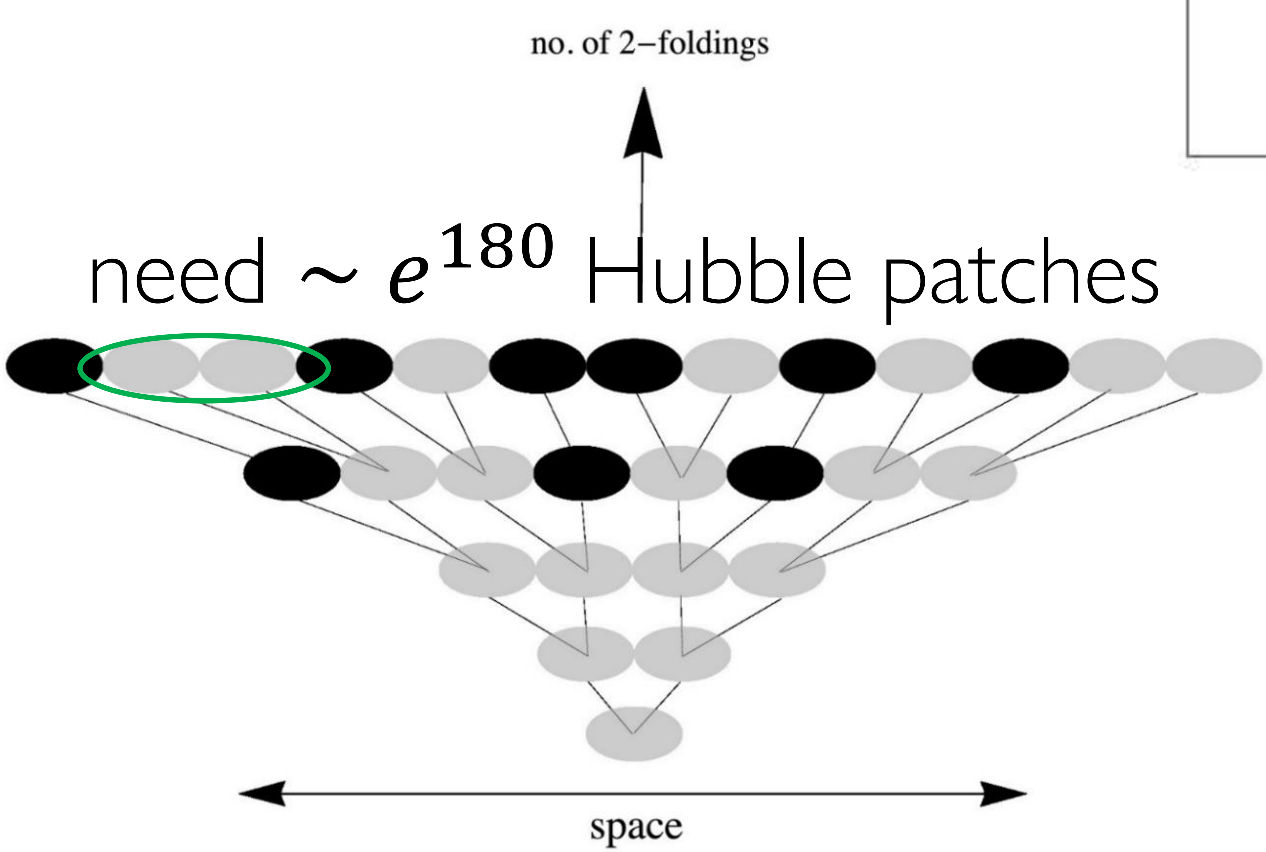
STATISTICS OF INFLATING ISLANDS

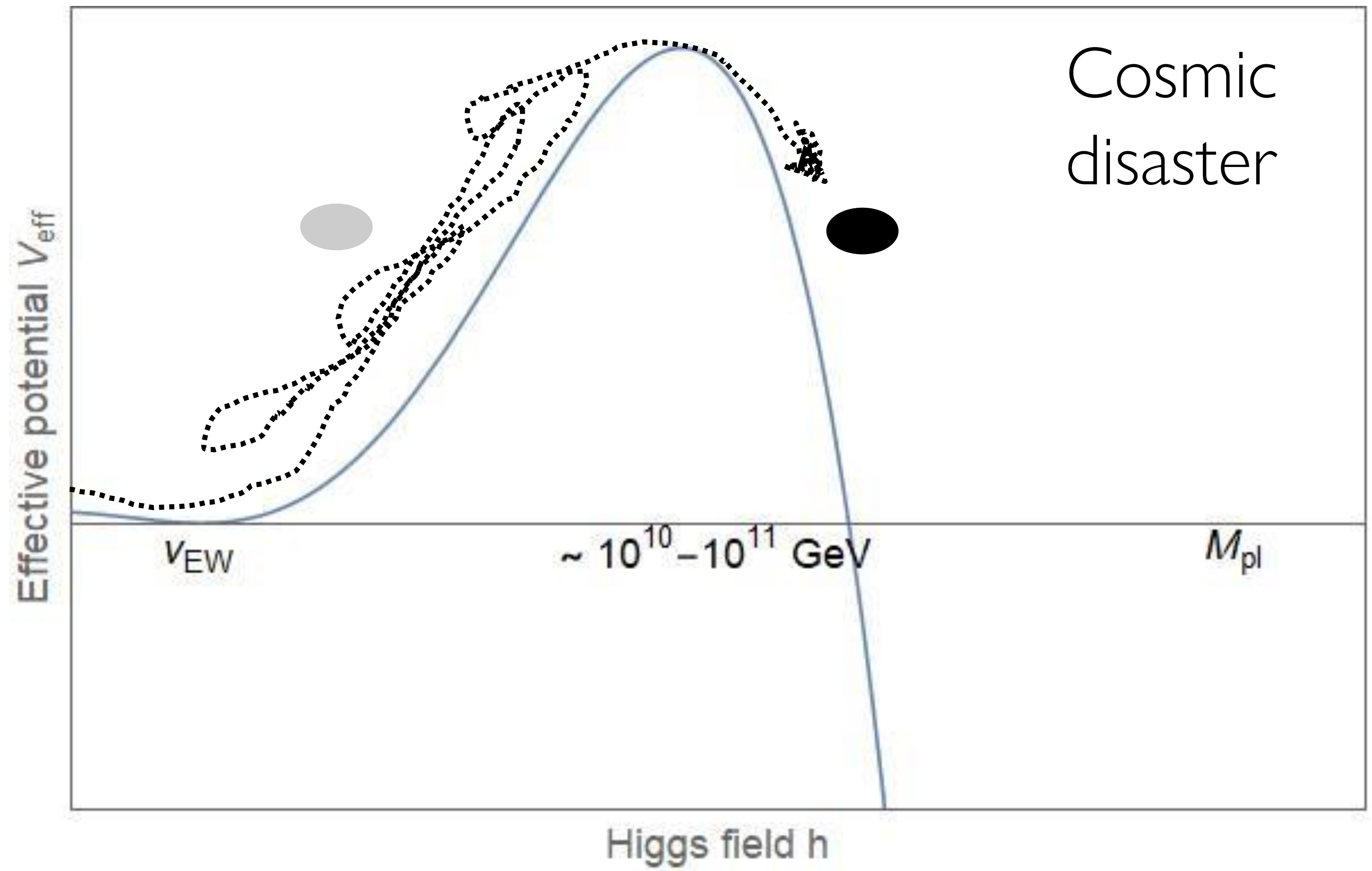
De-Sitter Fluctuations

$$\Delta h = \frac{H}{2\pi} \quad \text{(kick)}$$

per Hubble patch
per unit time
(a random walk behavior)

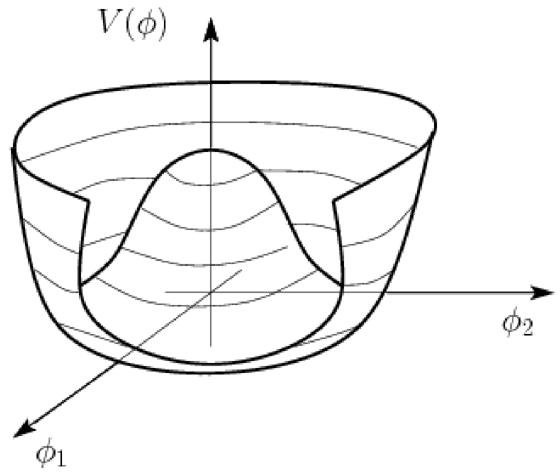
need $\sim e^{180}$ Hubble patches

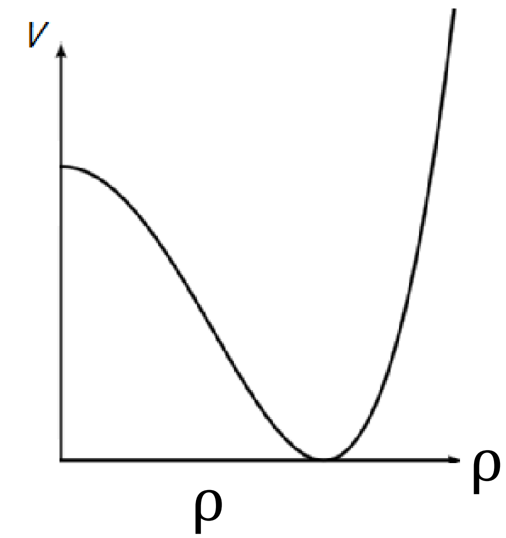




- Full distribution + eternal inflation
- But first, what's the correct probability measure in the QPF of the Higgs?

- Gauge redundancies, no Goldstones -> No spontaneous symmetry breaking?
- But if no SSB, then what is the probability measure?





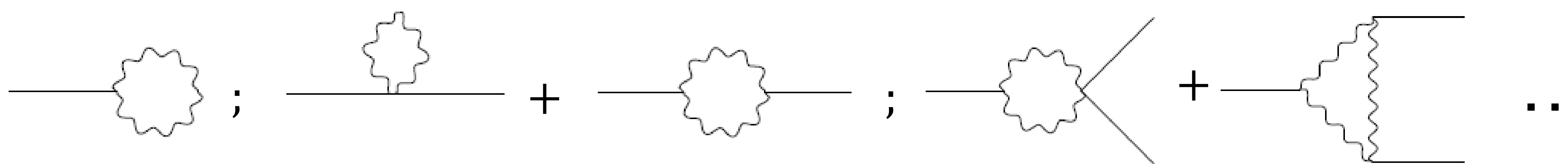
$$\int \boxed{D\rho\, \rho^3}\, D\theta\, DA^T e^{iS_c}$$

volume measure?

$$\int \boxed{D\rho}\, DA^L\, DA^T e^{iS_U}$$

line measure?

Quartic UV divergences



$$\mathcal{L}_{ct} = -3i\Lambda^4 \ln(v+h) = -3i\Lambda^4 \ln\rho$$

$$Z = \int \boxed{D\rho\, \rho^3}\, DA^L\, DA^T e^{iS_U}$$

volume measure

(Hertzberg, MJ 1807.05233)

LAUNCHING HIGGS INTO (ETERNAL) INFLATIONARY UNIVERSE

STATISTICS OF INFLATING ISLANDS

De-Sitter Fluctuations

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per Hubble patch
per unit time
(a random walk behavior)

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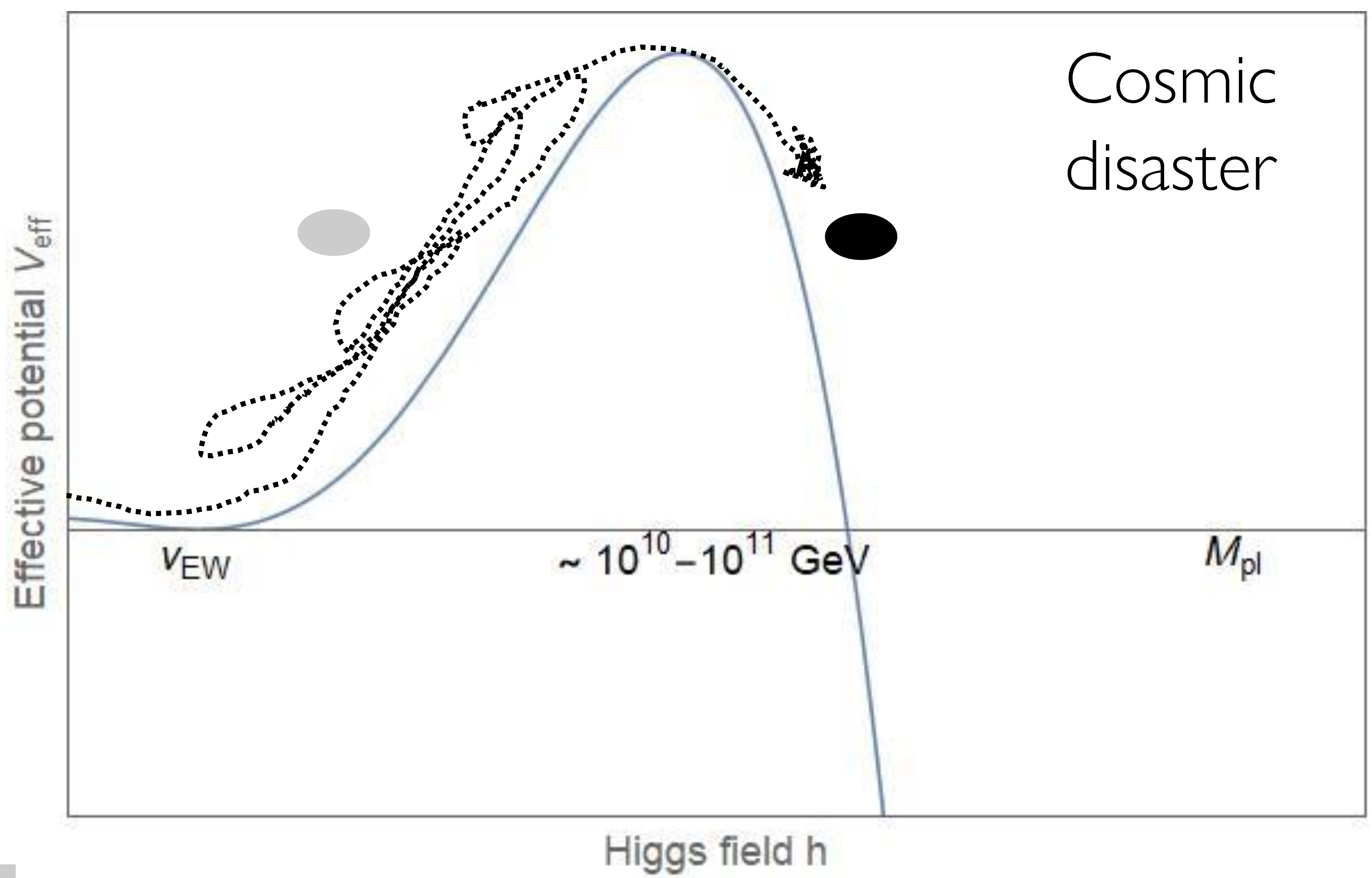
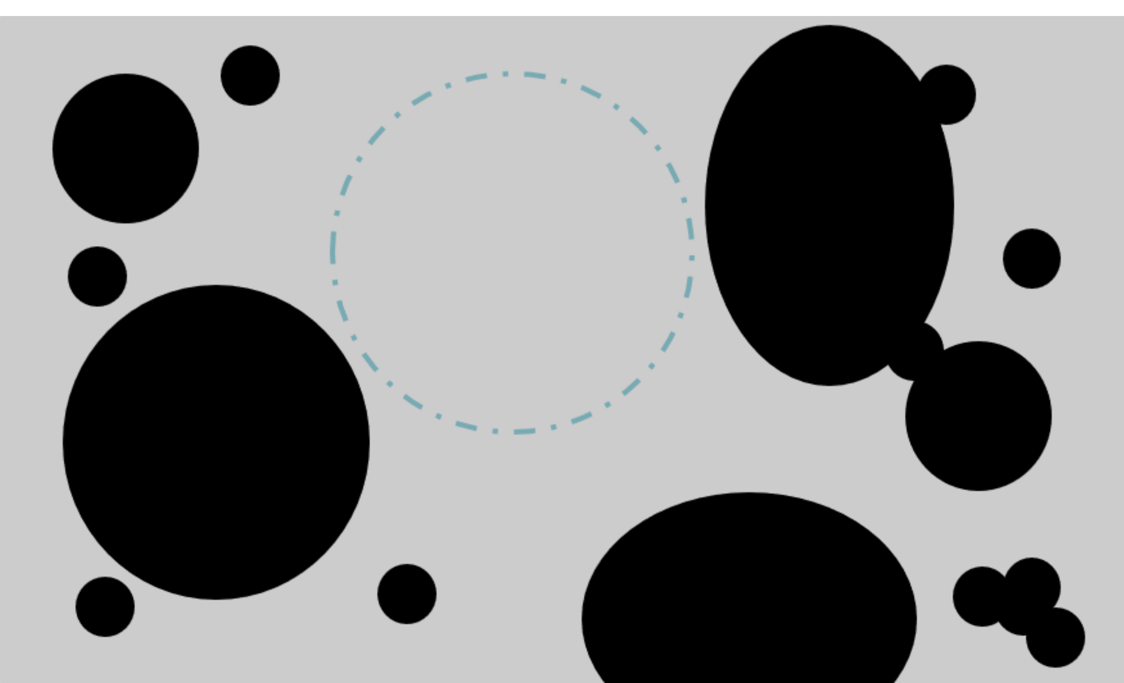




- Full distribution + eternal inflation

LANGEVIN ⟺ INTEGRAL EVOLUTION

drift (classical)

kick (quantum diffusion)

multiple fields (volume measure)

$$\frac{\vec{\varphi}_i - \vec{\varphi}_{i-1}}{\epsilon} + \frac{1}{DH^2}\frac{\partial V}{\partial \vec{\varphi}}\left(|\vec{\varphi}_{i-1}|\right) = \kappa\, \vec{\eta}_i$$

gaussian r.v.

Inflation within $\rho \in [0, \rho_{end})$

kernel

$$p_i\left(\rho_i\right) = \prod_{s=0}^{i-1} \int_0^{\rho_{\rm end}} d\rho_s\, K\left(\rho_{s+1}, \rho_s, \epsilon\right)\, p\left(\rho_0, 0\right)$$

(absorbed the radial field measure into the distribution)

convergence to the dominant eigenstate is clear

Steady state (constant) distribution

(dominant eigenstate of the kernel)

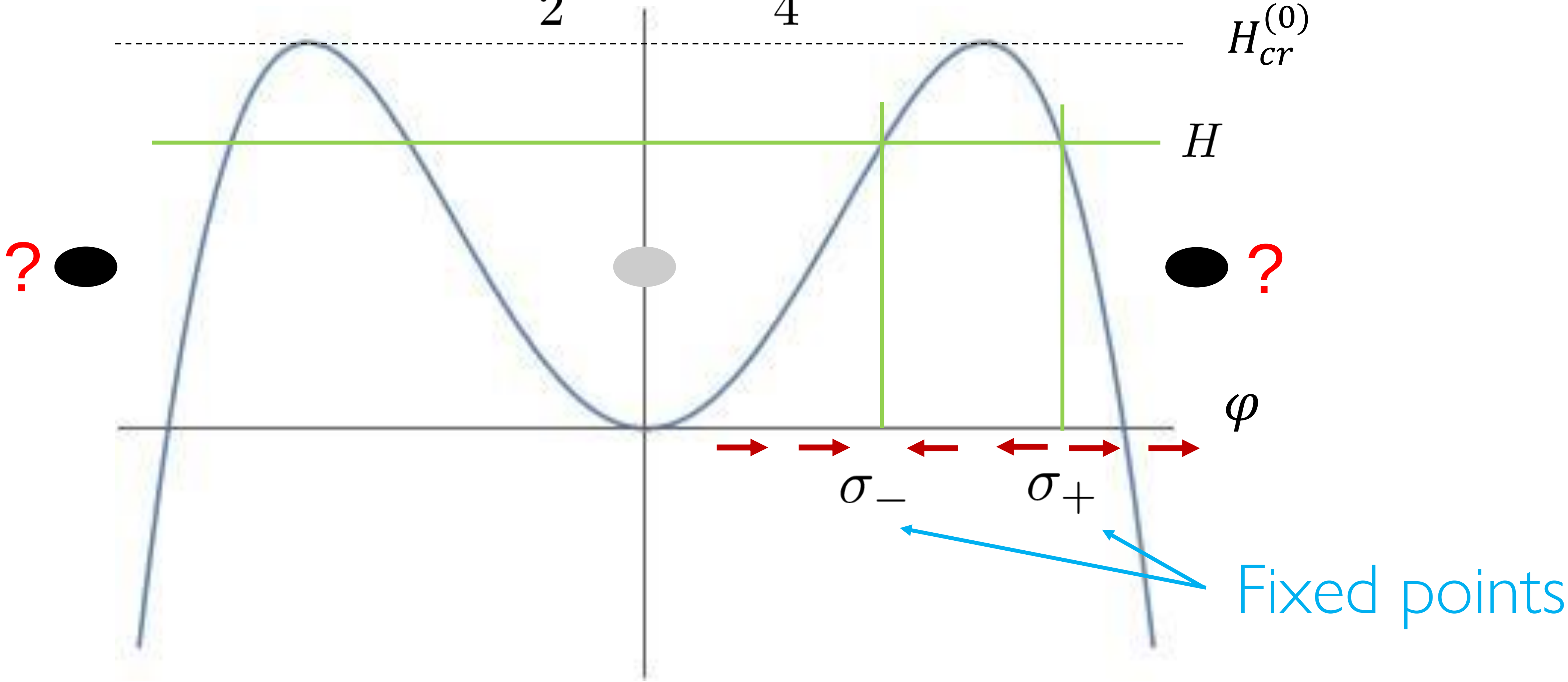
$$\tilde{p}(\rho) \equiv \frac{p(\rho, \infty)}{\int_0^{\rho_{\rm end}} d\rho\, p(\rho, \infty)}$$

(ρ_{end} is in the fast roll regime)

GAUSSIAN APPROXIMATION

A toy example of Higgs like 'M' potential

$$V = \frac{1}{2}m^2\varphi^2 - \frac{1}{4}\lambda\varphi^4$$

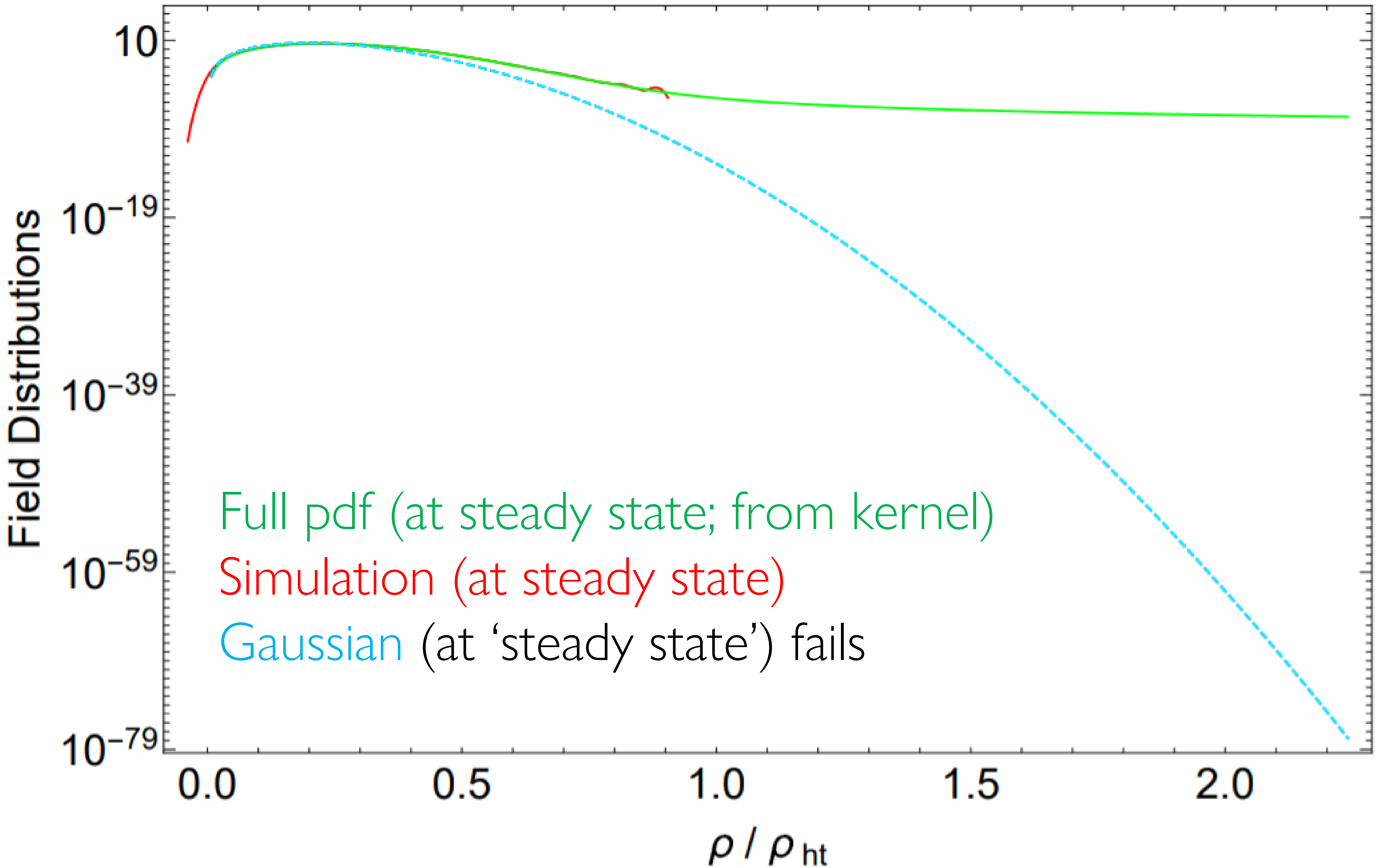


from Fokker-Planck $$\frac{d}{dN}\sigma^2 + \frac{2\lambda}{H^2}\left(\sigma^2 - \sigma_+^2\right)\left(\sigma^2 - \sigma_-^2\right) = 0$$

- $H \leq H_{cr}^{(0)}$ at least gives stationarity
- $H > H_{cr}^{(0)}$ does not even fetch this; flattening of distribution

COMPARISON of DISTRIBUTIONS with 1D SIMULATIONS
(large number of foldings)

$$H < H_{cr}^{(0)}$$

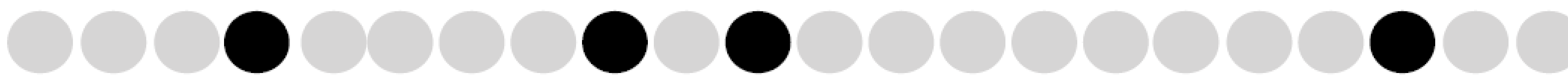


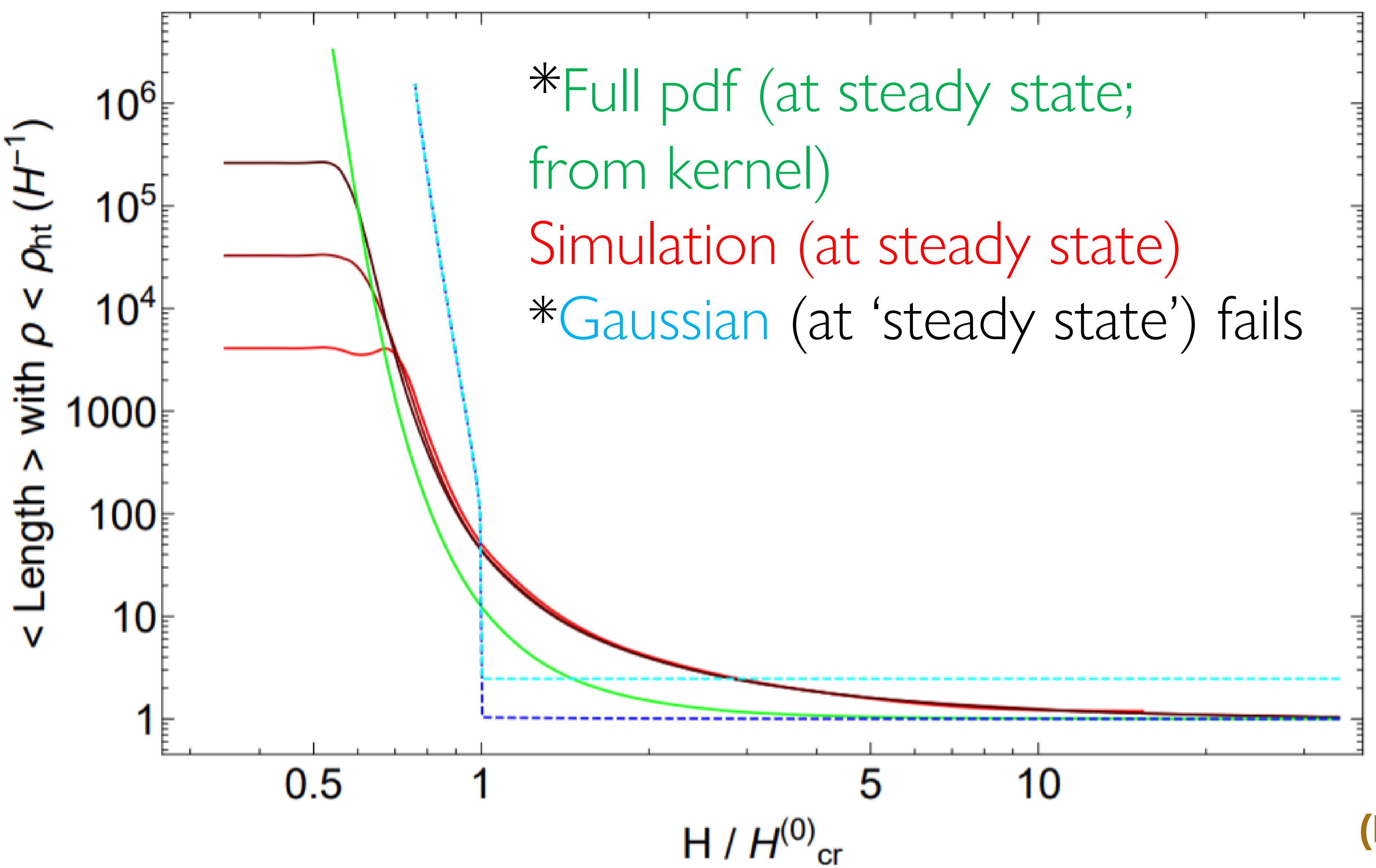
(MJ, Hertzberg 1910.04664)

AVERAGE SIZE OF HILLTOP CONTAINED REGIONS

*Under the assumption that each patch could be treated independently, we have (in 1D for instance)

$$\langle\,\text{Length}\,\rangle = \frac{1}{1-f}\,H^{-1}; \quad f = \int_0^{\rho_{\rm ht}} d\rho\;\tilde{p}(\rho)$$

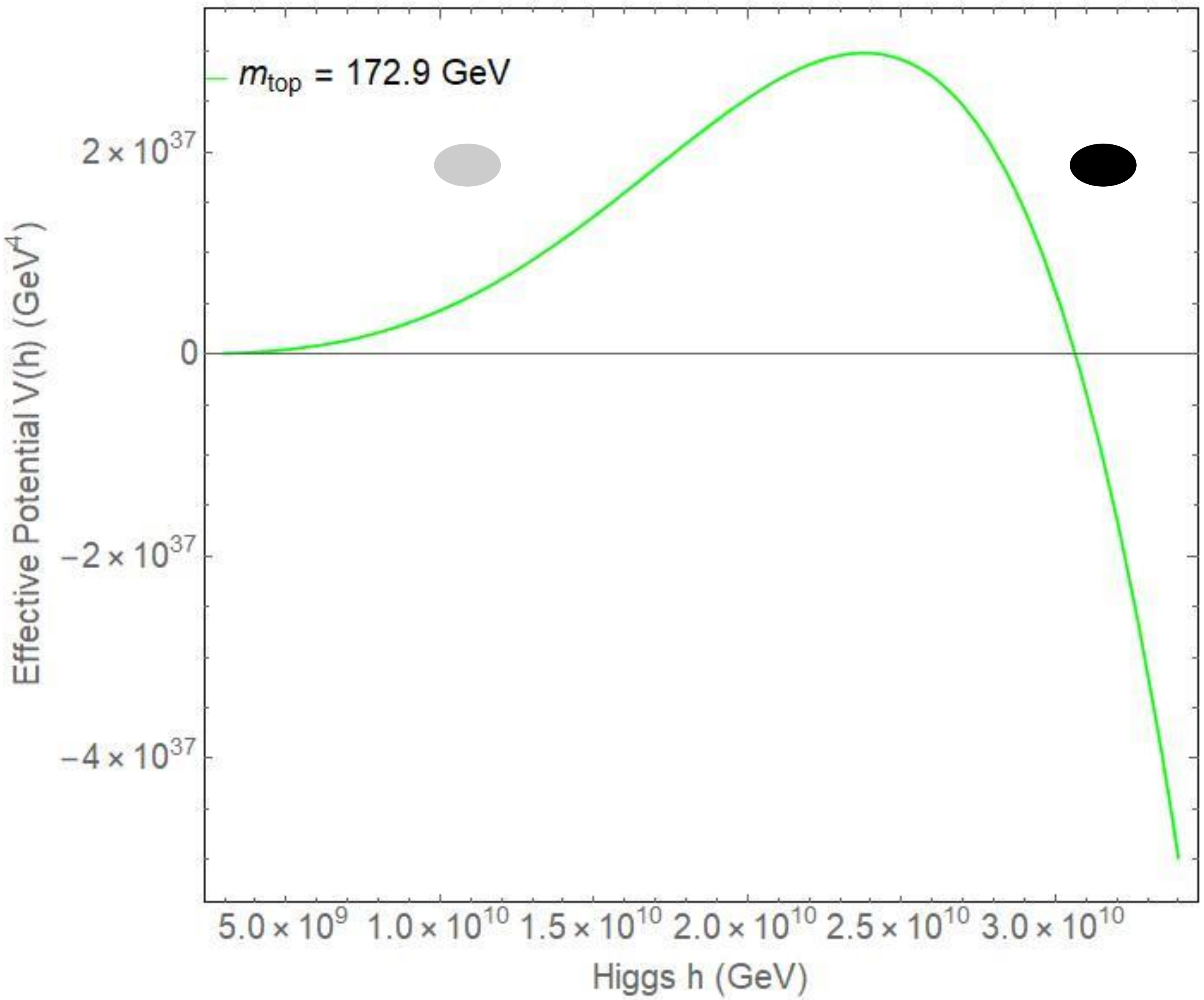




- Suggests that independent treatment is indeed valid for large average inflating lengths

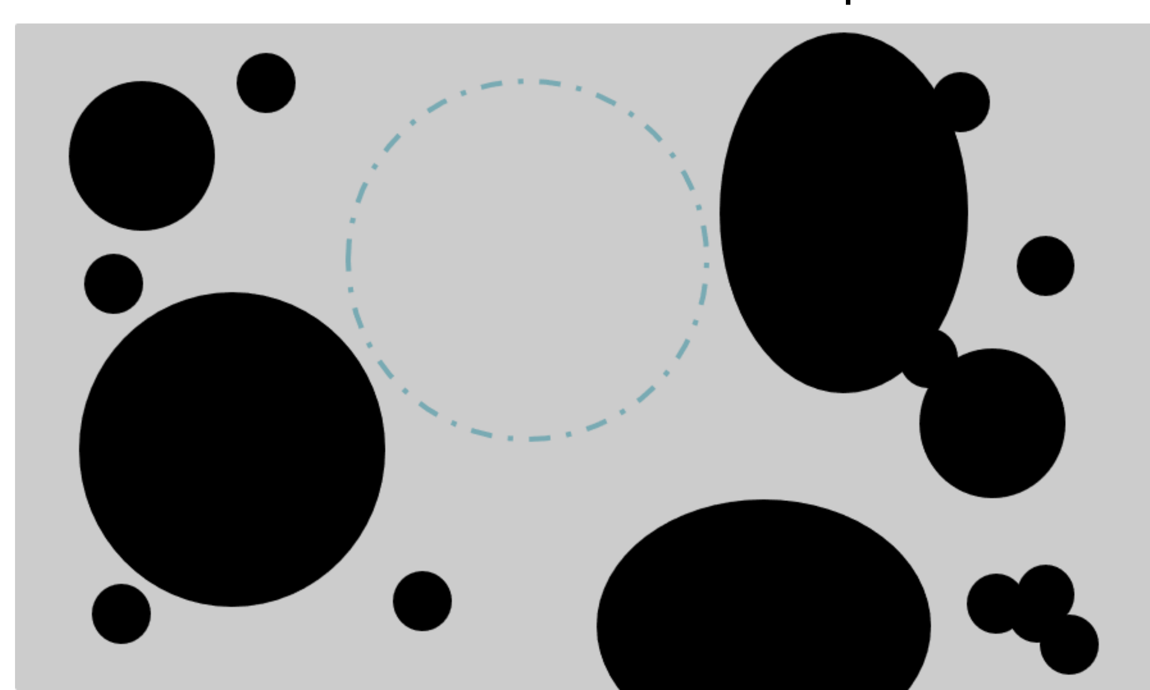
(MJ, Hertzberg 1910.04664)

ANALYSIS WITH HIGGS

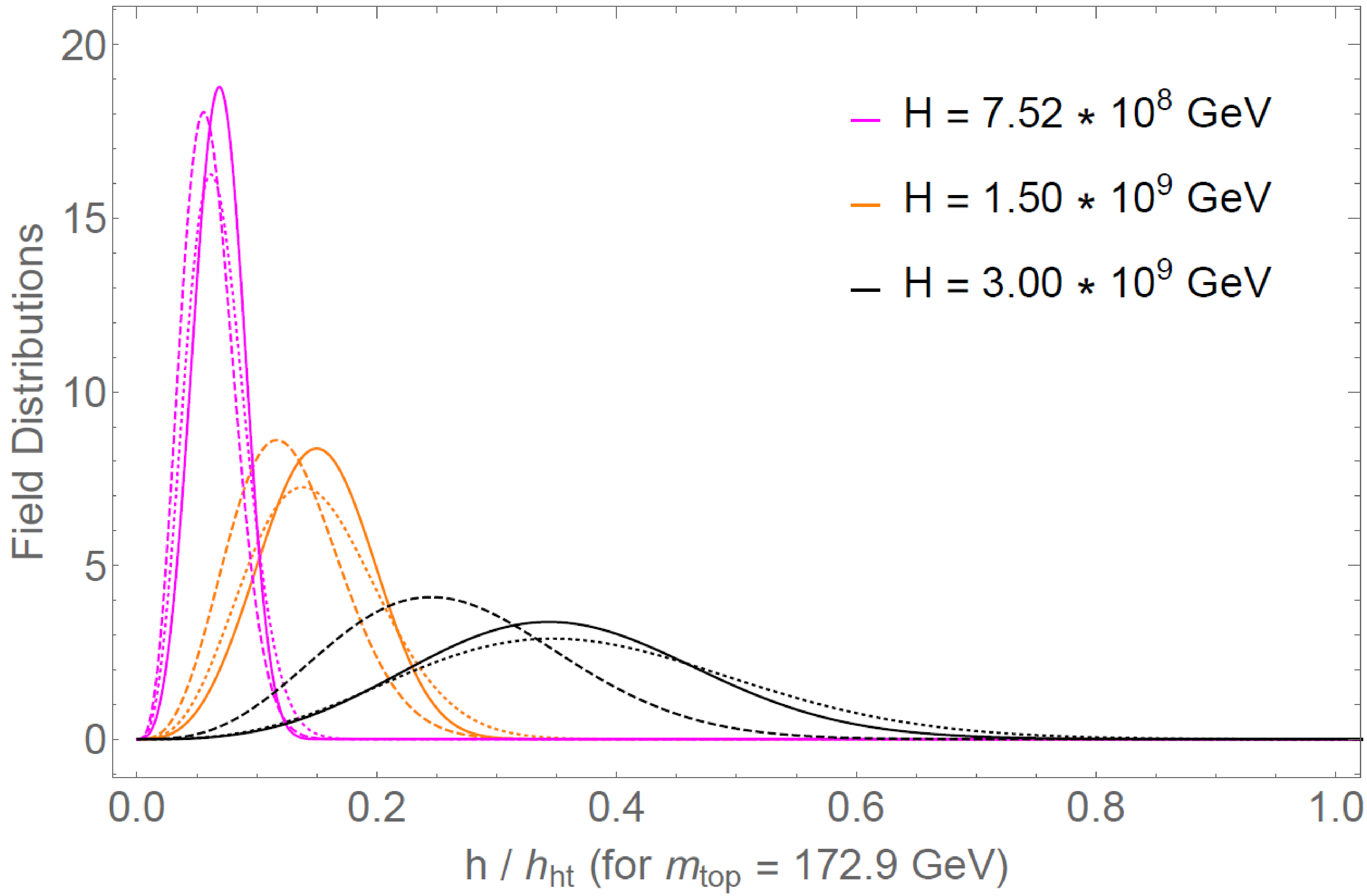


$$V(h) = \frac{\lambda(h)}{4} G(h)^4 h^4$$

need $\sim e^{180}$ Hubble patches



Higgs field distributions



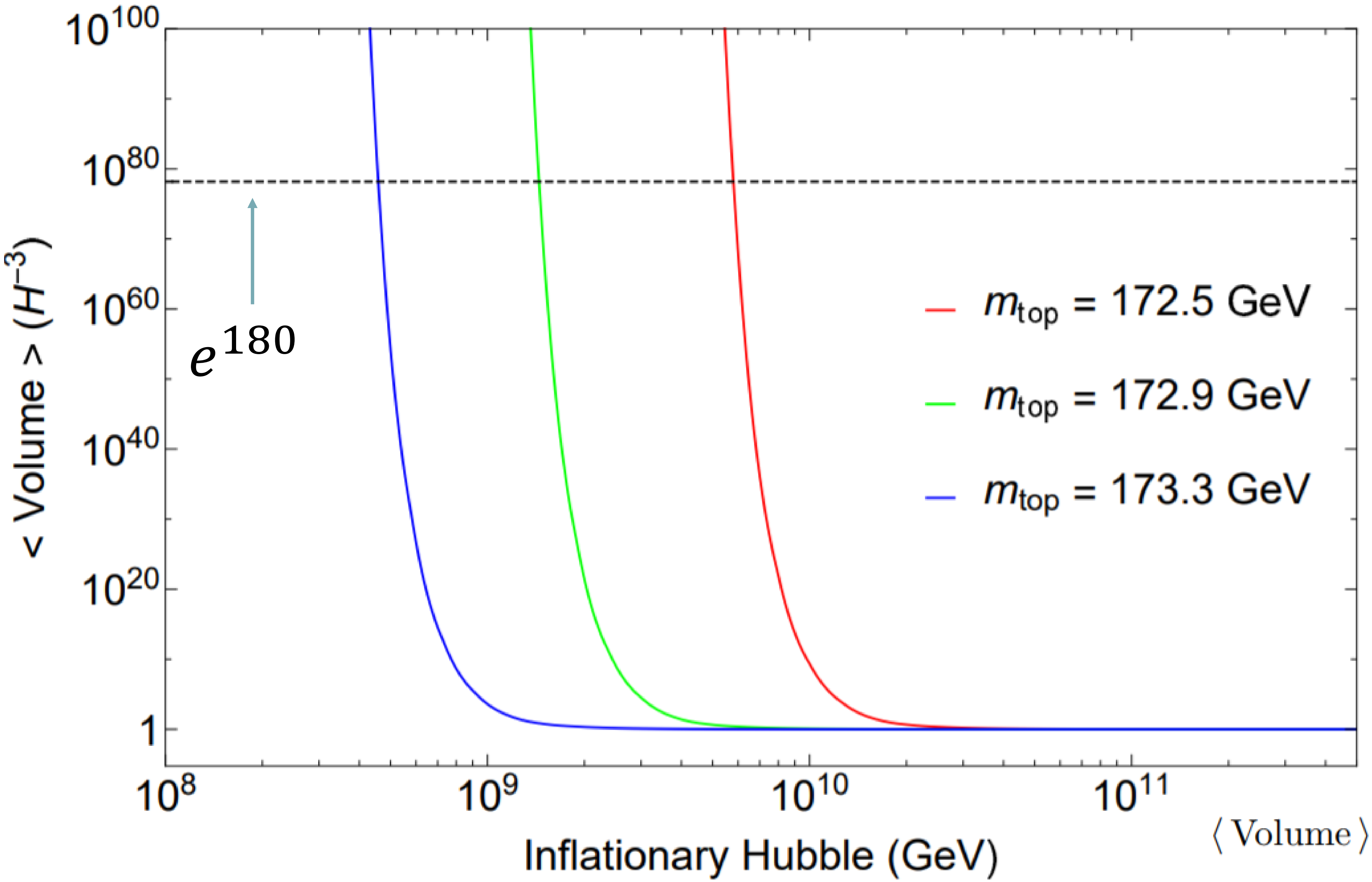
Full, steady state (from kernel)

Gaussian; steady state

Gaussian; N=60

(MJ, Hertzberg 1910.04664)

Average size of inflating regions, and a bound on inflationary Hubble



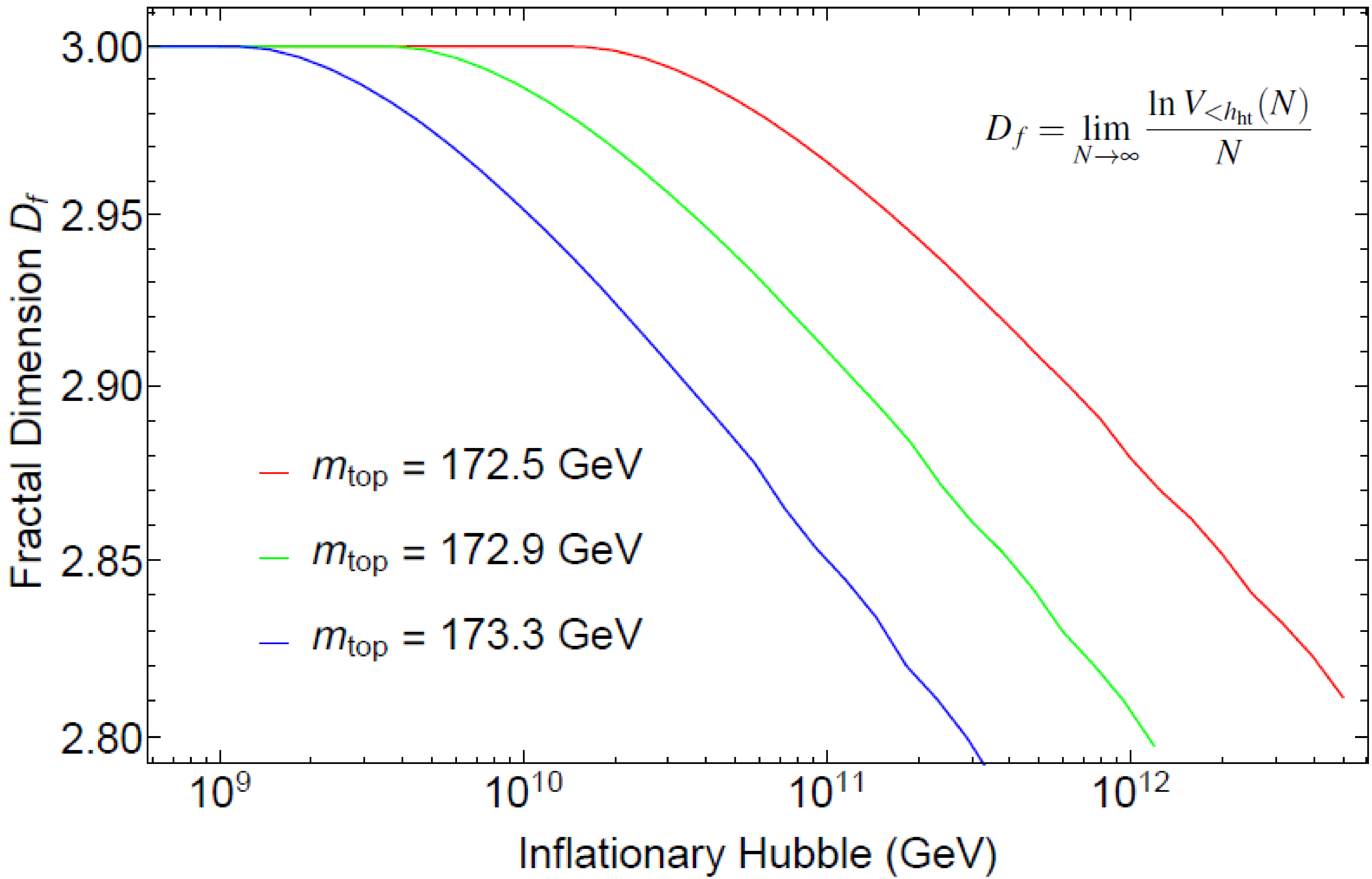
$$\langle \text{Volume} \rangle = \frac{1}{1-f_h} H^{-3};$$

$$f_h = \int_0^{h_{\rm ht}} dh\, \tilde{p}(h)$$

$$H_{\max} = \begin{cases} 5.8 \times 10^9 \text{ GeV} & \text{for } m_{\rm top} = 172.5 \text{ GeV} \\ 1.4 \times 10^9 \text{ GeV} & \text{for } m_{\rm top} = 172.9 \text{ GeV} \\ 4.6 \times 10^8 \text{ GeV} & \text{for } m_{\rm top} = 173.3 \text{ GeV} \end{cases}$$

(MJ, Hertzberg 1910.04664)

Fractal dimension of within hilltop contained regions



(MJ, Hertzberg 1910.04664)

STORY AFTER INFLATION
(Bridging the gap)

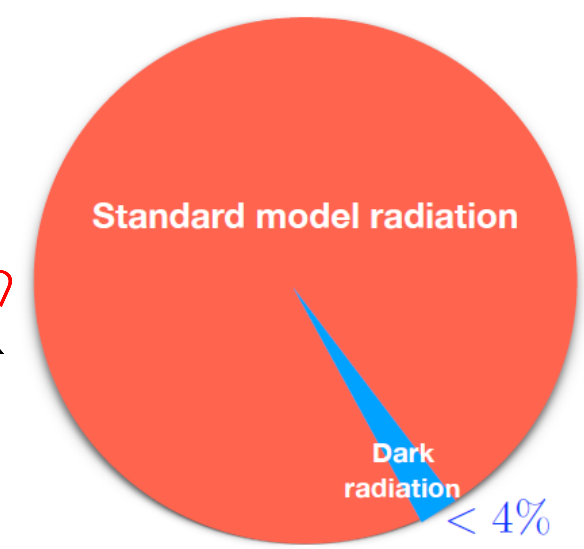
- Inflaton ϕ must couple to (at-least) the SM d.o.f. <=> (Reheating)
  e.g. direct coupling, non-minimal coupling
  $$\kappa\phi H^{\dagger}H, \quad g\phi^{2}H^{\dagger}H, \quad \xi R H^{\dagger}H$$

(Ema, Mukaida, Nakayama; Enqvist, Karciauskas, Lebedev, Rusak, Zatta;...)
[Moduli: Amin, Fan, Lozanov, Reece]

- But, parametric resonance? (Preheating)
  => **small** / fine tuned couplings

- Dark matter? (String theory suggests numerous hidden sectors
  $$\ldots\{SM\} \times \cdots \times SU(N_D) \times \ldots$$

Tension ↔



at ~ BBN
or $\Delta N_{eff} < 0.3$

- Baryogenesis?

A ‘NATURAL’ AND CONSERVATIVE MODEL

Natural
(unprotected masses are huge)

Explosive resonance?

$$\Delta\mathcal{L}_{UV} = -\kappa\phi H^{\dagger}H - \sum_i^{\text{Dark}} \frac{\phi}{4M_i} Tr\left[G_{i\,\mu\nu}G_i^{\mu\nu}\right] + \frac{\mu}{2}\phi\chi^2 + \cdots$$

Push κ to natural values ($\sim m_\phi$), but use the existence of a light Higgs

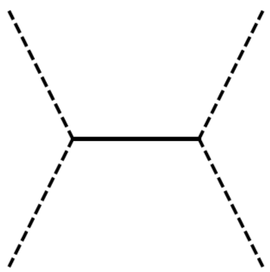
some high mass scale like $\sim M_{pl}$

m_χ $huge$; projected out

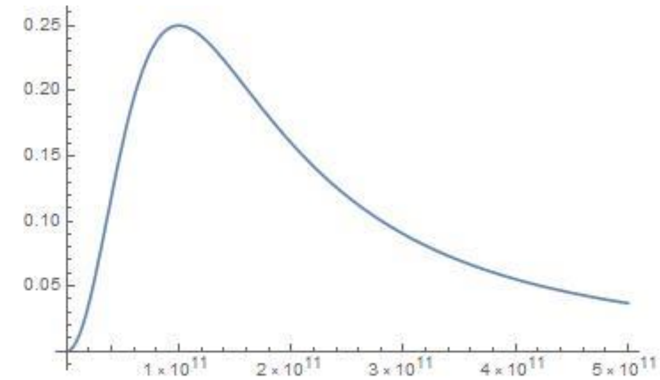
The first two are in fact the leading couplings

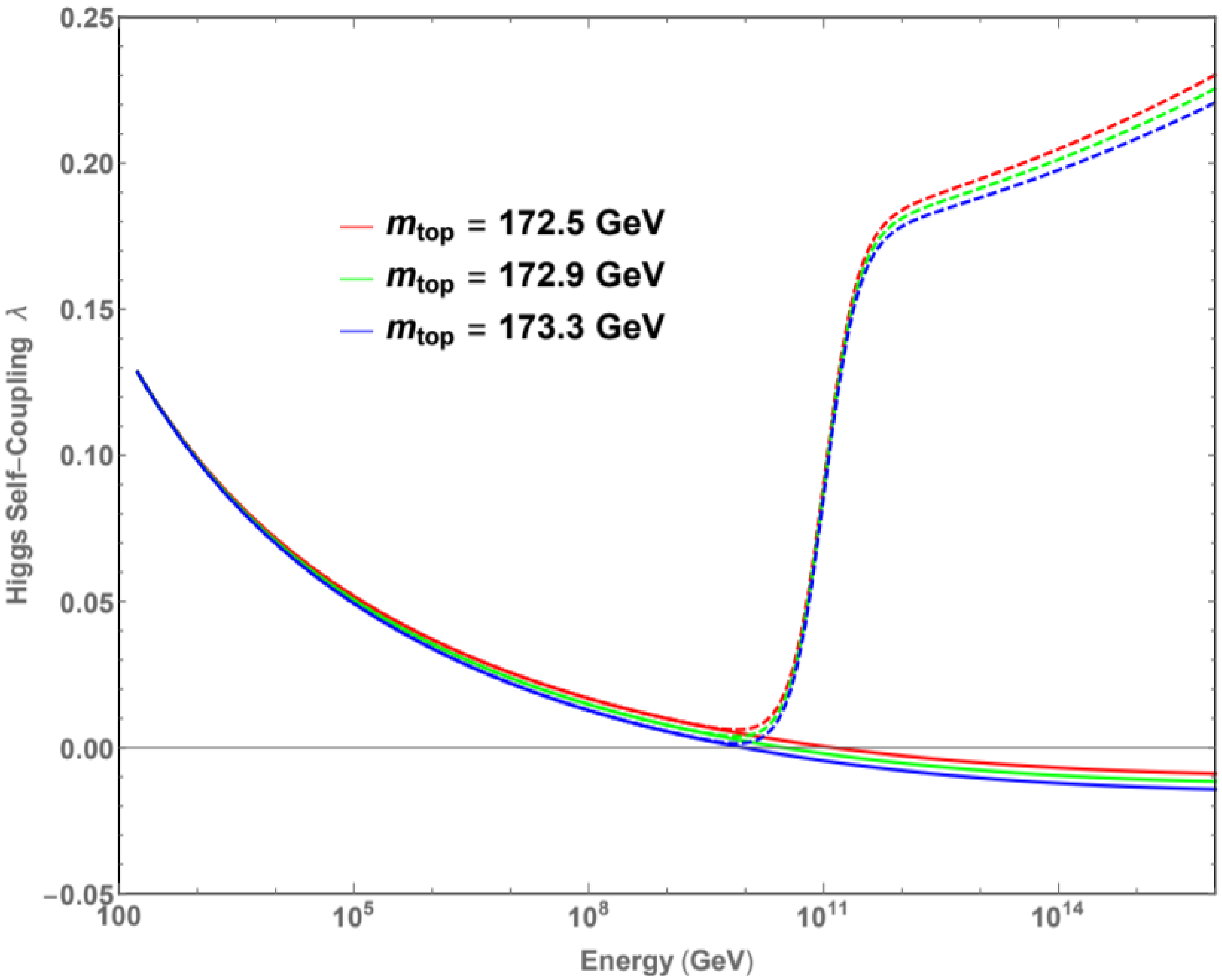
CORRECTION TO HIGGS SELF-COUPLING

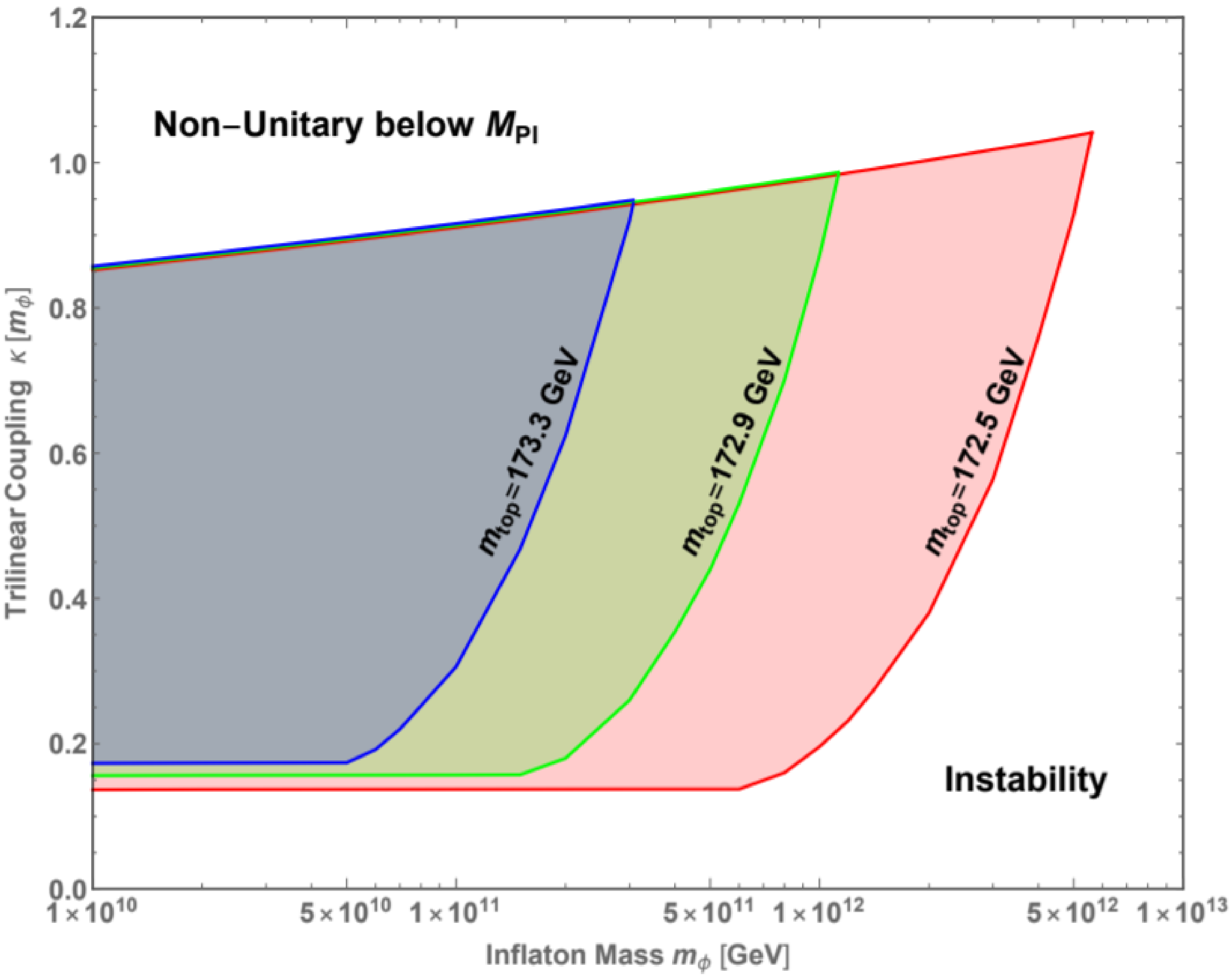
$$\Delta\mathcal{L} = -\kappa\phi H^\dagger H$$



$$\beta_\lambda = \frac{d\lambda}{d\ln E} = \beta_\lambda^{SM} + \frac{\kappa^2 E^2}{\left(m_\phi^2 + E^2\right)^2}$$





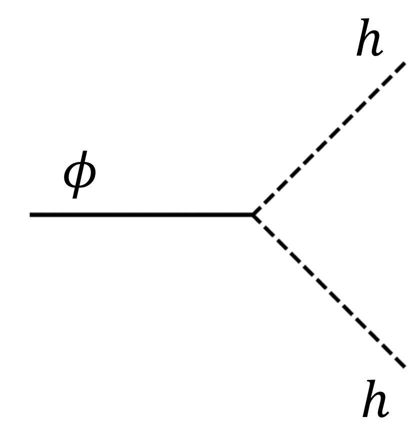


(Hertzberg, MJ 1911.04648)

INSTABILITY CURED; STORY AFTER INFLATION

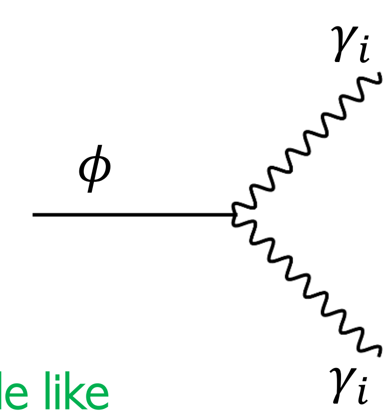
- Inflation ends ⇒ Preheating (no problem) + gradual/perturbative reheating…
- Reheating:

$$\Gamma(\phi \to h\, h) = \frac{\kappa^2}{8\pi\, m_\phi}$$



Number of (dark) gauge d.o.f.

$$\Gamma_\mathrm{i}(\phi \to \gamma_i\, \gamma_i) = \frac{g_i {m_\phi}^3}{128\pi\, M_i^2}$$



some high mass scale like

1. Perturbative decay dominant into the Higgs

2. Universe reheats to temperature $T_{reh} \approx 0.5\sqrt{\Gamma M_{pl}}$

   by the time $t_{reh} \sim \Gamma^{-1}$

SKETCHING THE SCENARIO

Inflation ends ⇒ Preheating + gradual/perturbative reheating

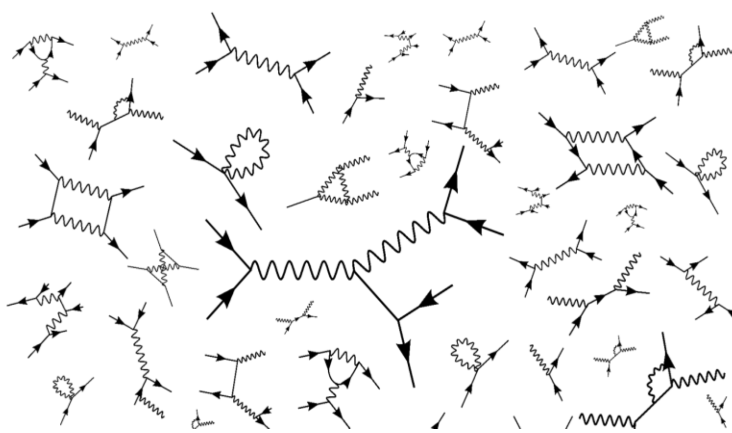
$T \sim T_{reh} \approx 0.5\sqrt{\Gamma M_{pl}}$ (room for baryogenesis, e.g. $T_{reh}{\sim}10^{14}$ GeV)

$T \sim m_\varphi$ (inflaton becomes Boltzmann suppressed from here)

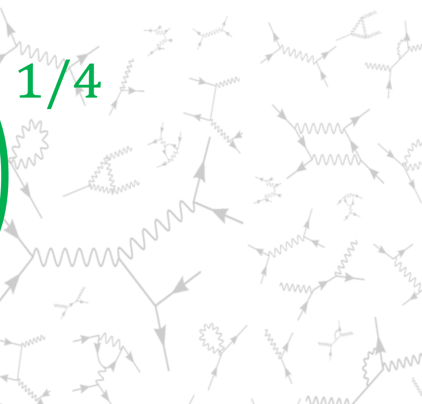
Inflaton + SM in equilibrium

dark sector(s) out of equilibrium

g_*



$$\xi_i \equiv \frac{T_i}{T} \approx 0.23\left(\frac{g_i\, m_\phi}{g_{i*} M_{pl}}\right)^{1/4} \ll 1$$



T_i $\quad g_{i*}$

$T \sim O(10)$ MeV $- O(100)$ TeV (possible dark sector confinement)

(Hertzberg, Sandora 1908.09841)

$T \sim O(1)\, MeV$ (BBN)

$\tilde{g}_*$ $\quad \tilde{g}_{i*}$

$$\Delta N_{eff} = \frac{4}{7}\sum_i \xi_i^4 \tilde{g}_{i*}\left(\frac{g_{i*}\,\tilde{g}_*}{\tilde{g}_{i*}\, g_*}\right)^{4/3} < 0.3$$

$MR\ equality \sim CMB$

$Today$ $\quad \Omega_d \approx 0.26\ (easily\ allowed)$

SUMMARY

- Took SM seriously to high energies because it is allowed by Unitarity.
- Instability in the Higgs at high energies. Dangerous during inflationary and post inflationary (preheating) eras. Went beyond Gaussian approximation + eternal inflation.
- Hint for new physics, especially when looked in broader setting of dark matter, BBN, CMB etc.

  Presented a 'natural' model to explain the dominance of visible sector during early eras, avoids catastrophes during inflation and post-inflation eras, leaving enough room for dark matter, baryogenesis.

BACKUP SLIDES

Langevin $\leftrightarrow$ Fokker Planck

$$\frac{d\vec{\varphi}}{dN} + \frac{1}{DH^2}\frac{\partial V}{\partial \vec{\varphi}} = \kappa \vec{\eta}_N$$

$$\frac{\partial p}{\partial N} = \frac{1}{DH^2}\frac{\partial}{\partial \vec{\varphi}}\left[\frac{\partial V}{\partial \vec{\varphi}} p\right] + \frac{\kappa^2}{2}\frac{\partial^2 p}{\partial \vec{\varphi}^2}$$

GAUSSIAN APPROXIMATION (?)

Ansatz

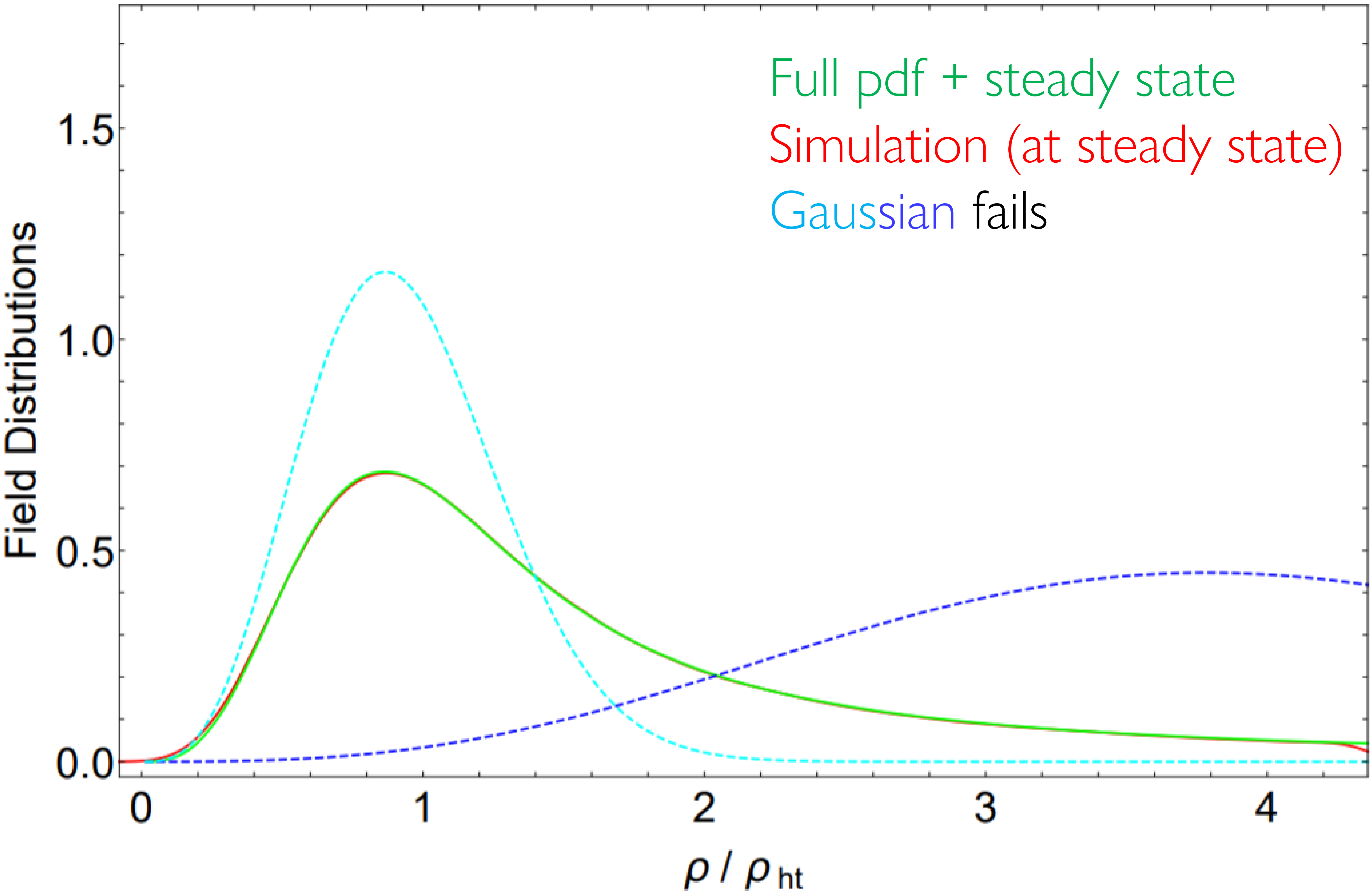
$$p(\vec{\varphi}, N) = \frac{1}{\sqrt{2\pi\sigma^2(N)}} e^{-\frac{\vec{\varphi}^2}{2\sigma^2(N)}}$$

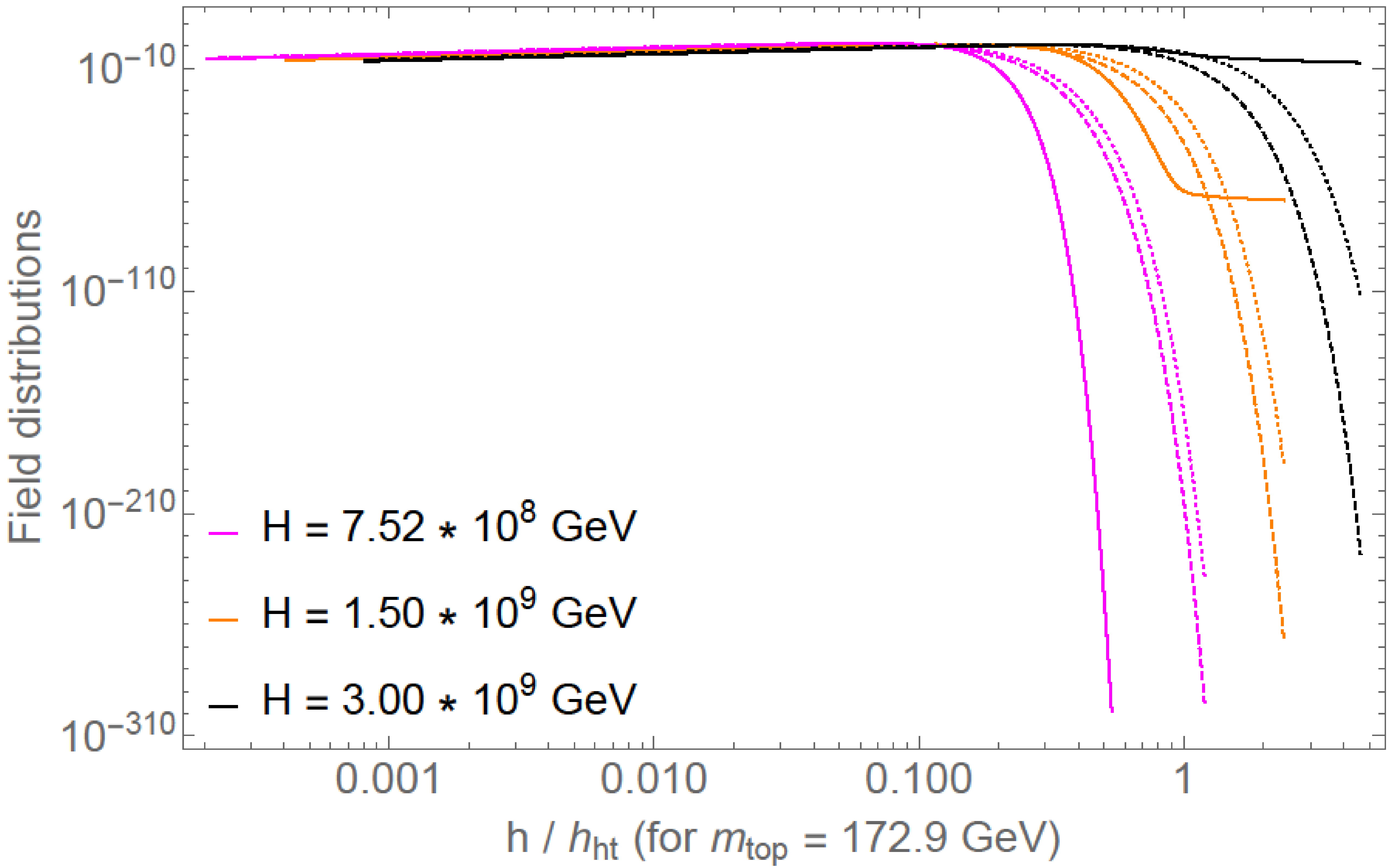
Fokker Planck

$$\frac{d}{dN}\sigma^2 + \frac{2}{DH^2}\left\langle \vec{\varphi}.\frac{\partial V}{\partial \vec{\varphi}} \right\rangle = \kappa^2$$

COMPARISON of DISTRIBUTIONS with 1D SIMULATIONS

$$H > H_{cr}^{(0)}$$





Field distributions
10^{-10}
10^{-110}
10^{-210}
10^{-310}
0.001
0.010
0.100
1
h / h_{ht} (for m_{top} = 172.9 GeV)
H = 7.52 * 10^8 GeV
H = 1.50 * 10^9 GeV
H = 3.00 * 10^9 GeV

A MEASURE OF FAST ROLL

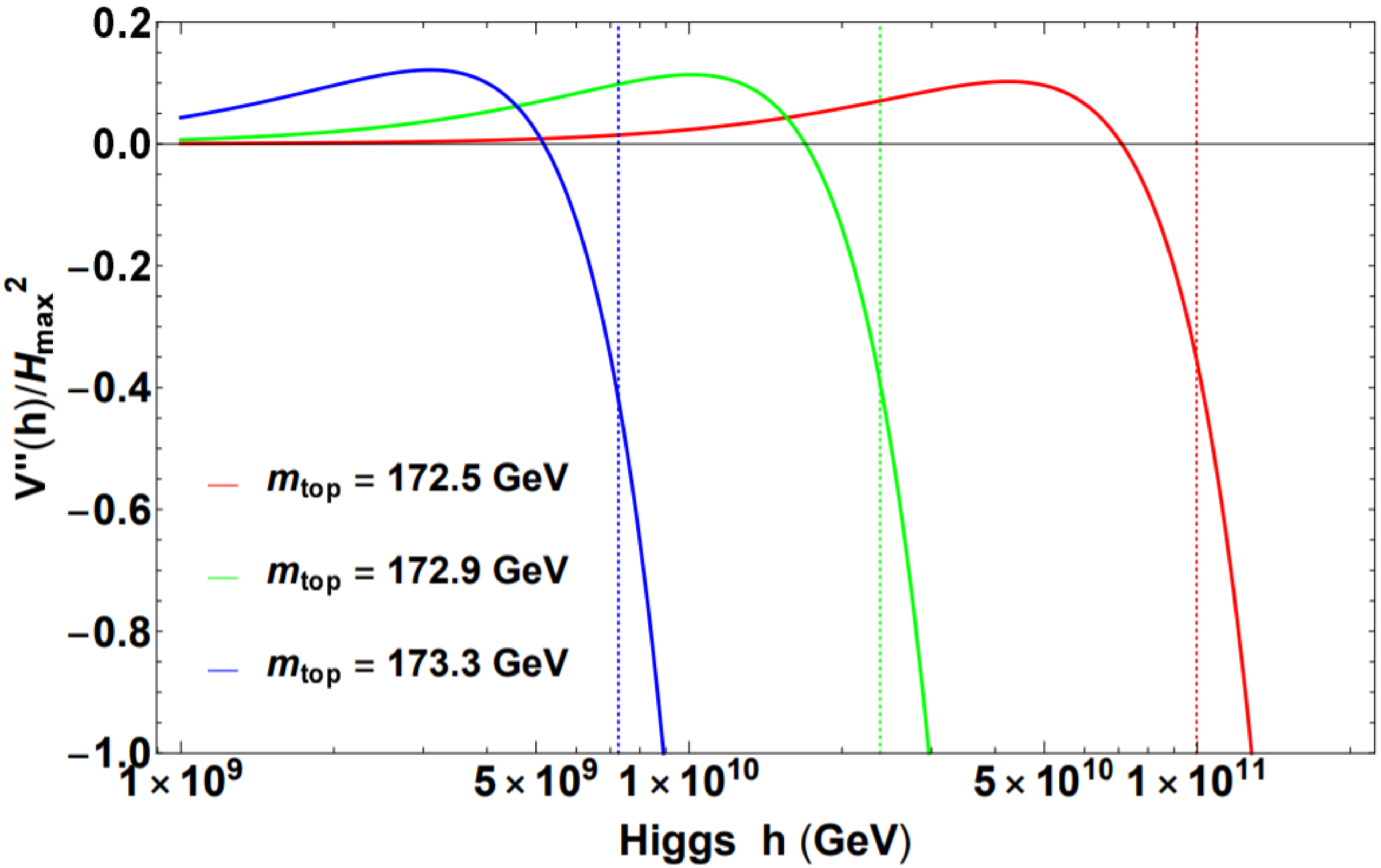
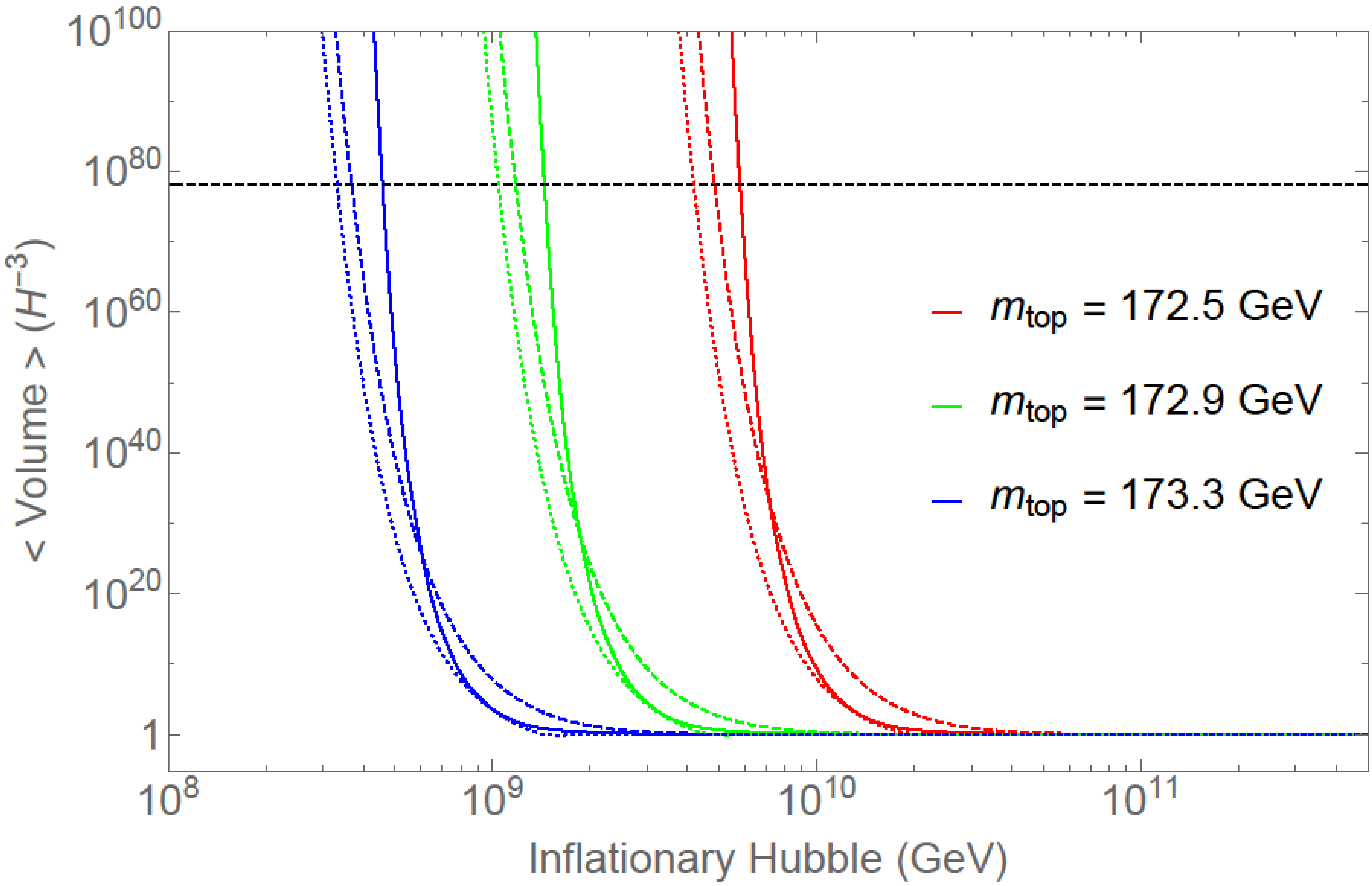


FIG. 9. A measure of the fast-roll $V''(h)/H^2$, with $H = H_{max}$ (the maximum H value to allow for a large observable universe). The dashed vertical lines are $h = h_{\rm ht}$ are the hill-top values for the Higgs. This plot shows that for $h > h_{\rm ht}$ the field is about to undergo fast-roll and we expect it to readily head towards an AdS crunch or other catastrophe. This is within the framework of the minimal SM in 3+1-dimensions for 3 different values of the top mass.



10^{100}
10^{80}
10^{60}
10^{40}
10^{20}
1
< Volume > (H^{-3})
10^{8}
10^{9}
10^{10}
10^{11}
Inflationary Hubble (GeV)
m_{top} = 172.5 GeV
m_{top} = 172.9 GeV
m_{top} = 173.3 GeV

Tree level potential function

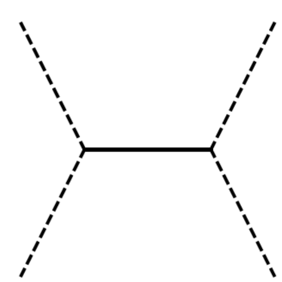
UV potential ⟶

$$V = \frac{m_\phi^2}{2}\phi^2 + \kappa\phi\left(H^\dagger H\right) + \lambda\left(H^\dagger H\right)^2 + \cdots$$

$$= \frac{1}{2}\left(m_\phi \phi + \frac{\kappa}{m_\phi}\left(H^\dagger H\right)\right)^2 + \left(\lambda - \frac{\kappa^2}{2m_\phi^2}\right)\left(H^\dagger H\right)^2 + \ldots$$

IR λ
(‘Effective’)

interactions



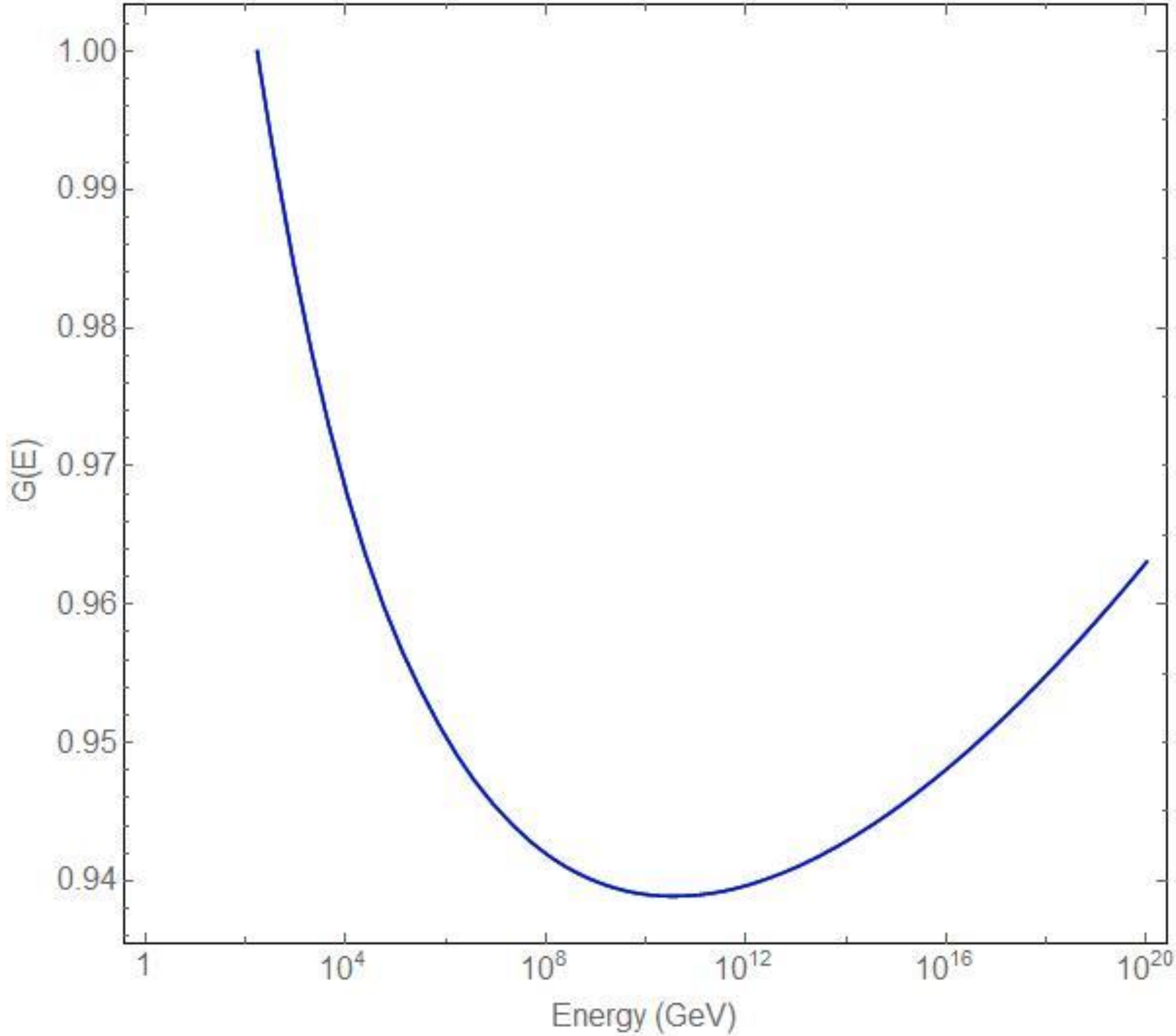
$$\Delta\mathcal{L} = -\kappa\phi H^\dagger H$$

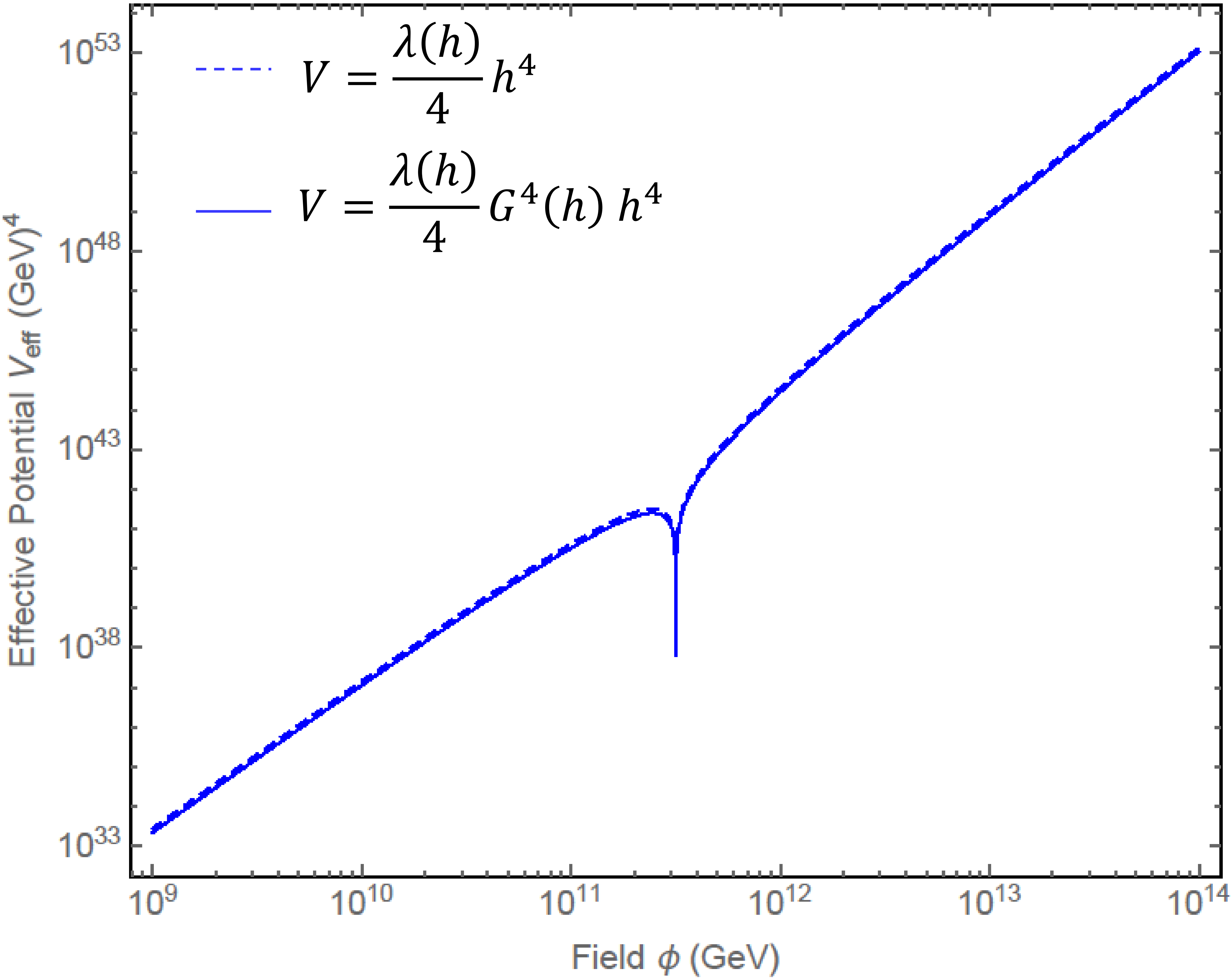
$$\beta_\lambda = \frac{d\lambda}{d\ln E} = \frac{\kappa^2 E^2}{\left(m_\phi^2 + E^2\right)^2}$$

$$+ \quad \beta_\lambda^{SM}$$

Loops

G(E ⟷ h): from field strength renormalization





$V = \frac{\lambda(h)}{4} h^4$
$V = \frac{\lambda(h)}{4} G^4(h)\, h^4$
Effective Potential V_{eff} (GeV)^4
10^{53}
10^{48}
10^{43}
10^{38}
10^{33}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
Field ϕ (GeV)