



Bounding the relative profitability of price discrimination

David A. Malueg^a, Christopher M. Snyder^{b,*}

^a *Department of Economics and the A. B. Freeman School of Business, Tulane University, 206 Tilton Hall, New Orleans, LA 70118, United States*

^b *Department of Economics, Dartmouth College, 301 Rockefeller Hall, Hanover, NH 03755, United States*

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Abstract

We derive bounds on the ratio of a monopolist's profit from third-degree price discrimination to that from uniform pricing. If the monopolist serves N independent markets, demand is continuous, and the cost function is superadditive, then the profit ratio is bounded by N . A linear-demand example is provided coming arbitrarily close to this bound. We provide examples showing the profit ratio can be unboundedly large when marginal cost is decreasing, demand is discontinuous, or fixed cost is positive. If the monopolist has access to certain demand-rationing strategies under uniform pricing, we can bound the profit ratio even for discontinuous demand functions and multiproduct cost functions.

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1. Introduction

How valuable is the ability to price discriminate? This question is central, for example, to the debate on parallel imports and international exhaustion of intellectual property rights. If allowing parallel imports eliminates price discrimination across countries, even consumers who benefit from a lower price may ultimately lose through a reduction in firms' ex ante investment

* Corresponding author.

E-mail addresses: dmalueg@tulane.edu (D.A. Malueg), Christopher.M.Snyder@dartmouth.edu (C.M. Snyder).

incentives.¹ The size of this incentive effect depends on the relative profitability of price discrimination.

Recently, the issue of parallel imports has received particular attention in the international market for pharmaceuticals (see [Danzon and Towse, 2003](#)). In order to preserve low prices in poor countries, the European Union recently tightened restrictions on the re-importation on malaria, tuberculosis, and HIV/AIDS drugs from poor countries, enhancing pharmaceutical manufacturers' ability to engage in price discrimination across rich and poor countries. Recent U.S. policy moved in the opposite direction: by relaxing restrictions on the re-importation of pharmaceuticals from Canada, the United States reduced pharmaceutical manufacturers' ability to engage in price discrimination across the two countries. These policies were motivated by a desire to reduce prices in certain markets with little regard to what [Danzon and Towse \(2003\)](#) note may be potentially significant effects on firm's incentives to undertake pharmaceutical research and development.² Our results on the relative profitability of price discrimination will provide bounds on the effect of such policies on firms' ex ante incentives to discover and develop pharmaceuticals as a function of the number of markets (countries) involved.

To date, the economics literature has been silent on the profitability of price discrimination, focusing instead on the effect of price discrimination on static social welfare (defined as consumer surplus plus producer surplus). We address this gap in the literature by examining the effect of third-degree price discrimination on a monopolist's profit. Typically it is of little interest to ask *whether* third-degree price discrimination increases a monopolist's profit—because all price vectors feasible under uniform pricing are also feasible under discrimination, the monopolist's profit under price discrimination is weakly higher than under uniform pricing.³ In this paper we ask *how much* price discrimination can increase a monopolist's profit.

We show that if a monopolist faces N independent markets, demand is continuous, and the cost function is superadditive (related to diseconomies of scale), then the profit ratio is bounded by N . In cases where under price discrimination some markets are not served or are charged equal prices, we provide the tight bound that is strictly less than N . If the preceding conditions on demand and cost do not hold, the ratio of profit under third-degree price discrimination to profit under uniform pricing can be arbitrarily large, as we demonstrate in a series of examples.

Our results relate to the public policy question of the effect of parallel importation of drugs and other goods on firms' ex ante investment incentives, as mentioned above. Our results have other practical implications for firm strategy and public policy. Firms' incentives to facilitate third-degree price discrimination by developing technologies to separate consumers into different markets and prevent arbitrage among them depend on the profitability of price discrimination. For example, [Odlyzko \(2004\)](#) notes the increasing complexity of price discrimination schemes in transportation, citing among other examples the evolution of canal tolls from simple uniform fees per boat for early canals in England to fees which varied not only

¹ [Rey \(2003\)](#) and [Valletti and Szymanski \(2004\)](#), for example, model the effect of parallel imports on manufacturers' investments in quality.

² For the development of new drugs, research is essential—and it does not come cheap. The average research and development expenditure on a new drug is an estimated \$400 million out-of-pocket, capitalized to \$800 million (2000 U.S. dollars) by the time of government approval ([DiMasi et al., 2003](#)).

³ In departures from a static model with passive final-good consumers—the model studied in this paper—uniform pricing may be more profitable than price discrimination. For example, a durable-good monopolist may benefit if it can commit to a constant price over time ([Coase, 1972](#)). For another example, an upstream firm that sells inputs to competing downstream firms using secret contracts may benefit from establishing a reputation for offering uniform contracts ([McAfee and Schwartz, 1994](#)).

by volume and weight of cargo but also by the type of commodity shipped and even the intended end use of the commodity for later canals in England and the United States. Shippers took various measures to evade the tolls including hiding high-toll commodities under low-toll ones; canal operators took various countermeasures including the use of books “listing canal boats, and the weight of cargo aboard as a function of how deeply in the water they lay” (Odlyzko, 2004, p. 331).

Our results also add to economists’ general theoretical understanding of price discrimination. To our knowledge, our paper is the first to derive bounds on the profitability of price discrimination relative to uniform pricing. Previous work has focused on conditions under which third-degree price discrimination decreases social welfare. Robinson (1933) showed that, if under uniform pricing a monopolist serves two independent markets with linear demands, allowing third-degree price discrimination leaves total output unchanged and therefore reduces welfare if discriminatory prices are different, because marginal benefits of consumption are not equalized across markets. Subsequent authors generalized this result. Schmalensee (1981) showed that for a monopolist with constant marginal cost facing independent demands, third-degree price discrimination raises social welfare only if it increases total output. Varian (1985) generalized Schmalensee’s result to allow for interdependent demands and nondecreasing marginal cost, while Schwartz (1990) allowed for interdependent demands and any cost function that depends only on total output. Motivated by the prescriptions of U.S. antitrust law, which applies to price discrimination in the sale of intermediate goods, Katz (1987), DeGraba (1990), Yoshida (2000), and O’Brien (2003) studied discrimination by a monopoly producer of an input used by (possibly) competing downstream firms. Collectively, these authors showed that allowing discrimination has an ambiguous effect on input prices. Our paper is closest in spirit to Malueg (1993) and Armstrong (1999). Malueg (1993) provided quantitative bounds on the social welfare given particular restrictions on market demands. As emphasized above, the present paper differs because our bounds are on relative profit rather than relative social welfare. Armstrong (1999) bounded the profitability of simple two-part tariffs relative to optimal nonlinear tariffs charged by a multiproduct monopolist. He showed that the profit from simple contracts converges to that from optimal contracts as the number of goods grows. The present paper differs because we study third-degree price discrimination rather than second-degree. The nature of our results is quite different as well: profit from the suboptimal pricing scheme (uniform pricing) does not necessarily converge to profit from the optimal one (third-degree price discrimination); rather, we show the gap can grow without bound as the number of markets grows. The proof that our bound is tight for linear demands relies on some novel analytical arguments.

The remainder of this paper is organized as follows. Section 2 bounds the relative profitability of price discrimination in the benchmark case, in which the monopolist is unable to ration demand. That is, the monopolist must serve all demand at price it sets. In Section 3, we argue that in many real-world markets the monopolist may easily be able to ration demand under uniform pricing. We thus turn to bounding the relative profitability of price discrimination under various plausible rationing assumptions. If the monopolist can ration demand by allowing itself to stock out, we show that the bounds provided in Section 2 hold even if demand is discontinuous. If the monopolist can ration demand by picking and choosing the markets in which it wishes to sell, then the bounds on the relative profitability of price discrimination can be extended to the case of a multiproduct monopolist whose costs are a function of the vector of market outputs rather than the sum of market outputs. This last extension has useful applications; for example, it covers the case in which a monopolist produces a homogeneous good in a central

plant and then incurs different transportation costs for delivering the good to different markets. Section 4 concludes.

2. Relative profits without rationing

In this section, we provide bounds on the relative profitability of price discrimination in the benchmark case in which the monopolist is assumed to be unable to ration demand. That is, the monopolist must serve all demanders at the chosen price. It turns out that the ability to ration demand can increase the profitability of uniform pricing. The reasons why this is so are subtle, so we defer a discussion of rationing to Section 3. We start with the case of no rationing because it is a natural benchmark and is good for building intuition.

2.1. The model

Suppose a monopolist faces N markets, with the demand function in market $i=1, \dots, N$ given by $q_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$; $q_i(p_i)$ denotes the quantity demanded in market i at the per-unit price p_i . Each q_i is a nonincreasing function of price. The markets are independent in that the quantity demanded in market i depends on the price of the good in market i and not the monopolist's price in any other market. Assume resale is impossible, thus ruling out arbitrage between markets. Aggregate demand at a uniform price p is given by $Q(p) = \sum_{i=1}^N q_i(p)$. Throughout Section 2 we assume that the monopolist cannot ration demand, so that at a uniform price p the monopolist sells $Q(p)$ units.

The monopolist's total cost function is given by $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$; $C(Q)$ denotes total cost when total sales in the N markets equals Q .

Let Π^d denote the firm's maximum profit under third-degree price discrimination:

$$\Pi^d \equiv \max_{p_1, \dots, p_N} \left\{ \sum_{i=1}^N p_i q_i(p_i) - C \left(\sum_{i=1}^N q_i(p_i) \right) \right\}. \quad (1)$$

To rule out uninteresting cases, we assume $\Pi^d > 0$. Let Π^u denote the firm's maximum profit under uniform pricing:

$$\Pi^u \equiv \max_p \{ pQ(p) - C(Q(p)) \}. \quad (2)$$

Let Π_i^s denote the monopolist's stand-alone profit in market i , that is, the maximum profit it could earn in market i if (counterfactually) there were no other markets:

$$\Pi_i^s \equiv \max_p \{ p q_i(p) - C(q_i(p)) \}. \quad (3)$$

We assume demands and cost are such that there exists a solution to each of the maximization problems (1), (2), and (3). Our analysis does not require these solutions to be unique.

2.2. Profit bound

Given certain conditions, we can bound the profit ratio Π^d / Π^u by the number of markets, N . In fact, we will be able to obtain an even tighter bound, N^* , equal to the minimum number of distinct prices that the monopolist charges in markets that are served under price discrimination. The difference between the bounds N^* and N is a somewhat technical point, the discussion of

which is deferred until after Proposition 1. It will suffice to note now that $N^* = N$ in many cases, and so in many cases there will be no distinction between the two bounds; there will only be a distinction between the two if, under price discrimination, two or more markets happen to be charged the same price or some markets happen not to be served.

The intuition for this bound is clear in the special case in which marginal cost is constant, there is no fixed cost, and discrimination requires each market to be served at a distinct price. In this case, $N^* = N$, and the notion of profit in an individual market is well-defined, implying, in particular, that one can identify the most profitable market under price discrimination. The monopolist could set the uniform price to equal the price in the most profitable market under price discrimination. It follows that Π^u is at least as great as the profit in the most profitable market under discrimination, which in turn is at least as great as the average discriminatory profit across the markets, Π^d/N ; i.e., $\Pi^u \geq \Pi^d/N$.

The arguments from the previous paragraph can be extended with some modification to the case of diseconomies of scale (more precisely, superadditive costs). If the uniform-pricing monopolist tried to mimic the price charged on the market with the greatest stand-alone profit, other markets might also have some demand at this price, and output would thus be greater than on the stand-alone market. In the presence of diseconomies of scale, it might be very unprofitable to serve this extra demand. However, if demand is continuous, the monopolist can raise the uniform price to a point at which the quantity sold under uniform pricing equals quantity sold on the market with the greatest stand-alone profit. The profit under this uniform price is at least as great as the profit in the most profitable stand-alone market because quantity is the same, cost is the same, and price is no lower. The profit under uniform pricing is thus at least as great as the average profit across stand-alone markets. With diseconomies of scale, the average profit across stand-alone markets exceeds the average discriminatory profit Π^d/N because the cost of serving all markets simultaneously (as is done under price discrimination) is greater than the cost of serving them independently (as is done in the stand-alone problem). Hence, $\Pi^u \geq \left(\sum_{j=1}^N \Pi_j^s \right) / N \geq \Pi^d/N$.

The argument does not extend to the case of economies of scale. Mimicking the market with the greatest stand-alone profit under uniform pricing need no longer generate the average discriminatory profit across markets, Π^d/N , because the cost of serving all markets simultaneously (as is done under price discrimination) is less than the cost of serving them independently (as is done in the stand-alone problem) if there are economies of scale. It is easy to see that the profit ratio Π^d/Π^u can be made arbitrarily large when economies of scale are present in the form of a positive fixed cost. The denominator of the profit ratio can be forced to zero by having the fixed cost approach the producer surplus under uniform pricing.⁴

Even if fixed cost is zero, the profit ratio can be made arbitrarily large when economies of scale are present in the form of decreasing marginal costs. Consider an example with two markets, the first with unit demand at a reservation price of 1 and the second with unit demand at a reservation price of v , $v < 1/2$.⁵ The monopolist's cost function is $C(Q) = c \cdot \min\{Q, 1\}$. In this case, $\Pi^d = 1 + v - c$, as the monopolist sets price in each market equal to the reservation value there. Under uniform pricing the monopolist will set the price to either 1 or v . At a price of 1,

⁴ If fixed cost is positive, the bound from our main result, Proposition 1 below, applies to the producer-surplus ratio if the variable-cost function satisfies the superadditivity condition.

⁵ Our main result, Proposition 1 below, requires demand to be continuous, but demand is discontinuous in this example. The particular demands were chosen to simplify the calculations. The discontinuities do not drive the result in the example: we could derive the same result using demand curves that were slightly modified to remove discontinuities.

profit is $1 - c$, which exceeds the profit of $2v - c$ at a price of v because it was assumed that $v < 1/2$. Consequently, $\Pi^u = 1 - c$. Therefore,

$$\frac{\Pi^d}{\Pi^u} = \frac{1 + v - c}{1 - c},$$

which can be made arbitrarily large by taking c sufficiently close to 1.

As the example suggests, generalizing the bound on relative profits beyond the case of constant marginal cost will require some notion of diseconomies of scale, superadditive costs to be precise.

Definition. $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is superadditive if $C(Q') + C(Q'') \leq C(Q' + Q'')$ for all $Q', Q'' \in \mathbb{R}_+$.

It can be shown that if C is convex and there is no fixed cost, then the cost function is superadditive. In general, a superadditive cost function must have a fixed cost of zero,⁶ but need not be continuous or convex.⁷

The following proposition, proved in the Appendix, is the main result of Section 2.

Proposition 1. *If $Q(\cdot)$ is continuous and $C(\cdot)$ is superadditive, then $\Pi^d / \Pi^u \leq N^*$.*

We provided intuition for the proof previously. The remaining loose end to tie up is that the bound in the proposition is in terms of N^* , the number of distinct prices used in the markets that are served under price discrimination, rather than N , the number of markets. First, suppose two markets happen to be charged the same price under price discrimination. Then the two markets can be combined into a single market without affecting the profit under either price discrimination or uniform pricing. But combining the two markets reduces the total number of markets to $N - 1$. Thus profit under discrimination can be at most $N - 1$ times that under uniform pricing. Arguments along these lines establish that the number of distinct prices provides a tighter bound on relative profitability than the number of markets. Second, suppose that a market is not served under price discrimination. Eliminating that market would leave profit under price discrimination unchanged. But then profit under discrimination could be at most $N - 1$ times the profit under uniform pricing on the $N - 1$ remaining markets, which is weakly less than the profit under uniform pricing on the original N markets if demand is continuous. Arguments along these lines establish that the number of markets that are served under price discrimination provides a tighter bound on relative profitability than the total number of markets.

We demonstrated in an earlier example that the condition on the cost function (superadditivity) is necessary to bound relative profits in Proposition 1. We can show that continuity of demand is also needed in Proposition 1 by providing an example with discontinuous demand in which the profit ratio can be arbitrarily large. The example is illustrated in Fig. 1. The first market has unit demand at a reservation price of 1. The second has a discontinuity at a price of 1, jumping from $1/v^2$ to \tilde{q} , where $\tilde{q} > 1$ and $v > 1$. Assume marginal cost equals zero for output up to $1 + 1/v^2$ and equals v for output above $1 + 1/v^2$. A price-discriminating monopolist would set

⁶ Superadditivity implies $C(0) + C(0) \leq C(0 + 0) = C(0)$. Therefore, $C(0) \leq 0$, which with nonnegativity of C implies $C(0) = 0$.

⁷ The following superadditive cost function is discontinuous and has strictly decreasing marginal cost for $Q > 1$:

$$C(Q) = \begin{cases} 0 & Q < 1 \\ 2Q - \frac{1}{Q} & Q \geq 1. \end{cases}$$

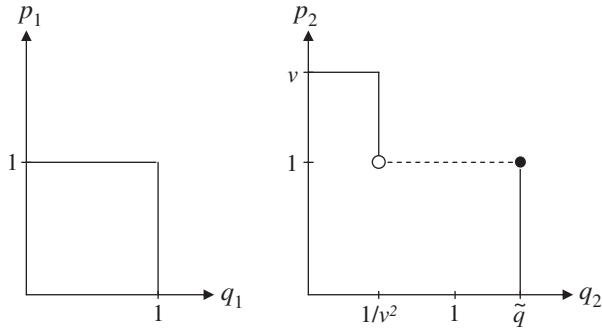


Fig. 1. Example with discontinuous demand.

a price of 1 in market 1 and v in market 2, yielding $\Pi^d = 1 + v(1/v^2) = 1 + 1/v$. The optimal uniform price is either v , yielding profit $v(1/v^2) = 1/v$, or 1, yielding profit $1 + \tilde{q} - v[\tilde{q} - (1 + 1/v^2)]$. For sufficiently large v , the latter profit is negative, in which case the optimal uniform price is v and profit is $\Pi^u = 1/v$. Therefore, $\lim_{v \uparrow \infty} \Pi^d / \Pi^u = \lim_{v \uparrow \infty} (1 + 1/v) / (1/v) = \infty$.

Proposition 1’s bound on the profit ratio stands in stark contrast to the behavior of the social-welfare bound. Examples with two independent markets and constant marginal cost show that the ratio of social welfare under third-degree price discrimination to social welfare under uniform pricing can range from zero to infinity (Malueg, 1993, Examples 1 and 2).

2.3. Tightness of the bound

We next prove the bound in Proposition 1 is tight. The proof is constructive: we construct demands in each of the N markets such that the profit under price discrimination is arbitrarily close to N times the profit under uniform pricing. Since our construction involves linear demands and costless production, the proposition has the interesting implication that, even in the textbook case of linear demands and constant marginal costs, profit under price discrimination can be considerably higher, nearly N times higher, under price discrimination than under uniform pricing. The proof of Proposition 2 is provided in the Appendix.

Proposition 2. For every $\epsilon > 0$, an example can be constructed with linear demands in each of the N markets such that $\Pi^d > N\Pi^u - \epsilon$.

The logic of the proof can be seen in Fig. 2. Assume production is costless. For any demand curve $Q_N(p)$ that intersects the horizontal axis, a linear demand $q_{N+1}(p) = a - bp$ can be

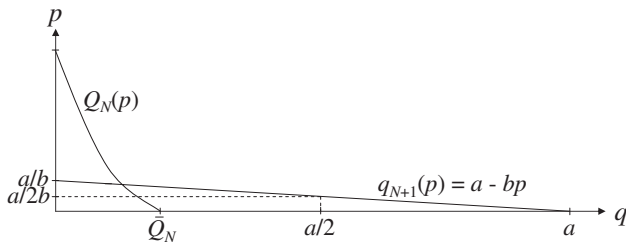


Fig. 2. Intuition for the proof of Proposition 2.

constructed such that the stand-alone profit from serving demand $q_{N+1}(p)$ is arbitrarily close to the stand-alone profit from serving demand $Q_N(p)$. If $q_{N+1}(p)$ is made sufficiently flat relative to $Q_N(p)$ —which can be accomplished by making the vertical intercept a/b sufficiently low and the horizontal intercept a sufficiently high—then the stand-alone profit from serving demand $q_{N+1}(p)$ can be made arbitrarily close to the profit from serving both markets under uniform pricing. As $q_{N+1}(p)$ is flattened, the optimal uniform price charged to both markets becomes so low that sales to the $Q_N(p)$ market, which as the figure shows can be no more than \bar{Q}_N , provides a negligible contribution to the uniform profit beyond the extremely high sales to the $q_{N+1}(p)$ market. Taking $Q_N(p)$ to be the aggregation of N linear demand curves, we can assume as our inductive hypothesis that profit from serving the N markets under price discrimination is nearly N times the profit under uniform pricing. (This is trivially true for $N=1$.) It follows that the profit from serving the $N+1$ independent markets (the N markets represented by $Q_N(p)$ along with market $q_{N+1}(p)$) under price discrimination is arbitrarily close to $N+1$ times the profit under uniform pricing, establishing our proof by induction.

3. Relative profits with rationing

In this section, we extend the bound on the relative profitability of price discrimination derived in Proposition 1 to a wider range of cases by allowing the monopolist to have at least a limited ability to ration demand. Rationing demand adds an extra degree of control that can increase the profitability of uniform pricing. Rationing can increase the profitability of uniform pricing enough that bounds can be established in some cases where otherwise price discrimination may have been unboundedly more profitable without rationing.

3.1. Rationing by stocking out

The first type of rationing we consider we call “stocking out.” This type of rationing allows the monopolist to sell any amount \bar{Q} less than or equal to the quantity demanded at a given uniform price $Q(p)$. In many settings, there would be little to prevent a monopolist from engaging in this sort of rationing if it so desired. It can simply produce \bar{Q} , sell this many units, and then refrain from producing more to serve the remaining demand.⁸ Such rationing has been implicitly assumed in a number of influential industrial-organization studies.⁹

If the monopolist can ration by stocking out, then the bound on the relative profitability of price discrimination, which we showed in Proposition 1 holds for continuous demand, also holds for discontinuous demand. The potential problem raised by discontinuous demand is that a jump in demand may put the uniform-pricing monopolist in a range of outputs for which its cost function is sharply increasing, thus reducing the profitability of uniform pricing. This problem can be avoided if the monopolist is allowed to stock out before reaching the costly range of outputs.

The following proposition, proved in the Appendix, essentially repeats the statement of Proposition 1 but substitutes demand rationing for demand continuity.

⁸ Rationing demand by stocking out would be made more difficult if consumers’ anticipation of the possibility caused them to reduce their demand, say because of unmodeled costs of traveling to the store, or if there were a legal requirement to issue rain checks.

⁹ Perhaps most notably, in contestability theory (Baumol et al., 1982), to break a sustainable industry configuration, an entrant is not required to satisfy all demand at the undercutting price it chooses.

Proposition 3. *Suppose the monopolist can ration demand by stocking out. If $C(\cdot)$ is superadditive, then $\Pi^d / \Pi^u \leq N^*$.*

It is immediate from Proposition 2 that the bound in Proposition 3 is tight.

3.2. Rationing by bypassing markets

The second type of rationing we consider we will call “bypassing markets.” With this type of rationing, the monopolist can decide simply not to serve certain markets (we will not assume it can ration demand in markets in which it sells a positive quantity). This sort of rationing could be implemented by setting up distribution networks only in those markets the monopolist would like to serve. If the monopolist can ration demand by bypassing markets we can extend our bound on the relative profitability of price discrimination to the case of a multiproduct cost function, that is, a function of the vector of market outputs rather than the sum. The potential problem created by the monopolist’s cost being a function of the vector of market outputs is that there may be some markets that are exceedingly costly to serve, but there would otherwise be no way to avoid serving such markets under uniform pricing unless the monopolist could bypass them. Under price discrimination, by contrast, there is no such problem since the monopolist can control how much is sold in any market by setting a sufficiently high price in it. For example, suppose a pizza monopolist charged a uniform delivered price to both customers living on a mountain and those living on a plain. It may be extremely expensive to deliver pizza to the mountain, but under uniform pricing with no rationing, the monopolist would be forced to serve mountain customers if they called for a pizza. The monopolist might be able to increase profit simply by refusing to serve mountain customers.

The leading case we have in mind in this section is indeed that of third-degree price discrimination involving geographically distinct markets, where it is likely that transportation costs differ across markets.¹⁰ If transportation costs to each market are common and linear in output, then the results of Proposition 1 apply—the transportation costs can simply be added to the production cost function. However, if the transportation costs are nonlinear or differ across destinations, then total costs depend on the distribution of output across markets. In such a case, Proposition 1 does not apply, requiring a new proposition, which we provide below, to bound the relative profitability of price discrimination.

Let $C : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$ denote the monopolist’s cost function, with $C(q_1, \dots, q_N)$ denoting the total cost of output vector (q_1, \dots, q_N) . This is a standard multiproduct cost function. Such a cost function captures the case in which the monopolist sells different goods on the different markets. It also captures the case in which the same product is sold but the costs depend on sales in individual markets perhaps because, as suggested in the previous paragraph, transportation costs

¹⁰ One of the motivations for studying uniform pricing in this paper is that there may be legal rules against price discrimination. Charging different delivered prices across geographically distinct markets might not violate such rules if the prices reflect the underlying transportation cost differences. Still, authorities might require uniform delivered prices to avoid creating a loophole that would allow firms to price discriminate freely. Also, firms might set uniform delivered prices in order to avoid having to prove to authorities that price differences are cost-justified. These considerations motivate our study of uniform delivered prices in the present section. Firms might respond to a requirement that delivered prices be uniform by charging a uniform FOB price and having consumers transport the good themselves, but this strategy may be infeasible if consumer transport costs are high. Furthermore, our earlier analysis in Section 2 can capture the case in which the monopolist only considers FOB pricing—under both discrimination and uniform pricing—with a suitable reinterpretation of demands to reflect transportation costs borne by consumers.

differ across markets. In this setting, we will redefine the functions Π^d , Π^u , and Π_i^s in the obvious way.

Formally, the monopolist is able to ration demand by bypassing markets if, under uniform pricing at price any p , it can restrict the quantity it sells on individual markets i to $\bar{q}_i \in \{0, q_i(p)\}$, for $i=1, \dots, N$.¹¹ An example serves to show that it is necessary to assume the monopolist can ration demand by bypassing markets to extend the bound on the profit ratio to multiproduct cost functions. Recall the example introduced in Section 2 in which there were two markets, the first of which had unit demand at a reservation price of 1 and the second of which had unit demand at a reservation price of v , $v < 1/2$. Reinterpret the assumed cost function as a multiproduct cost function $C(q_1, q_2) = cq_1$; i.e., the marginal cost in market 1 is c and in market 2 is zero. Calculations identical to those in Section 2 show that Π^d/Π^u is unbounded as $c \uparrow 1$. The problem arising with uniform pricing in this example is that if consumers cannot be rationed, then the monopolist must serve the high-cost market (market 1) along with the low-cost market (market 2). Excluding market 1 would allow the monopolist to serve the low-cost market alone and earn profit approaching that under discrimination.

A side benefit of assuming that the monopolist can ration demand by bypassing markets is that it allows us to relax the condition on cost in Proposition 1, replacing superadditivity with a condition involving diseconomies of scope. We first must define these cost concepts in a multiproduct-cost-function setting. Let $\mathbf{e}_i \in \mathbb{R}^N$ denote the vector having a one in component i and zeros everywhere else.

Definition. $C : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$ exhibits weak market-by-market diseconomies of scope if, for all vectors of market output $(q_1, \dots, q_N) \in \mathbb{R}_+^N$ and all $j=1, \dots, N$,

$$C(q_j \mathbf{e}_j) + C\left(\sum_{i \neq j} q_i \mathbf{e}_i\right) \leq C\left(\sum_{i=1}^N q_i \mathbf{e}_i\right)$$

Informally, the definition says that the total cost of producing (q_1, \dots, q_N) weakly exceeds the cost of producing q_j alone plus the cost of producing all outputs in (q_1, \dots, q_N) but q_j . This definition is a bit weaker than standard diseconomies of scope, which requires costs to rise at least weakly when any partition of the N markets is combined together. Weak market-by-market diseconomies of scope involves only one class of partitions, those dividing one market from the rest of the $N - 1$ markets.

We next extend our earlier definition of superadditivity to the multiproduct case in the obvious way.

Definition. $C : \mathbb{R}_+^N \rightarrow \mathbb{R}_+$ is superadditive if $C(\mathbf{Q}') + C(\mathbf{Q}'') \leq C(\mathbf{Q}' + \mathbf{Q}'')$ for all $\mathbf{Q}', \mathbf{Q}'' \in \mathbb{R}_+^N$.

Superadditivity clearly implies weak market-by-market diseconomies of scope, but the converse is not true. To see this, consider the cost function $C(q_1, q_2) = \sqrt{q_1} + \sqrt{q_2} + q_1 q_2$. Because $C(q_1, 0) + C(0, q_2) = \sqrt{q_1} + \sqrt{q_2} \leq C(q_1, q_2)$, this function exhibits weak market-by-market diseconomies of scope. It is not everywhere superadditive; for example, $C(1, 0) + C(1, 0) = 2 > \sqrt{2} = C(2, 0)$.

¹¹ The strength of rationing by bypassing markets is not directly comparable to rationing by stocking out assumed in the previous section. Rationing by bypassing markets is a stronger concept in that it deals with individual markets. It is weaker concept in that the monopolist cannot finely tune market output but can only choose to serve market demand or not serve the market at all.

The first proposition in this section is a general result about profit bounds for multiproduct cost functions.

Proposition 4. *Suppose the monopolist can ration demand by bypassing markets. If the cost function exhibits weak market-by-market diseconomies of scope, then $\Pi^d / \Pi^u \leq N^*$.*

The proof is sketched in the Appendix. The logic behind the proof is similar to that behind the proofs of Propositions 1 and 3. It is immediate from Proposition 2 that the bound in Proposition 4 is tight.

We next use Proposition 4 to prove a corollary that covers the application that motivated this section, namely, third-degree price discrimination across geographic markets with heterogeneous transportation costs.

Proposition 5. *Suppose costs are given by*

$$C(q_1, \dots, q_N) = \hat{C} \left(\sum_{i=1}^N q_i \right) + \sum_{i=1}^N t_i(q_i), \quad (4)$$

where $\hat{C} \left(\sum_{i=1}^N q_i \right)$ is the cost of producing output $\sum_{i=1}^N q_i$ in a central plant and where $t_i(q_i)$ is the cost of transporting output q_i from the plant to market i . Suppose $\hat{C}(\cdot)$ is superadditive and $t_i(0) = 0$ for $i = 1, \dots, N$. Suppose the monopolist can ration demand by bypassing markets. Then $\Pi^d / \Pi^u \leq N^*$.

The proof, provided in the Appendix, shows that the conditions on \hat{C} and t_1 through t_N imply that C exhibits weak market-by-market diseconomies of scope. Proposition 4 thus applies, implying $\Pi^d / \Pi^u \leq N^*$.

4. Conclusion

We have shown that if a monopolist facing N independent markets has a superadditive cost function, then profit under third-degree price discrimination cannot exceed N times the profit under uniform pricing. Indeed, we derived an even tighter bound: N^* , the minimum number of distinct prices needed for the markets that are served under price discrimination. N^* is lower than N to the extent some of the N markets are not served, and to the extent discriminatory prices happen to be the same across several markets. We provided an example with linear demands showing the bound is tight. We provided further examples showing that the assumptions of superadditivity and continuity of demand are generally required for the bound.

In Section 3, we showed that the bounds derived in Section 2 hold under a broader set of conditions if the monopolist is assumed to have some ability to ration demand. If the monopolist can ration demand by stocking out, that is, electing to sell only a fraction of the quantity demanded at the chosen uniform price, then the assumption on the continuity of demand is no longer needed to bound the relative profitability of price discrimination. If the monopolist can ration demand by bypassing markets, that is, being able to choose which markets it serves and which not, then the bound on the relative profitability of price discrimination can be extended to the case of a multiproduct cost function. The bound holds under a condition related to diseconomies of scope, slightly weaker than superadditivity of the cost function.

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Appendix

Proof of Proposition 1. Let (p_1^d, \dots, p_N^d) be a solution to Eq. (1) having N^* distinct finite prices. Let $(\tilde{p}_1^d, \dots, \tilde{p}_{N^*}^d)$ be the distinct finite prices from (p_1^d, \dots, p_N^d) . For $i = 1, \dots, N^*$, define

$$S_i \equiv \{j = 1, \dots, N \mid p_j^d = \tilde{p}_i^d\}$$

and

$$\tilde{q}_i(p) \equiv \sum_{j \in S_i} q_j(p).$$

Thus, under third-degree price discrimination, all markets in S_i face the same price; \tilde{q}_i is the aggregate demand of this subset of markets. Define $\tilde{Q}(p) \equiv \sum_{i=1}^{N^*} \tilde{q}_i(p)$. Let $\tilde{\Pi}^u$ and $\tilde{\Pi}_i^s$ denote the profit values associated with problems (2) and (3), given demands $\tilde{q}_1, \dots, \tilde{q}_{N^*}$ and cost function C .

We next establish that

$$\Pi^u \geq \tilde{\Pi}^u \tag{5}$$

$$\geq \tilde{\Pi}_i^s. \tag{6}$$

Begin by observing that $\lim_{p \uparrow \infty} Q(p) = 0$; otherwise, profit under uniform pricing would be infinite, contradicting the assumption that there exists a solution to the uniform-pricing problem (2). Therefore, because $Q(\cdot)$ is continuous and $Q(p) \geq \tilde{Q}(p)$ for all p , there is some $p' \geq \tilde{p}^u$ such that $Q(p') = \sum_{j=1}^{N^*} \tilde{q}_j(\tilde{p}^u)$. In the original uniform pricing problem, the profit at price p' then equals $\tilde{\Pi}^u$; the maximum uniform-pricing profit must be at least this large, yielding Eq. (5).

To prove Eq. (6), let p_i' denote a solution to Eq. (3) given demands $\tilde{q}_1, \dots, \tilde{q}_{N^*}$ and cost function C , $i = 1, \dots, N^*$. One can show that because the individual market demands are all nonincreasing, aggregate demand $Q(\cdot)$ is continuous if and only if the individual market demands q_1, \dots, q_N are continuous. Therefore, if $Q(\cdot)$ is continuous, then so too is $\tilde{Q}(\cdot)$. For all p and i , $\tilde{Q}(p) \geq \tilde{q}_i(p)$; this fact together with $\lim_{p \uparrow \infty} Q(p) = 0$ and the continuity of $\tilde{Q}(\cdot)$ imply there exists $p_i'' \geq p_i'$ such that $\tilde{Q}(p_i'') = \tilde{q}_i(p_i')$. Therefore, for all $i = 1, \dots, N$,

$$\tilde{\Pi}^u \geq p_i'' \tilde{Q}(p_i'') - C(\tilde{Q}(p_i'')) \tag{7}$$

$$= p_i'' \tilde{q}_i(p_i') - C(\tilde{q}_i(p_i')) \tag{8}$$

$$\geq p_i' \tilde{q}_i(p_i') - C(\tilde{q}_i(p_i')) \tag{9}$$

$$= \tilde{\Pi}_i^s, \tag{10}$$

where Eq. (7) holds because p_i'' is just one of the prices to be considered in the maximization problem implicit in the definition of $\tilde{\Pi}^u$, Eq. (8) holds because $\tilde{Q}(p_i'') = \tilde{q}_i(p_i'')$ by construction, Eq. (9) holds because $p_i'' \geq p_i'$, and Eq. (10) holds by the definitions of p_i' and $\tilde{\Pi}_i^s$.

Let $IC(Q'', Q')$ denote the incremental cost of producing Q'' units of output, given Q' units are already to be produced; i.e., $IC(Q'', Q') \equiv C(Q' + Q'') - C(Q')$. We have

$$\tilde{\Pi}^u \geq \tilde{\Pi}_j^s \tag{11}$$

$$\geq \tilde{p}_j^d \tilde{q}_j(\tilde{p}_j^d) - C(\tilde{q}_j(\tilde{p}_j^d)) \tag{12}$$

$$\geq \tilde{p}_j^d \tilde{q}_j(\tilde{p}_j^d) - IC\left(\tilde{q}_j(\tilde{p}_j^d), \sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d)\right), \tag{13}$$

for all $j=1, \dots, N^*$, where in Eq. (13) it should be understood that $\sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d) = 0$ for $j=1$. Condition (11) follows from Eq. (6), and Eq. (12) follows because \tilde{p}_j^d is just one of the prices to be considered in the maximization problem implicit in the definition of $\tilde{\Pi}_j^s$. Condition (13) can be seen as follows. The superadditivity of C implies $C(Q') + C(\tilde{q}_j(\tilde{p}_j^d)) \leq C(Q' + \tilde{q}_j(\tilde{p}_j^d))$ for all $Q' \geq 0$. Rearranging,

$$\begin{aligned} C(\tilde{q}_j(\tilde{p}_j^d)) &\leq C(Q' + \tilde{q}_j(\tilde{p}_j^d)) - C(Q') \\ &= IC(\tilde{q}_j(\tilde{p}_j^d), Q'), \end{aligned} \tag{14}$$

for all $Q' \geq 0$, where Eq. (14) follows from the definition of IC . Condition (13) thus follows because Eq. (14) holds for all $Q' \geq 0$, and in particular for $Q' = \sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d)$.

Observe that the sum over all markets of the incremental costs involved in condition (13) is simply total cost under price discrimination; i.e.,

$$\begin{aligned} \sum_{j=1}^{N^*} IC\left(\tilde{q}_j(\tilde{p}_j^d), \sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d)\right) &= \sum_{j=1}^{N^*} \left\{ C\left(\sum_{k=1}^j \tilde{q}_k(\tilde{p}_k^d)\right) - C\left(\sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d)\right) \right\} \\ &= C\left(\sum_{k=1}^{N^*} \tilde{q}_k(\tilde{p}_k^d)\right) - C(0) \\ &= C\left(\sum_{k=1}^{N^*} \tilde{q}_k(\tilde{p}_k^d)\right), \end{aligned} \tag{15}$$

where the last step holds because superadditivity of C implies $C(0)=0$. Summing Eq. (13) over $j=1, \dots, N^*$, we obtain

$$N^* \Pi^u \geq N^* \tilde{\Pi}^u \tag{16}$$

$$\geq \sum_{j=1}^{N^*} \left\{ \tilde{p}_j^d \tilde{q}_j(\tilde{p}_j^d) - IC\left(\tilde{q}_j(\tilde{p}_j^d), \sum_{k=1}^{j-1} \tilde{q}_k(\tilde{p}_k^d)\right) \right\} \tag{17}$$

$$= \sum_{j=1}^{N^*} \tilde{p}_j^d \tilde{q}_j(\tilde{p}_j^d) - C \left(\sum_{j=1}^{N^*} \tilde{q}_j(\tilde{p}_j^d) \right) \tag{18}$$

$$= \sum_{j=1}^{N^*} \tilde{p}_j^d \sum_{i \in S_j} q_i(\tilde{p}_j^d) - C \left(\sum_{j=1}^{N^*} \sum_{i \in S_j} q_i(\tilde{p}_j^d) \right) \tag{19}$$

$$= \sum_{j=1}^{N^*} \sum_{i \in S_j} p_i^d q_i(p_i^d) - C \left(\sum_{j=1}^{N^*} \sum_{i \in S_j} q_i(p_i^d) \right)$$

$$= \sum_{\substack{i=1, \dots, N \\ \text{s.t. } p_i^d < \infty}} p_i^d q_i(p_i^d) - C \left(\sum_{\substack{i=1, \dots, N \\ \text{s.t. } p_i^d < \infty}} q_i(p_i^d) \right)$$

$$= \Pi^d, \tag{20}$$

where Eq. (16) follows from Eq. (5), Eq. (17) follows from summing over Eq. (13), Eq. (18) follows from Eq. (15), and Eq. (19) follows from the definition of \tilde{q}_j . Rearranging Eq. (20) gives $\Pi^d / \Pi^u \leq N^*$. \square

Proof of Proposition 2. We will prove the proposition by induction on N . Throughout the proof, attention will be restricted to examples with costless production.

Notice that, for $N=1$, $\Pi^d = \Pi^u$; so the proposition is trivially true. Suppose the proposition holds for N markets. We will show it also holds for $N+1$ markets.

Let $\epsilon > 0$. Since the proposition holds for N markets, an example can be constructed with N linear demands such that the profit under price discrimination is within $\epsilon/3$ of N times the profit under uniform pricing. Let Π_N^d denote the profit under price discrimination, Π_N^u the profit under uniform pricing, and $Q_N(p)$ aggregate demand under uniform pricing in this N -market example. We have

$$\Pi_N^d > N\Pi_N^u - \frac{\epsilon}{3}. \tag{21}$$

Take this example and construct a new example with $N+1$ linear demands by adding a market with demand $q_{N+1}(p) = a - bp$, where

$$a = \frac{4(N+1)(3N\Pi_N^u + \epsilon)\bar{Q}_N}{N\epsilon}, \tag{22}$$

$$b = \frac{12(N+1)^2(3N\Pi_N^u + \epsilon)\bar{Q}_N^2}{N\epsilon^2}, \tag{23}$$

and $\bar{Q}_N = Q_N(0)$ is the horizontal intercept of the demand curve $Q_N(p)$. Since $Q_N(p)$ is the aggregation of linear demands, $\bar{Q}_N \in (0, \infty)$, implying $a, b \in (0, \infty)$. Let Π_{N+1}^d denote the profit under price discrimination and Π_{N+1}^u the profit under uniform pricing in the new

example with $N+1$ markets. Let Π_{N+1}^s denote the stand-alone profit in the added market. We have

$$\begin{aligned} \Pi_{N+1}^s &= \frac{a^2}{4b} \\ &= \Pi_N^u + \frac{\epsilon}{3N}, \end{aligned} \tag{24}$$

where Eq. (24) holds by substituting for a and b from Eqs. (22) and (23) and simplifying. Eq. (24) implies that market $N+1$ is served under uniform pricing in the example with $N+1$ markets, in turn implying that the profit-maximizing uniform price must be below a/b , the vertical intercept of demand curve $q_{N+1}(p)$. Hence,

$$\begin{aligned} \Pi_{N+1}^u &= \max_{p \in [0, a/b]} \{pQ_N(p) + pq_{N+1}(p)\} \\ &\leq \max_{p \in [0, a/b]} \{p\} \max_{p \in [0, a/b]} \{Q_N(p)\} + \max_{p \in [0, a/b]} \{pq_{N+1}(p)\} \\ &= \left(\frac{a}{b}\right)\bar{Q}_N + \Pi_{N+1}^s \\ &= \Pi_N^u + \frac{(2N+1)\epsilon}{3N(N+1)}. \end{aligned} \tag{25}$$

Eq. (25) holds by substituting for a and b from Eqs. (22) and (23), substituting for Π_{N+1}^s from Eq. (24), and simplifying. We have

$$\begin{aligned} \Pi_{N+1}^d &= \Pi_{N+1}^s + \Pi_N^d \\ &> \left(\Pi_N^u + \frac{\epsilon}{3N}\right) + \left(N\Pi_N^u - \frac{\epsilon}{3}\right) \\ &\geq (N+1)\Pi_{N+1}^u - \epsilon. \end{aligned} \tag{26}$$

Condition (26) holds by Eqs. (21) and (24). Condition (27) holds by substituting for Π_{N+1}^u from the inequality (25) relating Π_{N+1}^u and Π_N^u . Thus, we have shown the proposition holds for the case of $N+1$ markets, completing the proof by induction. \square

Proof of Proposition 3. Continuity of demand was only used in the proof of Proposition 1 to establish conditions (5) and (6). We will show that these conditions hold for general (possibly discontinuous) demands assuming that the monopolist can ration demand by stocking out. The rest of the proof will be identical to that of Proposition 1 and is thus omitted.

We are left to prove $\Pi^u \geq \tilde{\Pi}_i^s$ for all $i=1, \dots, N^*$, where $\tilde{\Pi}_i^s$ is the profit value associated with problem (3) given market demand $\tilde{q}_i(p)$. Let \tilde{p}_i^s be the optimal price in the stand-alone profit-maximization problem yielding $\tilde{\Pi}_i^s$. Under uniform pricing, the monopolist can do no worse than charging \tilde{p}_i^s . If it charges this price, it can choose to sell any amount up to $Q(\tilde{p}_i^s)$ since it can ration demand by stocking out. In particular, it can sell $\tilde{q}_i(\tilde{p}_i^s)$ units. We know $\tilde{q}_i(\tilde{p}_i^s) < Q(\tilde{p}_i^s)$ because $Q(\cdot)$ is the sum of market demands including $\tilde{q}_i(\cdot)$ and others. Hence

$$\Pi^u \geq \tilde{p}_i^s \tilde{q}_i(\tilde{p}_i^s) - C(\tilde{q}_i(\tilde{p}_i^s)) = \tilde{\Pi}_i^s. \quad \square$$

Proof of Proposition 4 (sketch). For simplicity we sketch the proof for the case in which $N^*=N$. An approach analogous to that used in the proof of Proposition 1 would handle the case in which $N^* < N$. Because the monopolist can ration demand by bypassing markets, it can restrict

supply just to market i and charge the same price as is optimal in the stand-alone problem for this market. It thus follows that $\Pi^u \geq \Pi_j^s$ for all $j=1, \dots, N$. Steps similar to those in Eqs. (11)–(13) of the proof of Proposition 1 next yield

$$\Pi^u \geq p_j^d q_j (p_j^d) - C(q_j (p_j^d) \mathbf{e}_j). \tag{28}$$

Before proceeding, we generalize the earlier definition of incremental cost to apply to output vectors: for $\mathbf{Q}', \mathbf{Q}'' \in \mathbb{R}_+^N$, define $IC(\mathbf{Q}'', \mathbf{Q}') \equiv C(\mathbf{Q}' + \mathbf{Q}'') - C(\mathbf{Q}')$. We next show that weak market-by-market diseconomies of scope implies stand-alone cost is less than the incremental cost of producing for market j above that produced for markets $1, 2, \dots, j - 1$. Fix output vector (q_1, \dots, q_N) and $j \in \{1, \dots, N\}$. Then applying the definition of weak market-by-market diseconomies of scope to the vector $(q_1, \dots, q_{j-1}, q_j, 0, \dots, 0)$ implies

$$C(q_j \mathbf{e}_j) + C\left(\sum_{i=1}^{j-1} q_i \mathbf{e}_i\right) \leq C\left(\sum_{i=1}^j q_i \mathbf{e}_i\right),$$

which in turn yields

$$\begin{aligned} C(q_j \mathbf{e}_j) &\leq C\left(\sum_{i=1}^j q_i \mathbf{e}_i\right) - C\left(\sum_{i=1}^{j-1} q_i \mathbf{e}_i\right) \\ &= IC\left(q_j \mathbf{e}_j, \sum_{i=1}^{j-1} q_i \mathbf{e}_i\right). \end{aligned} \tag{29}$$

Together Eqs. (28) and (29) imply

$$\Pi^u \geq p_j^d q_j (p_j^d) - IC\left(q_j (p_j^d) \mathbf{e}_j, \sum_{i=1}^{j-1} q_i (p_i^d) \mathbf{e}_i\right),$$

the analogue of step (13) in the proof of Proposition 1. Arguments paralleling those in the remainder of the proof of Proposition 1 then establish $\Pi^d / \Pi^u \leq N$. \square

Proof of Proposition 5. The proof proceeds by showing that the conditions on C in the statement of the proposition imply that C exhibits weak market-by-market economies of scope. Since the monopolist is assumed to be able to ration demand by bypassing markets, Proposition 4 applies, implying $\Pi^d / \Pi^u \leq N^*$. For all $j=1, \dots, N$,

$$C(q_j \mathbf{e}_j) + C\left(\sum_{i \neq j} q_i \mathbf{e}_i\right) = \hat{C}(q_j) + \hat{C}\left(\sum_{i \neq j} q_i\right) + \sum_{i=1}^N t_i(q_i) \tag{30}$$

$$\leq \hat{C}\left(\sum_{i=j}^N q_i\right) + \sum_{i=1}^N t_i(q_i) \tag{31}$$

$$= C(q_1, \dots, q_N), \tag{32}$$

where Eq. (30) holds by Eq. (4) and $t_i(0)=0$ for all i , Eq. (31) holds because \hat{C} is superadditive, and Eq. (32) holds again by Eq. (4). But Eq. (32) is equivalent to weak market-by-market diseconomies of scope. \square

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