# A dynamic theory of countervailing power 

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In this article I develop a model of an infinitely repeated procurement auction with one buyer and several sellers. The buyer can accumulate a backlog of unfilled orders which, similar to a boom in demand, forces the sellers to collude on a low price to prevent undercutting. If the buyer's cost of shifting its consumption over time is low enough, then the extent of collusion is bounded away from the joint-profit-maximizing level even for discount factors approaching one. The model is extended to allow for multiple buyers. Large buyers are shown to obtain lower prices from the sellers. Buyer mergers increase profit for all buyers, not just the merging pair, at the expense of the sellers. In contrast, buyer growth through addition harms buyers that do not grow and benefits sellers.

## 1. Introduction

Conventional wisdom suggests that, relative to small buyers, large buyers have an advantage in obtaining price concessions from sellers-or, in Galbraith's (1952) terms, that size confers countervailing power. ${ }^{1}$ The conventional wisdom has been verified by a number of empirical studies. ${ }^{2}$ Interest in the existence of large-buyer discounts dates back before the passage of the Robinson-Patman Act of 1936. Recently, the issue has received renewed attention with the success of large retail chain stores. ${ }^{3}$

In the absence of economies of distribution (one obvious explanation for volume discounts), what explanation can be provided for discounts to large buyers? One literature explains discounts to large buyers using static bargaining models. Chipty and Snyder (1996) and Stole and Zwiebel (1996) study Nash bargaining between a single

[^0]firm and many small trading partners. In particular, they examine various cases that arise if the underlying production function exhibits nonconstant returns to scale. Horn and Wolinsky (1988) and McAfee and Schwartz (1994) show that product-market competition may affect downstream firms' negotiations with an input supplier. Another literature explains volume discounts as a possible feature of optimal nonlinear tariffs when the seller is imperfectly informed about the buyers' valuation of the good (Maskin and Riley (1984) and, in a bargaining model, Gertner (1989)).

These theories are appropriate for markets with a single large seller (labor union or dominant input supplier) and many buyers (customers or downstream firms). They do not capture the effects of upstream competition on prices. This latter feature often appears to be the driving force behind countervailing power in practice. Consider the following example from Scherer and Ross (1990, p. 307):
[D]uring 1955 and 1956 the five tetracycline producers settled down into a pattern of submitting identical $\$ 19.1884$ per one-hundred-capsule bottle in Veterans Administration transactions, the largest of which involved 30,000 bottles. Then, in October of 1956, the Armed Services Medical Procurement Agency (ASMPA) made its first tetracycline purchase, calling for 94,000 bottles. . . Two firms held to the established $\$ 19.1884$ price, but Bristol-Myers undercut to $\$ 18.97$ and Lederle cut all the way to $\$ 11.00$.

Scherer and Ross suggest that the ability of the tetracycline producers to sustain collusive prices was impaired by the appearance of a large buyer in the form of the ASMPA. The authors provide other cases in which the existence of large buyers and lumpy orders tended to erode tacit collusion by sellers including the cast-iron pipe industry of the 1880s (the industry involved in the Addyston Pipe and Steel Co. proceedings) and the turbogenerator industry in the 1960 s . Posner (1981) suggests that price discounts were offered by the railroads to Standard Oil in the 1880 s and 1890 s for similar reasons. Indeed, most of the industries in which there was evidence of countervailing power (see footnote 2 ) were oligopolies rather than monopolies at the upstream level.

These examples indicate that the theories based on static models with a monopoly seller may be inadequate to explain the prevalence of volume discounts in practice. The present article provides a formal theory in which buyer characteristics affect the ability of sellers to collude. The underlying model is an infinitely repeated procurement auction. In the constituent game, a single buyer seeks bids from many potential sellers, where a bid represents the price at which a seller is willing to supply the buyer. If the buyer were an inert demand curve, then the model would be identical to the oligopoly supergame studied by Friedman (1979) and later authors. Instead, we assume that the buyer is an autonomous player, capable of altering its intertemporal consumption pattern. In particular, the buyer receives a steady stream of consumption opportunities over time but may wait and satisfy several of these consumption opportunities at the same time. Ceteris paribus, the buyer would rather satisfy each consumption opportunity when it arises; but by accumulating a backlog of unfilled orders and purchasing all at once, the buyer may gain a strategic advantage over the sellers.

Intuition for why the accumulation of a large backlog of orders may benefit the buyer can be seen in the results of Rotemberg and Saloner (1986). The authors show that if demand varies over time, the collusive price may vary, too. Collusion is most difficult to sustain when current demand is high relative to expected future demand, since then the gain from deviating (the profit from undercutting the collusive price) is large relative to the punishment for deviating (the loss of future profits). In states of the world with high current demand, the sellers may be forced to charge a low price to limit the benefit from deviating. In short, there may be "price wars during booms." 4

[^1]In the model of this article, the buyer is able to generate "booms" endogenously by accumulating a backlog of orders, thereby obtaining a low price from the sellers. This off-equilibrium-path threat is enough to constrain the price the sellers charge the buyer even if the buyer purchases every period. ${ }^{5}$

The results turn out to depend on three key variables. The usual results hold for the number of firms $N$ and the discount factor $\delta$ : collusion becomes more difficult as $N$ rises and as $\delta$ falls. A new parameter is introduced, $\theta$, which measures the relative ease with which the buyer can transfer its consumption opportunities intertemporally. In general, $\theta$ is less than one, implying that the benefits from buying at the preferred time degrade with delay. The broad result is that as $\theta$ increases, the maximum collusive price falls. This result can be seen intuitively: the larger is $\theta$, the more valuable is any backlog of unfilled orders the buyer accumulates, i.e., the larger is the endogenous "boom" in demand generated by the buyer. The sellers are forced to lower the collusive price to prevent undercutting.

In the spirit of the folk theorems, the set of possible equilibria in the limit as $\delta$ approaches one is studied. A result of note is that for any $\delta$ less than one, there exists $\theta$ such that the surplus from collusion falls below a bound strictly below the monopoly level.

The model can be naturally extended to have an arbitrary number of buyers, $M$. The price offered to a certain buyer, say buyer $m$, depends on two factors: (i) the size of buyer $m$ relative to the market (the aggregate size of all buyers) and (ii) the surplus that the sellers extract from all other buyers. Concerning (i), the larger is buyer $m$ relative to the market, the relatively larger is any backlog that it accumulates. A seller's gain from undercutting and serving buyer $m$ 's backlog increases relative to the punishment for undercutting; so the collusive price charged to buyer $m$ must be relatively lower to maintain collusion. Concerning (ii), the more surplus the sellers are able to extract from other buyers, the more severe is the optimal punishment for undercutting the equilibrium price charged to $m$. This is true since the optimal punishment involves marginal-cost pricing-and thus the loss of all future profits-in transactions with all buyers, not just buyer $m$. The more severe the punishment for undercutting, the greater the collusive price that can be charged buyer $m$ while maintaining collusion.

The implications of buyer size, merger, and growth for equilibrium prices and profits can be viewed in the light of (i) and (ii). By (i), if one buyer is larger than another, it must be charged a lower price in equilibrium. Thus size confers countervailing power to a buyer. Merger increases the size of the merging buyers relative to the market. By (i), this tends to reduce the price paid by the merging buyers. By (ii), this reduction in price has an indirect, positive effect on the nonmerging buyers. Thus as a result of the merger, all buyers pay a lower price, the merging pair and nonmerging buyers. Seller profit falls. The result concerning mergers is particularly interesting since it contrasts the finding for buyer growth-growth thought of as an increase in the size of an existing buyer holding the size of other buyers constant. The growing buyer pays a lower price due to (i). Sellers gain from the growth: although the sellers receive a lower price from the growing buyer, this price is multiplied by a greater quantity, so on net the sellers obtain a larger surplus. By (ii), since seller surplus from other buyers increases, nongrowing buyers pay a higher price as a result of buyer growth. The fact that nongrowing buyers are harmed can also be seen in terms of (i): relative to the market, nongrowing buyers shrink as a result of growth, so the sellers can charge them a higher collusive price.

[^2]The article provides a theory explaining discounts for large buyers in which the upstream level is an oligopoly (in contrast to the literatures on bargaining and optimal nonlinear pricing cited above) and in which buyers purchase each period in equilibrium so that demand cycles are not observed (in contrast to the supergame literature cited above). Two remaining explanations for size discounts are the threat of backward integration by large buyers in the presence of upstream scale economies (Katz, 1987) and the existence of economies of distribution. The dynamic theory presented here could be distinguished empirically from these alternatives using evidence on the effect of buyer merger on the prices offered to other buyers in the market: the alternatives would imply that merger harms other buyers if buyers compete with each other on the product market (provided price discrimination is possible); the alternatives would imply that merger has no effect on other buyers if they do not directly compete.

The final section discusses extensions of the model to the case in which the buyer may purchase multiple units of the good to be stored for later use and to the case in which the buyer may sign a contract for delivery of the good over several periods. The seller may also be reinterpreted as a producer of an intermediate input and the buyer as a final-good producer. Thus the model has wide applicability to many industries. Besides the examples mentioned above, the potential to accumulate order backlogs or store large inventories may serve a strategic purpose in defense procurement, sales of aircraft to airlines, and sales of cars to rental-car companies. The potential to sign longterm contracts may serve a strategic purpose in industries such as cable television. According to the model, horizontal integration of local cable franchises should allow the franchises to obtain lower prices from program suppliers, a result supported by the empirical evidence in Chipty (1995). To mention another diverse example, the model would also support the contention (see, e.g., Greer (1993) and Miller (1993) for newspaper accounts) that the combination of firms into regional health alliances may enhance their bargaining position in negotiations with insurers and HMOs.

## 2. Model

■ The game has $N+1$ players: $N$ identical sellers indexed by $n=1, \ldots, N$ and one buyer. The sellers produce a good, which the buyer purchases. The leading interpretation of the game is that the sellers are upstream firms, the buyer is a downstream firm, and the good is an intermediate input that the downstream firm converts into the final product; however, the buyer can just as easily be taken as the consumer of the final good. Each period, the buyer has the opportunity to consume one unit of the good from which it obtains surplus $v$. In the leading interpretation, $v$ represents the profit from the sale of the final product, requiring one unit of the intermediate input to produce; however, $v$ could equally well represent the surplus obtained from the personal use of the good. The sellers have no fixed costs and constant marginal costs, normalized to zero.

There are an infinite number of periods in the game indexed by $t=1,2, \ldots$ Let $\delta \in[0,1)$ denote the per-period discount factor. Each period, the buyer may choose to hold a procurement auction. In an auction, each seller submits a bid to the buyer indicating the price at which it will supply the demand of the buyer.

If the buyer does not consume in period $t$, because it either does not hold an auction in the period or rejects the sellers' offers, in period $t+1$ it obtains a new consumption opportunity valued at $v$. The old consumption opportunity does not disappear: we assume that satisfying the period- $t$ consumption opportunity in period $t+1$ gives the buyer surplus $\theta v$ (in terms of period $t+1$ utility), where $\theta \in(0,1)$. In other words, $1-\theta$ represents the percentage loss in surplus from consumption one period later than the time the opportunity presents itself.

It is natural to suppose $\theta<1$, i.e., that surplus is lost if consumption is delayed. Interpreting the buyer as a downstream firm and the good as material used to make a final good, delay in purchasing the material could mean that the downstream firm misses a peak in demand for the final product. For instance, assuming a continuum of final-good consumers arrives on the market each period, it may be that within a given cohort a fraction $1-\theta$ cancel their orders each period there is a delivery delay. Alternatively, one could interpret the good as a capital input that depreciates and thus must be replaced periodically; if so, delaying replacement may mean lower-volume or lower-quality sales. To mention a specific application, consider car rentals (with auto manufacturers as the sellers and the rental-car company as the buyer): the rental-car company may lose sales if it does not replace old vehicles periodically.

Assume the rate of decline in the value of the consumption opportunity is constant over time so that one arriving in period $t$ is valued at $\theta^{k-1} v$ if served in period $t+k$ (in terms of period $t+k$ utility). For example, if the buyer refrains from consuming for two periods and then consumes three units in the next period, then it obtains surplus $v+\theta v+\theta^{2} v$ gross of the transfer price. In general, if it fills a $k$-period backlog, the buyer obtains gross surplus $v\left(1+\theta+\cdots+\theta^{k-1}\right)$. Define the series $s_{i}(x) \equiv \sum_{j-1}^{i} x^{j-1}$. Then the buyers's surplus from consuming a backlog of $k$ units can be written $s_{k}(\theta)$.

Turn now to the players' strategies. In brief, the buyer chooses the number of periods that elapse between auctions or, equivalently, the size of the backlog that it accumulates before seeking bids from the sellers. In an auction, the sellers submit simultaneous, secret bids representing the prices at which the sellers are willing to satisfy the entire $k$-unit demand of the buyer. The buyer accepts the lowest bid conditional on earning a nonnegative net surplus. The assumption that the buyer accepts the lowest bid automatically and has no ability to pursue other actions is made in the spirit of the subsequent analysis: the subsequent analysis focuses on equilibria that give the buyer the least surplus and the sellers the greatest. The assumptions made here imply that the buyer is a price taker in all respects except for its ability to accumulate a backlog of unfilled orders. If the buyer were allowed a richer strategy space, say allowing it to issue counteroffers to the sellers, the buyer's surplus would be higher and the sellers' lower in the resulting equilibria than is the case in the present model. ${ }^{6}$

Formally, let $h_{t}$ denote the history of players' observable actions up to and including period $t .^{7}$ Let $H_{t}$ be the set of all $h_{r}$. The buyer's strategy is a mapping $a_{t}: H_{t-1} \rightarrow\{0,1\}$, where the buyer conducts an auction in period $t$ if $a_{t}=1$ and not if $a_{t}=0$. Seller $n$ 's strategy is a mapping $p_{t}^{n}: H_{t-1} \times\{0,1\} \rightarrow \mathbf{R}^{+}$, where $p_{t}^{n}$ is its bid to serve the buyer's order conditional on $a_{t}=1$. (If $a_{t}=0$, then seller $n$ 's bid is immaterial; so we can set $p_{t}^{n}=0$ in that case without loss of generality.) The accounting convention is that $p_{t}^{n}$ is the bundle price, implying that the per-unit price for an order of size $k$ is $p_{t}^{n} / k$. The buyer publicly announces one of the sellers' bids-this is the winning bid, and the realized price paid by the buyer, unless the buyer rejects all bids. History $h_{\text {, consists }}$ of a sequence of indicator variables (indicating whether an auction had occurred in a given period) together with an announced price for each period in which an auction was conducted. The set of histories of observable actions can therefore be written

$$
H_{t}=\left\{\left(d_{\tau}, d_{\tau} r_{\tau}\right)_{\tau=1}^{t} \mid d_{\tau} \in\{0,1\}, r_{\tau} \in \mathbf{R}^{+}\right\}
$$

[^3]Price-taking behavior on the part of the buyer is modelled formally as follows. Suppose the buyer conducts an auction in period $t$. Let $k$ be the number of periods that have elapsed since the previous auction, implying that the buyer has a demand backlog of $k$ units in period $t$. Define $q_{t}$ to be the minimum bid in the auction, i.e., $q_{t}=\min \left\{p_{t}^{n} \mid n=1, \ldots, N\right\}$. First, we require that the seller bid for the task of supplying the buyer's entire demand backlog ( $k$ units). This requirement implies that the buyer cannot hold an inventory of unfilled orders after a successful auction. Second, we require the buyer to accept bid $q_{t}$ if and only if $q_{t} \leq v s_{k}(\theta)$. This requirement implies that the buyer accepts the lowest bid provided it earns nonnegative surplus from so doing. In case of ties, it is assumed the buyer chooses the winning bid at random among the lowest bidders. Bid $q_{t}$ is publicly announced during the auction.

The game is similar to the traditional supergame in prices except for the feature that the buyer can accumulate a backlog of orders over several periods. In contrast to supergames, here the stage games are not identical but depend on the size of the backlog. A feature that the game shares with supergames is the multiplicity of subgameperfect equilibria. Following the traditional practice in the supergame literature (see, e.g., Green and Porter (1984), Abreu (1986), Rotemberg and Saloner (1986), and Haltiwanger and Harrington (1991)), we will look for an upper bound on the level of seller collusion, focusing on the subgame-perfect equilibrium that yields the sellers the greatest profit, called the extremal equilibrium.

## 3. Extremal equilibrium

- Perfect collusion. The section begins with an analysis of the conditions under which the sellers are able to extract all the buyer's surplus. Recall the result from the canonical supergame in prices that the sellers are able to earn the joint monopoly profit if they are patient enough-i.e., if $\delta \geq 1-1 / N$ (Tirole, 1988). When the buyer can accumulate a backlog, there is a natural generalization of this condition, which we derive in the following discussion leading up to a formal statement in Proposition 1.

The joint surplus of the buyer and the sellers is greatest when the buyer consumes each period, in which case the net present value of joint surplus is $(1+\delta+\cdots) v=v /(1-\delta)$. I shall say the sellers engage in perfect collusion if they extract this whole surplus, $v /(1-\delta)$, from the buyer in the extremal equilibrium. If the sellers collude perfectly, in equilibrium the buyer must purchase every period at price $v$. Additionally, the buyer must not be able to earn positive surplus off the equilibrium path by conducting an auction after having accumulated a multiple-unit backlog. This would be a profitable deviation for the buyer, since it earns zero surplus along the equilibrium path.

Suppose the buyer conducts an auction for a backlog of $k$ units for some $k \in \mathbf{N}$. The sellers' joint continuation payoff can be no greater than

$$
\begin{equation*}
v s_{k}(\theta)+v\left(\frac{\delta}{1-\delta}\right) \tag{1}
\end{equation*}
$$

The first term of (1) is a bound on the profit the sellers can earn from serving the $k$-unit backlog; the second term is a bound on the payoff from serving the buyer in the continuation game following the $k$-unit auction (generated by the buyer's returning to purchasing each period at price $v$ ). Let $n$ index the seller that earns the lowest continuation payoff following the buyer's deviation. Seller $n$ can earn no more than

$$
\begin{equation*}
\frac{v}{N}\left[s_{k}(\theta)+\frac{\delta}{1-\delta}\right] \tag{2}
\end{equation*}
$$

Now, if seller $n$ bids $\epsilon$ less than $\nu s_{k}(\theta)$ in the auction for $k$ units, for all $\epsilon>0$ the buyer will accept the bid. Seller $n$ would deviate from the collusive equilibrium if there exists $\epsilon>0$ such that

$$
v s_{k}(\theta)-\epsilon>\frac{v}{N}\left[s_{k}(\theta)+\frac{\delta}{1-\delta}\right] ;
$$

i.e., if

$$
\begin{equation*}
\delta<\frac{(N-1) s_{k}(\theta)}{1+(N-1) s_{k}(\theta)} . \tag{3}
\end{equation*}
$$

For perfect collusion to be feasible, there can exist no $k \in \mathbf{N}$ such that (3) holds. The right-hand side of (3) in increasing in $k$; in the limit as $k \rightarrow \infty$, this term converges to ( $N-1$ )/( $N-\theta$ ). Thus, if $\delta<(N-1) /(N-\theta)$, there exists $k \in \mathbf{N}$ such that (3) holds, implying perfect collusion cannot be sustained.

If $\delta \geq(N-1) /(N-\theta)$, an equilibrium can be constructed in which there is perfect collusion. Namely, the buyer conducts an auction each period; the sellers bid $v$ to supply the unit. If the buyer deviates by accumulating a backlog of size $k$, the sellers bid $v s_{k}(\theta)$. If a seller deviates by undercutting the proposed equilibrium bids, the sellers revert to marginal-cost pricing (i.e., they bid zero) in all subsequent auctions. Given the sellers' strategies, the buyer does not gain from accumulating a backlog. The sellers have no incentive to deviate, since for all $k \in \mathbf{N}$ the gain from colluding exceeds the gain from deviating. To characterize the equilibrium fully, we need to specify the strategies that the players would pursue if, off the equilibrium path, the sellers' bids were all greater than $v s_{k}(\theta)$. Among other specifications, we can posit that the sellers revert to marginalcost pricing and the buyer conducts an auction in all subsequent periods. These strategies constitute a Nash equilibrium of the off-equilibrium-path subgame. Given the outcome (marginal-cost pricing), the sellers would not take the off-path subgame. We have proved the following proposition.

Proposition 1 (perfect collusion, single buyer). Perfect collusion is sustainable if and only if

$$
\begin{equation*}
\delta \geq \frac{N-1}{N-\theta} . \tag{4}
\end{equation*}
$$

Proposition 1 implies that perfect collusion is sustainable for large enough values of the discount factor. Thus the folk theorem for repeated games-that any outcome giving players at least their individually rational payoffs is possible if players are patient enough (see Fudenberg and Maskin (1986))-holds in the present case as well. Note that condition (4) is weaker the lower is $\theta$. Intuitively, if the consumption opportunities degrade fairly rapidly over time, then accumulating a backlog of orders does not help the buyer break the sellers' collusion. The value of orders accumulated early on in the backlog quickly becomes negligible, so even a backlog approaching infinite size provides too small a benefit to induce deviation. In the extreme case with $\theta=0$, condition (4) becomes $\delta \geq 1-1 / N$. As discussed above, this is the same condition for perfect collusion from the canonical supergame in prices; the model presented here can thus be viewed as a natural generalization of the canonical supergame. As $\theta$ approaches one, the minimum discount factor needed to sustain perfect collusion also approaches one. As $N$ increases, the minimum discount factor needed to sustain perfect collusion increases.

- Intermediate levels of collusion. For $\delta<1-1 / N$, the unique equilibrium involves the sale of the good at marginal cost (a price of zero) each period. To see that collusion is not sustainable, note that the buyer has a richer strategy space in the present model than in the canonical supergame in prices, so if the sellers cannot collude in the canonical supergame, they cannot collude in the present model. For $\delta \geq(N-1) /(N-\theta)$, as was shown in Proposition 1, perfect collusion is sustainable. I shall show that the interval between $(N-1) / N$ and $(N-1) /(N-\theta)$ is characterized by a level of collusion intermediate between perfect competition and perfect collusion. For conciseness, let $\Delta^{\text {int }}$ be the interval $((N-1) / N,(N-1) /(N-\theta))$.

The following lemma, proved in the Appendix, will simplify the analysis.
Lemma 1. There exists an extremal equilibrium in which the buyer purchases each period at a time-invariant price bid by all the sellers.

Let $q^{*}$ denote the equilibrium price bid each period in an extremal equilibrium. Since the sellers' bids are the same in this equilibrium, they obtain an equal share $1 / N$ of the expected surplus each period. I shall measure the level of collusion as $S$, the fraction of total surplus accruing to the sellers in equilibrium; i.e., $S=q^{*} / v$. To compute $S$, I need to compute the equilibrium price $q^{*}$ explicitly. I first derive an upper bound on $q^{*}$ and then construct an equilibrium in which this bound is attained.

For $q^{*}$ to be the equilibrium price, the buyer must obtain a higher payoff from purchasing each period than from accumulating a backlog of $k$ units before conducting an auction. Let $q_{k}$ be the minimum bid in the $k$-unit auction. To allow for the possibility that $q_{k}>v s_{k}(\theta)$ (implying that $q_{k}$ is rejected by the buyer) define $\lambda_{k} \equiv \min \left[q_{k}, v s_{k}(\theta)\right]$. We must have

$$
\begin{equation*}
\delta^{k-1}\left[v s_{k}(\theta)-\lambda_{k}\right] \leq\left(\frac{1-\delta^{k}}{1-\delta}\right)\left(v-q^{*}\right) . \tag{5}
\end{equation*}
$$

If $q_{k}>v s_{k}(\theta)$, then the left-hand side of (5) is zero. Since the right-hand side is nonnegative, (5) is automatically satisfied. On the other hand, if $q_{k} \leq v s_{k}(\theta)$, then $\lambda_{k}=q_{k}$. Therefore the left-hand side of (5) represents the present value of the buyer's payoff from purchasing in the $k$-unit auction (in terms of period-1 utility). The right-hand side represents the present value of the buyer's payoff from purchasing each period at price $q^{*}$. For the buyer not to deviate from equilibrium, (5) must hold.

For the equilibrium to be subgame perfect, we need a condition guaranteeing that the sellers would not undercut $q_{k}$ in the $k$-unit auction off the equilibrium path. There are two cases to consider. First, assume $q_{k} \leq v s_{k}(\theta)$, implying that the buyer accepts bid $q_{k}$ in the $k$-unit auction and that $\lambda_{k}=q_{k}$. The most the sellers can jointly earn if they do not undercut $q_{k}$ is $\lambda_{k}+\delta q^{*} /(1-\delta)$. The second term is the present value of the greatest possible continuation payoff. The most the worst-off seller can earn is $1 / N$ times the joint payoff. By undercutting $q_{k}$ by $\epsilon>0$, this seller can earn $\lambda_{k}-\epsilon$. To guarantee the worst-off seller does not deviate, ${ }^{8}$

$$
\begin{equation*}
\lambda_{k} \leq \frac{1}{N}\left(\lambda_{k}+\frac{\delta q^{*}}{1-\delta}\right) . \tag{6}
\end{equation*}
$$

Rearranging, this condition becomes

[^4]\[

$$
\begin{equation*}
\lambda_{k} \leq \frac{\delta q^{*}}{(1-\delta)(N-1)} \tag{7}
\end{equation*}
$$

\]

Second, assume that $q_{k}>v s_{k}(\theta)$, implying that the buyer does not purchase in the $k$-unit auction and that $\lambda_{k}=\nu s_{k}(\theta)$. It can be shown that the collusive profit for the worst-off seller in this case is bounded above by $\delta q^{*} /[(1-\delta)(N-1)] .{ }^{9}$ By undercutting, this seller can earn at least $\lambda_{k}$. Therefore, (7) is a necessary condition for $q_{k}$ not to be undercut in this case as well.

It is worth digressing from the analysis to discuss conditions (5) and (7), since they capture much of the intuition behind the later results. Condition (5) can be thought of as putting a lower bound on $\lambda_{k}$ : i.e., the sellers cannot offer the buyer too attractive a price if it accumulates a backlog, or the buyer would not buy each period. Condition (7) can be thought of as putting an upper bound on $\lambda_{k}$ : i.e., the price following the buyer's accumulating a $k$-unit backlog cannot be too high, or else a seller would undercut the collusive price. This is the Rotemberg and Saloner (1986) insight: prices may have to be limited in booms to sustain collusion. The difference here is that the buyer generates the "boom" endogenously by accumulating a backlog.

Depending on the size of the backlog $k$ and the per-period collusive price $q^{*}$, there may be no value of $\lambda_{k}$ satisfying (5) and (7) simultaneously: by lowering the price to prevent undercutting, the sellers may induce the buyer to accumulate a backlog of orders. In that case, the equilibrium value of $q^{*}$ has to be reduced. Reducing $q^{*}$ increases the buyer's benefit from purchasing each period relative to accumulating a backlog. ${ }^{10}$ In sum, even though the buyer does not accumulate a backlog in equilibrium, the off-equilibrium-path outcomes serve to constrain the value of $q^{*}$.

Returning to the analysis, substituting for $\lambda_{k}$ from (7) into (5) and rearranging yields

$$
\begin{equation*}
\left(1-\frac{N \delta^{k}}{N-1}\right) q^{*} \leq\left[1-\delta^{k}-\delta^{k-1}(1-\delta) s_{k}(\theta)\right] v \tag{8}
\end{equation*}
$$

For $q^{*}$ to be the per-period price in a subgame-perfect equilibrium, (8) must hold for all $k \in \mathbf{N}$. Simple calculations show that the right-hand side is positive. Thus (8) holds trivially if the left-hand side is not strictly positive. Define

$$
\hat{k} \equiv \frac{\ln (N-1)-\ln N}{\ln \delta} .
$$

It can be shown that the left-hand side of (8) is strictly positive if and only if $k>\hat{k}$. Therefore, the requirement that (8) must hold for all $k \in \mathbf{N}$ is equivalent to the following condition:

[^5]\[

$$
\begin{equation*}
q^{*} \leq \min _{k \in \hat{K}}\left\{v\left[\frac{1-\delta^{k}-\delta^{k-1}(1-\delta) s_{k}(\theta)}{1-N \delta^{k} /(N-1)}\right]\right\}, \tag{9}
\end{equation*}
$$

\]

where $\hat{K} \equiv\{i \in \mathbf{N} \mid i>\hat{k}\}$.
An equilibrium can be constructed in which $q^{*}$ attains the upper bound in (8). In any auction of one unit, the sellers bid $q^{*}$ equal to the bound in (9). In any auction of $k$ units, the sellers bid $\lambda_{k}$ satisfying (5) and (7) (since $q^{*}$ satisfies (9), such a $\lambda_{k}$ exists). The buyer conducts an auction each period as long as the minimum bid is $q^{*}$. If any seller undercuts the specified prices, the sellers revert to marginal cost (zero) pricing. Given these strategies, it is easily verified that (5) is a sufficient condition to prevent the buyer from deviating by accumulating a backlog and (7) is a sufficient condition to prevent a seller from undercutting. ${ }^{11}$ This equilibrium must be extremal, since it attains the bound. Dividing by $v$ in (9) yields the following proposition.

Proposition 2 (intermediate collusion, single buyer). Suppose $\delta \in \Delta^{\text {int. }}$. Then the level of collusion in the extremal equilibrium is given by

$$
\begin{equation*}
S=\min _{k \in \hat{K}}\left[\frac{1-\delta^{k}-\delta^{k-1}(1-\delta) s_{k}(\theta)}{1-N \delta^{k}((N-1)}\right] . \tag{10}
\end{equation*}
$$

Although $S$ does not have a closed-form solution, it is possible to compute $S$ by minimizing the right-hand side of (10) numerically. Figure 1 graphs $S$ as a function of $\delta$ fixing $\theta$ and $N$. The figure shows that $S$ increases with $\delta$ until $\delta=.8$, at which point $S=1 .^{12}$ The intuition for the slope of the curve is slightly complicated by the interaction of several factors. As $\delta$ increases, one effect is that the sellers value the future relatively more, so the threat of punishment for undercutting is relatively more severe. Formally, increasing $\delta$ relaxes constraint (7). Thus the sellers are able to collude more effectively, implying that the curve in Figure 1 should be upward sloping. An offsetting effect is that the buyer is more patient and so loses less surplus if it delays consumption until later periods. Formally, increasing $\delta$ increases the first term on the left-hand side of (5). This effect would tend to lower $4^{*}$, since the buyer would require more surplus to induce it to purchase each period rather than to accumulate a backlog. The fact that all players share the same discount factor convolutes the two effects, but it is apparent that the dominant effect is the one regarding the sellers.

It is straightforward to show $\partial S / \partial \theta<0$ for $\delta \in \Delta^{\text {int }} .{ }^{13}$ Intuitively, the higher is $\theta$, the greater is the buyer's payoff from accumulating a backlog relative to its payoff from consuming each period (see the first term on the left-hand side of (5)). The sellers are forced to reduce $q^{*}$ to prevent it from accumulating a backlog, reducing $S$.

It is also true that $\partial S / \partial N<0$ for $\delta \in \Delta^{\mathrm{im}} .{ }^{14}$ Intuitively, the higher is $N$, the lower is the price that can be sustained following the buyer's accumulating a backlog (see the right-hand side of (7)), increasing the buyer's payoff from accumulating a backlog. To induce the buyer to purchase each period, $q^{*}$ must be reduced.

[^6]FIGURE 1
EXTREMAL LEVEL OF COLLUSION FOR INCREASING $\delta(N=2, \theta=.75)$


The result for the canonical supergame in prices states that perfect collusion is possible for values of the discount factor close enough to one. The remainder of the section presents a contrasting result in the context of repeated auctions: no matter how high the discount factor, if the decline in value of the buyer's consumption opportunities is slow enough, collusion is bounded away from perfection. Formally, define $\bar{S} \equiv \sup _{s \in(0,1)}\left(\inf _{\theta \in(0,1)} S\right)$. We have the following proposition, proved in the Appendix.
Proposition 3 (bounds on extremal collusion). $\bar{S}<1$.
The proposition can be stated in another way: there exists $\bar{S}<1$ such that, for all $N \geq 2$ and $\delta<1, S<\bar{S}$ for some $\theta<1$. The proof of Proposition 3 provides expressions that can be used to compute $\bar{S}$ numerically. For $N=2, \bar{S}=.797$. As shown in Figure 2, $\bar{S}$ falls precipitously with increasing $N$.

Proposition 3 does not controvert the folk theorem, since the folk theorem supposes all the parameters of the model to be fixed except for $\delta$. Indeed, if $N$ is large enough, collusion can be rendered impossible no matter how high the discount factor, even in the canonical supergame in prices. However, $\theta$ differs from $N$ in that $\theta$ is a characteristic of the buyer, whereas $N$ is a characteristic of the sellers. Restating Proposition 3, we could say that, maintaining seller characteristics, for all $\delta<1$ there exist parameters not associated with the sellers for which collusion falls below some bound strictly less than one.

## 4. Multiple buyers

- In Section 3, $\theta$ is interpreted as an index of countervailing power. The buyer has no actual bargaining power in any extremal equilibrium in the sense that the buyer

FIGURE 2
BOUND ON THE EXTREMAL LEVEL OF COLLUSION, $\bar{S}=\sup _{\delta_{\epsilon}(0,1)}\left(\inf _{\theta \in(0,1)} S\right)$, FOR INCREASING $N$

cannot make counteroffers to the sellers; however, the buyer is capable of tempting a seller to deviate by accumulating a large backlog of unfilled orders. The parameter $\theta$ measures the cost of accumulating the backlog. In empirical work it may be difficult to quantify $\theta$, although in principle proxies for $\theta$ could be found, including a firm's inventory policy, the durability of the good, etc. The model can be extended to relate countervailing power to readily observable economic variables and events. In particular, I shall examine how buyer size and merger between buyers influence downstream countervailing power.

To achieve this end, the model can be extended to allow for $M$ buyers indexed by $m=1, \ldots, M$. I abstract from issues involved in buyer competition and suppose that they operate in separate geographic markets. I allow buyers to have different sizes, a realistic assumption since, for example, the size of the buyer may depend on the mass of consumers in its product market. Let $\phi_{m}$ measure the size of the buyer; formally, $\phi_{m}$ is the number of new consumption opportunities that buyer $m$ has each period. Rank the buyers by market size so that $\phi_{1} \leq \phi_{2} \leq \cdots \leq \phi_{M}$. The number of consumption opportunities across buyers, the aggregate size of the market $\Sigma_{m=1}^{M} \phi_{m}$, is denoted $\Phi$.

To formalize players' strategies, let $h_{t}$ denote the history of players' observable actions in the extended model with $M$ buyers and $H_{t}$ denote the set of $h_{r}$. Buyer $m$ 's strategy is a mapping $a_{t}^{m}: H_{t-1} \rightarrow\{0,1\}$, where it conducts an auction in period $t$ if $a_{t}^{m}=1$ and not if $a_{t}^{m}=0$. Seller $n$ 's strategy is a vector of mappings (one for each $m$ ) $p_{t}^{n, m}: H_{2-1} \times\{0,1\}^{M} \rightarrow \mathbf{R}^{+}$, where $p_{t}^{n, m}$ is its bid to serve buyer $m$ 's order if $a_{t}^{m}=1$. As before, if $a_{t}^{m}=0$, I can set $p_{t}^{n . m}=0$ without loss of generality. Note that I allow $p_{t}^{n, m}$ to be conditioned on the other buyers' auction decisions in addition to buyer $m$ ' $s$. The accounting convention is that $p_{l}^{n, m}$ is the bid per unit of buyer size to serve buyer
$m$ 's order. ${ }^{15}$ Following the logic of the single-buyer case in Section 2, in the extended model the set of histories of observable actions can be written

To formalize price-taking by the buyer, suppose that the buyer conducts an auction in period $t$ after having accumulated a backlog for $k$ periods. Define $q_{t}^{m}$ to be the minimum bid in the auction; i.e., $q_{t}^{m}=\min \left\{p_{t}^{n, m} \mid n=1, \ldots, N\right\}$. Buyer $m$ is required to accept $q_{t}^{m}$ if and only if $q_{t}^{m} \leq v s_{k}(\theta)$. As before, the buyer randomizes over low bidders in case of ties. Bid $q_{t}^{m}$ is publicly announced during the auction.

The remainder of the section will proceed in the same manner as the single-buyer case, first examining the conditions under which the sellers can maintain perfect collusion and then calculating the level of collusion that can be attained if perfect collusion is impossible. The logic of the analysis is identical: a bound on the level of collusion is established, and then an equilibrium is constructed attaining the bound.

- Perfect collusion. In the model with multiple buyers, the sellers will be said to engage in perfect collusion if the sellers are able to extract surplus $\nu \Phi /(1-\delta)$ in an extremal equilibrium. This entails that the buyers purchase each period at price $v$ in equilibrium. It cannot be the case that a buyer obtains positive surplus by conducting an auction in a period $t$ after accumulating a backlog of $k>1$ units. Letting $q_{i}^{m}$ be the minimum seller bid in the $k$-unit auction, $q_{t}^{m} \geq v s_{k}(\theta)$. To ensure subgame perfection, $q_{t}^{m}$ must not be undercut. Let $n$ index the seller that obtains the lowest share of the continuation payoff following a buyer's accumulation of a $k$-unit backlog. The most seller $n$ can earn if it does not undercut is bounded by

$$
\begin{equation*}
\frac{1}{N}\left[v \phi_{m} s_{k}(\theta)+Z+\frac{v \Phi \delta}{1-\delta}\right] \tag{11}
\end{equation*}
$$

The first term is the maximum profit from serving buyer $m$ in the $k$-unit auction; the second term $Z$ denotes the profit from serving the other buyers ( $i \neq m$ ) in the current period; the third term is the maximum possible profit in subsequent periods. By undercutting, seller $n$ can earn at least

$$
\begin{equation*}
v \phi_{m} s_{k}(\theta)+\max (Z, 0) \tag{12}
\end{equation*}
$$

For perfect collusion to be sustainable, (11) must exceed (12); a necessary condition for this can be found by setting $Z=0$; i.e.,

$$
(N-1) v \phi_{m} s_{k}(\theta) \leq \frac{v \Phi \delta}{1-\delta}
$$

or, rearranging,

$$
\begin{equation*}
\delta \geq \frac{(N-1) \phi_{m} s_{k}(\theta)}{\Phi+(N-1) \phi_{m} s_{k}(\theta)} . \tag{13}
\end{equation*}
$$

A necessary and sufficient condition for (13) to hold for all $k \in \mathbf{N}$ and $m=1, \ldots, M$ is

[^7]\[

$$
\begin{equation*}
\delta \geq \frac{N-1}{N-1+(1-\theta) \Phi / \phi_{M}} \tag{14}
\end{equation*}
$$

\]

If (14) holds, we can construct an equilibrium in which there is perfect collusion. Namely, the buyers conduct an auction each period, and the sellers bid $v$ per unit to supply the buyers. If some buyer $m$ deviates by accumulating a backlog for $k>1$ periods, this buyer becomes the target buyer as distinct from the rest of the buyers, called nontarget buyers. When the target buyer conducts its auction, the sellers bid $v s_{k}(\theta)$ to supply it. The strategy of the nontarget buyers is to continue to conduct an auction each period. The sellers offer the nontarget buyers a zero price each period up to and including the period in which the target buyer stops accumulating a backlog and conducts an auction. If one of the nontarget buyers begins to accumulate a backlog, it becomes the new target buyer. The original target buyer, if it purchases before the new target buyer, is offered a zero price for its backlog. If it delays its auction until after the new target buyer purchases, it retains its status as the target buyer. After the target buyer purchases, the players revert to the initial equilibrium strategies, implying that the buyers again purchase at price $v$ each period. If a seller undercuts any of the specified prices, the sellers revert to marginal-cost pricing thereafter.

In view of expressions (11) and (12), it is apparent why the nontarget buyers are given the good for free while the target buyer is accumulating its backlog. Giving the good away reduces the $Z$ term to zero, effectively reducing the surplus from undercutting relative to the benefit from colluding, true since (11) increases in $Z$ slower than (12) for $Z \geq 0$.

It is a simple matter to verify that the proposed strategies form an equilibrium. If no buyer is currently the target, buyers are indifferent between purchasing each period and delaying their purchase. If a buyer becomes a target off the equilibrium path, nontarget buyers strictly prefer to purchase each period, since they obtain a positive surplus from so doing; by delaying they become the target buyer and obtain no surplus. The buyers' strategies thus constitute an equilibrium. Given the buyers' strategies, condition (14) guarantees that the sellers do not deviate from equilibrium. We have proved the following.
Proposition 4 (perfect collusion, multiple buyers). Perfect collusion is sustainable if and only if (14) holds.

Compared to the one-buyer case, the new variable in the proposition is the share of the largest buyer relative to the aggregate; if the buyers are firms, this variable is the downstream one-firm concentration ratio $D C R_{1}$. The higher is $D C R_{1}$, the more difficult it is for the sellers to collude perfectly. The intuition for this result is that the larger a buyer relative to the market, the more tempting is its accumulated backlog to deviating sellers. If any buyer can break the sellers' collusion, it is the largest, buyer $M$. The benefit from undercutting (12) rises faster with the market share of buyer $M$ than the benefit from colluding (11).

Consider the comparative-statics exercise of adding a buyer of size $\phi^{\prime}$ to the existing group of $M$ buyers. If $\phi^{\prime} \leq \phi_{M}$, then this addition clearly reduces $D C R_{1}$ and so makes perfect collusion easier for the sellers in the sense that (14) is satisfied for higher $N$ and $\theta$ and lower $\delta$. Even if $\phi^{\prime}>\phi_{M}$, the addition of a buyer reduces $D C R_{1}$ as long as $\phi^{\prime}<\phi_{M} \Phi /\left(\Phi-\phi_{M}\right)$. Only if the additional buyer is substantially larger than buyer $M$ does adding a buyer make perfect collusion more difficult. If $M=1$, then adding a buyer always reduces $D C R_{1}$ and so always enhances the ability of sellers to collude perfectly. This can be seen formally by noting that condition (14) is weaker than (4). Indeed, the right-hand side of (4) is the limit of the right-hand side of (14) as $D C R_{1}$ approaches one.

- Intermediate levels of collusion. Suppose that $\delta \in \Delta_{M}^{\mathrm{int}}$, where

$$
\Delta_{M}^{\mathrm{im}} \equiv\left(\frac{N-1}{N}, \frac{N-1}{N-1+(1-\theta) \Phi / \phi_{M}}\right) .
$$

Perfect collusion is impossible by Proposition 4, but the sellers may be able to sustain some intermediate level of collusion in an extremal equilibrium. The following lemma, an extension of Lemma 1 to the case of multiple buyers, serves to simplify the analysis. Its proof is in the Appendix.
Lemma 2. In the model with multiple buyers, there exists an extremal equilibrium in which each buyer purchases each period at a time-invariant price bid by all the sellers.
In view of Lemma 2, to quantify the degree of seller collusion we need only compute the constant per-period price the sellers charge buyer $m$, denoted $q^{m *}$.

Following the logic of the last section, we can compute a bound on $q^{m *}$ and then construct an equilibrium in which the bound is attained. The condition guaranteeing that buyer $m$ does not accumulate a backlog in equilibrium is identical to (5), with the exception that the price $q^{*}$ is now superscripted by $m$ :

$$
\begin{equation*}
\delta^{k-1}\left[\nu s_{k}(\theta)-\lambda_{k}\right] \leq\left(\frac{1-\delta^{k}}{1-\delta}\right)\left(v-q^{m *}\right) \tag{15}
\end{equation*}
$$

The condition analogous to (7) is

$$
\begin{equation*}
\phi_{m} \lambda_{k} \leq \frac{\delta \sum_{i-1}^{M} \phi_{i} q^{i *}}{(1-\delta)(N-1)} . \tag{16}
\end{equation*}
$$

Note, first, that the left-hand side of (16) is increasing in $\phi_{m}$. The larger is the buyer that accumulates a backlog, the greater a seller's benefit from undercutting a given collusive price. Note, second, that the right-hand side of (16) involves the sellers' surplus from transactions with buyers $i \neq m$. The greater is this surplus, the greater is the punishment for undercutting, allowing the sellers to collude on a higher price in transactions with buyer $m$. Condition (16) identifies an externality among the buyers that leads to several interesting results (Propositions 7 through 9).

Putting (15) and (16) together and rearranging implies

$$
\begin{equation*}
q^{m *} \leq \min \left\{v, \min _{k \in \bar{X}} v\left[\alpha(k)+\frac{\beta(k)}{\phi_{m}} \sum_{i \neq m} \phi_{i} \frac{q^{i *}}{v}\right]\right\}, \tag{17}
\end{equation*}
$$

where

$$
\alpha(k) \equiv \frac{1-\delta^{k}-\delta^{k-1}(1-\delta) s_{k}(\theta)}{1-N \delta^{k}((N-1)} \quad \text { and } \quad \beta(k) \equiv \frac{\delta^{k}}{N-1-N \delta^{k}} .
$$

Note that $\alpha(k)$ and $\beta(k)$ are functions of $\theta$ and $N$, but this dependence has been suppressed in the notation.

An equilibrium can be constructed in which $q^{m *}$ attains the bound in (17) for each $m=1, \ldots, M$. The construction is identical to the construction in the case above of perfect collusion with multiple buyers, with the exception that a buyer is offered price $\lambda_{k}$ satisfying (15) and (16) if it accumulates a backlog of $k>1$ units and thereby becomes the target buyer. Such a $\lambda_{k}$ exists, since $q^{m *}$ satisfies (17). Arguments analogous to those used in the previous sections can be used to verify that the proposed
strategies yield an equilibrium. Since the upper bound on collusive surplus is attained by the equilibrium, it is extremal.

To quantify the level of seller collusion in the presence of multiple buyers, define $S_{m} \equiv q^{m *} / v . S_{m}$ is the share of surplus generated by trades between the sellers and buyer $m$ accruing to the sellers in an extremal equilibrium. Dividing (17) by $v$, we have the following proposition.

Proposition 5 (intermediate collusion, multiple buyers). Suppose $\delta \in \Delta_{M}^{\text {int }}$. Then the extremal level of collusion with respect to buyer $m$ is given by

$$
\begin{equation*}
S_{m}=\min \left\{1, \min _{k \in \tilde{K}}\left[\alpha(k)+\frac{\beta(k)}{\phi_{m}} \sum_{i \neq m} \phi_{i} S_{i}\right]\right\} . \tag{18}
\end{equation*}
$$

One can think of the right-hand side of (18) as defining a system of equations, the highest fixed point of which gives the extremal equilibrium. Since $S_{i} \in[0,1]$ for all $i=1, \ldots, M$ and the right-hand side of (18) is nondecreasing in $S_{i}$ for $i \neq m$, Theorem 4 of Milgrom and Roberts (1994) applies, guaranteeing the existence of a unique highest fixed point.

The right-hand side of (18) is identical to the right-hand side of (10), with the addition of the last term. If $\phi_{i}=0$ for all $i \neq m$ and $\delta \in \Delta^{\text {int }}$, then (18) reduces to $S_{m}=\min _{k \in \dot{K}} \alpha(k)$, an equation equivalent to (10).

Equation (18) shows that $S_{m}$ varies with $\phi_{m}$ (the size of buyer m) and $\Sigma_{i \neq m} \phi_{i} S_{i}$ (the surplus accruing to the sellers in their transactions with other buyers). The intuition for the dependence of $S_{m}$ on these two factors is contained in the discussion of condition (16) above. It bears emphasizing that although buyers operate independently from each other, there is a complementarity among them. The more surplus the sellers extract from buyers $i \neq m$, the more they can extract from $m$. This complementarity comes from the fact that the maximal punishment for a seller's undercutting in an auction to serve buyer $m$ is marginal-cost pricing in transactions with all buyers, not just $m$. The idea that sellers can use the surplus generated in one independent market to boost the level of collusion in another is familiar from Bernheim and Whinston (1990). I shall refer to the fact that $S_{m}$ depends on $\Sigma_{i \neq m} \phi_{i} S_{i}$ as complementarity among buyers.

- Buyer size, growth, number, and merger. Expression (18) can be used to obtain propositions predicting the effect of buyer size, growth, and merger, and the effects of changes in the number of buyers, on firm profits and on prices. Propositions 6 through 9 are proved in the Appendix. The proofs are complicated by the fact that calculating $S_{m}$ involves the solution of a system of $M$ equations and involves a minimization step where the argmin of (18), $k_{m}^{*}$, is restricted to be in the subset of the integers $\hat{K}$. To circumvent these complications, we employ the robust comparative-statics results of Milgrom and Roberts (1994).

Although the formal proofs are slightly complicated, it is possible to construct heuristic proofs of the propositions. Holding all other variables fixed, the right-hand side of (18) is nonincreasing in $\phi_{m}$. Therefore, we should expect that sellers obtain a larger share of the surplus the smaller the buyer.

Proposition 6 (buyer size). $S_{1} \geq S_{2} \geq \cdots \geq S_{M}$.
Proposition 6 states that size confers countervailing power to buyers: large buyers are able to obtain lower prices from the sellers. The intuition for this result comes from a consideration of an off-equilibrium-path subgame following the accumulation of a backlog by a buyer: the larger the buyer, the greater the gain from a seller's undercutting the collusive price relative to the loss of future profits. In a sense, large buyers are
capable of generating larger endogenous "booms" in demand. The sellers must offer the larger buyers relatively lower per-period prices to induce them not to delay purchase.

Another implication of the fact that (18) is nonincreasing in $\phi_{m}$ is that buyer $m$ should benefit ( $S_{m}$ should decline) if it grows. Condition (16) makes it clear that sellers benefit from the growth of buyer $m$ as well: if $\phi_{m}$ increases, the sellers can always decrease $\lambda_{k}$ leaving the left-hand side of (16) constant. But since the right-hand side of (16) is increasing in $\phi_{m}$, the seller can in fact allow $\phi_{m} \lambda_{k}$ to increase, thereby increasing $\phi_{m} S_{m}$. Since there is complementarity among buyers, an increase in $\phi_{m} S_{m}$ causes $S_{i}$ to rise for all buyers $i \neq m$, implying that these buyers face (weakly) higher prices.

Proposition 7 (buyer growth). Consider the extremal equilibria in two economies, where the second economy is identical to the first except that buyer $m$ is larger in the second. ${ }^{16}$ Relative to the first economy, buyer $m$ obtains a larger fraction of the surplus from its transactions with the sellers in the second economy. Buyer $m$ 's total surplus is greater. The surplus of each of the other buyers is lower. Seller profit is greater.

The effect on buyer $m$ of an increase in its size is obvious from Proposition 6: increasing size makes the threat of accumulating a backlog more costly to the sellers, so they offer the buyer a lower price to induce the buyer to purchase each period. As discussed above, sellers cannot be harmed by the growth of buyer $m$. Though they obtain less surplus per unit from $m$, they sell more units. Other buyers ( $i \neq m$ ) are harmed by buyer $m$ 's growth, the harm coming from complementarity among buyers. Another way to view the effect is that the growth of buyer $m$ increases the size of the market, causing all other buyers to shrink relative to the market and lose some of their countervailing power.

The addition of a buyer can be viewed as a special case of buyer growth-namely, the buyer grows from size zero to some positive size. Hence, the effect on equilibrium of increasing the number of buyers is an immediate consequence of Proposition 7.

Proposition 8 (addition of a buyer). Consider the extremal equilibria in two economies, where the second economy is identical to the first except that there is an additional buyer. Relative to the first economy, the original buyers' surplus is lower and the sellers' is higher in the second economy.

Proposition 7 deals with growth of buyer $m$ holding constant the size of all other buyers. Such growth might occur if $m$ were a final-good producer that began serving a new region or market niche. A second way for buyers to grow is through merger. Merger leads to an increase in the size of a buyer without increasing the aggregate size of the market. By the above reasoning, the merging buyers should receive lower prices. This unambiguously harms the sellers, since the lower price is not compensated by an increase in quantity. Other buyers should benefit as a result of the complementarity among buyers. These results are borne out formally as follows.

Proposition 9 (buyer merger). Consider the extremal equilibria in two economies, where the second economy is identical to the first except that two of the buyers have merged in the second. Relative to the first economy, the per-unit surplus for all buyers is greater in the second economy. Seller profit is lower.

[^8]It is important to note that the term "merger" is broadly interpreted in this subsection. A merger here is a combination of buyers into one coordinated purchasing unit. One way to accomplish the coordination is through profit sharing, but other methods of coordination are possible. ${ }^{17}$

## 5. Conclusion

- Perhaps the most interesting empirical implications of the model regard the effect of buyer growth on other buyers' profits, an effect stemming from the complementarity among buyers. The effect differs depending on how the buyer grows. If a buyer grows through merger, then all buyers in the industry benefit, since merger leaves the total size of the market unchanged. On the other hand, if buyer growth increases the size of the market-i.e., if growth is through addition rather than merger-then the buyers that do not grow pay higher prices and earn lower profit in response to buyer growth. Significantly, buyer growth affects other buyers even if they do not compete with each other on the product market.

The model discussed in the text applies to markets in which buyers can accumulate demand backlogs. It is also possible to construct variants of the model applying to other markets, e.g., markets in which buyers can purchase several units of the good and store it for future consumption needs or markets in which buyers can sign longterm contracts for future deliveries of the good. The set of additional applications covered by these variants is broad. It would include the purchase by skiers of lift tickets (contracts the duration of which varies from a day to a season). As reported by Meyers (1994), skiers have formed clubs in order to increase their bargaining power with ski resorts in Colorado. It would include the purchase of health insurance by firms. An issue raised in the recent health care debate is that the formation of regional health care alliances might improve the bargaining power of firms in their negotiations with insurers and HMOs (see, e.g., Greer (1993) and Miller (1993)). It would include the supply of cable programs (such as movie or sports channels) to local cable operators. The results of Chipty (1995) suggest that larger, regionally integrated cable operators obtain lower prices from the program suppliers. Under certain assumptions (regarding the storage technology or the cost of signing long-term contracts), the result obtained from the variants of the basic model are qualitatively similar to those in Sections 3 and $4 .{ }^{18}$

The analysis raises several issues for future research. One issue is the effect of early buyer mergers on the incentives for later, bandwagoning mergers. A second issue is the effect of changes in buyer size and of buyer merger on the surplus of final-good consumers and on social welfare. It would be useful to relax the assumption of unit demands, allowing increases in the price charged by the sellers to lead to a reduction in downstream sales and in social welfare. A third issue is the interaction between buyer-size effects analyzed in the article and business-cycle effects modelled in Bagwell and Staiger (forthcoming), Haltiwanger and Harrington (1991), and Kandori (1991).

## Appendix

- Proofs of Lemmas 1 and 2 and Propositions 3, 6, 7, and 9 follow.

[^9]Proof of Lemma 1. Throughout the proof I shall suppose that $\delta \geq 1-1 / N$ without loss of generality. If $\delta<1-1 / N$, collusion is not sustainable even if the buyer cannot accumulate a backlog of orders. Allowing the buyer a richer strategy space can only impair the sellers' ability to collude. Therefore, if $\delta<1-1 / N$, the lemma is trivially satisfied because an extremal equilibrium exists, with the buyer purchasing each period at a price of zero.

Consider an equilibrium in which the buyer delays for $k>1$ periods before conducting an auction. I shall derive an equilibrium in which the buyer conducts an auction each period and that provides the sellers with greater surplus than the original equilibrium. Let $U_{\mathrm{I}}^{p}$ (respectively, $U_{i}^{s}$ ) denote the present discounted value of the buyer's payoff (respectively, sellers' joint payoff) in the continuation game starting in period $t$. Consider a new equilibrium (the variables associated with which are indicated with tildes) in which the buyer conducts an auction each period and the sellers charge price $\tilde{p}_{1}=v-(1-\delta) U_{1}^{b}$. Note that $\tilde{p}_{1}$ is computed to give the buyer the same payoff as in the original equilibrium: $\tilde{U}_{1}^{b}=\left(\nu-\tilde{p}_{1}\right)(1-\delta)=U_{1}^{b}$. Since the original equilibrium involved delay, $U_{\mathrm{i}}+U_{1}^{b}<v /(1-\delta)$. In the new equilibrium, $\tilde{U}_{\mathrm{i}}+\tilde{U}_{1}^{b}=\tilde{U}_{1}^{s}+U_{1}^{b}=\nu /(1-\delta)$, implying $\bar{U}_{\mathrm{i}}>U_{\mathrm{i}}$. Hence the new equilibrium dominates the original in terms of seller surplus.

The equilibrium outcome can be supported by the following off-path strategies for the players. If any seller undercuts $\tilde{p}_{1}$, the sellers respond with the grim strategy (price equal to zero in all future periods). Since $\delta>1-1 / N$, no seller undercuts in equilibrium. If the buyer does not conduct an auction in a period, the sellers revert to the strategies used in the original equilibrium. The buyer thus earns the same if it deviates or if it purchases each period ( $U_{1}^{h}$ in either case). Thus, there exists a subgame-perfect equilibrium in which the buyer purchases each period at $\tilde{p}_{1}$.

Similar arguments can be used to show that, corresponding to any equilibrium in which the price charged to the buyer varies over time, there exists an equilibrium providing the sellers with at least as much surplus in which the sellers charge a constant price to the buyer over time. Q.E.D.
Proof of Proposition 3. Since $\partial S / \partial \theta \leq 0, \inf _{\theta \in(0,1)} S=\lim _{\theta \rightarrow 1} S$. By l'Hôpital's Rule, $\lim _{\theta \rightarrow 1} s_{k}(\theta)=k$. Thus,

$$
\lim _{\hat{\theta} \rightarrow 1} S=\min _{k \in \hat{K}}\left[\frac{1-\delta^{k}-k(1-\delta) \delta^{k-1}}{1-N \delta^{\gamma} /(N-1)}\right]
$$

Consider the change in variables $\delta^{k}=\gamma$. The restriction $k \in \hat{R}$ is then equivalent to the restriciton $\gamma \in \Gamma_{s} \equiv\left\{\delta^{\chi} \mid k \in \mathscr{K}\right\}$. Additionally, since $\ln \gamma=k \ln \delta, k(1-\delta) \delta^{-1}=\gamma \ln \gamma g(\delta)$ where $g(\delta) \equiv(1-\delta) /(\delta \ln \delta)$. Hence,

$$
\begin{equation*}
\bar{S}=\sup _{\delta \in(0,1)}\left\{\min _{\gamma \in \Gamma_{8}}\left[\frac{1-\gamma-\gamma \ln \gamma g(\delta)}{1-N \gamma /(N-1)}\right]\right\} \tag{A1}
\end{equation*}
$$

The fact that $\gamma$ is constrained to be in a subset of $(0,1)$ complicates the proof slightly. The proof proceeds by computing the unconstrained solution $\gamma^{4}$, found by allowing $\gamma$ to vary freely on ( 0,1 ), and then noting that the constrained minimizer must be in the interval ( $\left.\gamma^{u} / \delta, \gamma^{u}\right]$. These calculations produce an upper bound on $\bar{S}$. A lower bound on $\bar{S}$ can also be computed and shown to equal the upper bound, establishing the exact value of $\bar{S}$.

The first-order condition for the unconstrained minimizer $\gamma^{4}$ is

$$
\begin{equation*}
\frac{1}{g(\delta)}=N-1+N \gamma^{a}+(N-1) \ln \gamma^{u} \tag{A2}
\end{equation*}
$$

Substituting from (A2) implies, after some algebraic manipulation,

$$
\frac{1-\gamma^{u}-\gamma^{u} \ln \gamma^{\prime \prime} g(\delta)}{1-N \gamma^{u} /(N-1)}=1+\gamma^{u} g(\delta)
$$

Thus,

$$
\begin{aligned}
\bar{S} & \leq \sup _{\delta \in(0,1)}\left\{\sup _{\gamma \in\left(\gamma^{\prime \prime} \delta, \gamma^{u}\right]}[1+\gamma g(\delta)]\right\} \\
& =\sup _{\delta \in(0,1)}\left[1+\frac{\gamma^{u}}{\delta} g(\delta)\right] \\
& =\lim _{\delta \rightarrow 1}\left[1+\frac{\gamma^{u}}{\delta} g(\delta)\right] \\
& =1-\lim _{\delta \rightarrow 1} \gamma^{u}
\end{aligned}
$$

The second step holds because $g(\delta) \leq 0$, so the supremum is achieved at the left endpoint of the interval
( $\left.\gamma^{\mu} / \delta, \gamma^{u}\right]$. To see the third step define $F(\delta) \equiv 1+\gamma^{u} g(\delta) / \delta$. Applying the implicit function theorem to (A2), it can be shown that $d \gamma^{\prime \prime} / d \delta<0$ for $\delta \in(0,1)$. It can also be shown that $g^{\prime}(\delta)>0$ for $\delta \in(0,1)$. As stated above, $g(\delta) \leq 0$ for $\delta \in(0,1)$. Putting these facts together implies $F^{\prime}(\delta)>0$ for $\delta \in(0,1)$. Thus the supremum is equal to the limit as $\delta \rightarrow 1$. The last step follows because $\lim _{\delta \rightarrow 1} g(\delta)=-1$ by l'Hôpital's Rule.

The above calculations produced an upper bound on $S$. A lower bound can also be constructed. In view of (A1),

$$
\begin{align*}
\bar{S} & \geq \lim _{\delta \rightarrow 1}\left\{\min _{x \in(0,1)}\left[\frac{1-\gamma-\gamma \ln \gamma g(\delta)}{1-N \gamma /(N-1)}\right]\right\} \\
& =\min _{\gamma \in(0.1)}\left[\frac{1-\gamma+\gamma \ln \gamma}{1-N \gamma /(N-1)}\right] . \tag{A3}
\end{align*}
$$

But (A3) can be shown to equal $1-\lim _{\delta \rightarrow 1} \gamma^{\prime \prime}$. Putting the bounds together thus implies $\bar{S}=1-\lim _{\tilde{\delta} \rightarrow 1} \gamma^{\mu}$.
We can compute $\lim _{\delta \rightarrow 1} \gamma^{\prime \prime}$ by first taking the limit of (A2) as $\delta \rightarrow 1$ and second numerically solving the resulting equation, $N\left(1+\gamma^{\mu}\right)+(N-1) \ln \gamma^{u}=0$, for $\gamma^{\mu}$. For $N=2, \bar{S}=.797$ to three decimal places. Since $d \tilde{S} / d N<0, \bar{S} \leq .797$ for all $N \in \mathbf{N}$. (To show $d \tilde{S} / d N<0$, calculations similar to those in footnote 13 can be applied to the expression in (A3).) Q.E.D.
Proof of Lemma 2. Following the discussion in the proof of Lemma 1, I shall assume throughout that $\delta>1-1 / N$. Consider an equilibrium in which buyer $m$ delays for $k>1$ periods before conducting an auction. Let $U_{i}^{b m}$ (respectively, $U_{i}^{m}$ ) be the present discounted value of buyer $m$ 's payoff (respectively, sellers' joint payoff from transactions with buyer $m$ ) in the continuation game starting in period $t$. A new equilibrium can be constructed (the variables associated with which are indicated with tildes) in which all buyers purchase each period at price $\tilde{p}_{1}^{m}$ given by $\phi_{m} \tilde{p}_{1}^{m}=\phi_{m} v-(1-\delta) U_{1}^{b m}$. Following the proof of Lemma 1, it can be shown that $\tilde{U}_{\mathrm{p}}^{\mathrm{pm}}=U_{\mathrm{i}}^{\mathrm{pm}}$ and $\tilde{U}_{\mathrm{i}}^{m}>U_{\mathrm{i}}^{\mathrm{s} m}$.

The equilibrium outcome can be supported by the following off-path strategies for the players. If any seller undercuts $\tilde{p}_{1}^{m}$, the sellers respond with the grim strategy. Since $\delta>1-1 / N$, no seller undercuts in equilibrium. If a buyer (say buyer $m$ ) deviates by accumulating a backlog, the players' strategies are as follows.

Suppose first that buyer $m$ 's period-1 action in the original equilibrium was to delay purchase. Then the players pursue the exact same strategies as in the original equilibrium. The only issue is that there may exist other buyers that would have delayed purchase in the original equilibrium (note that these buyers conduct an auction each period in the new equilibrium). The sellers can simulate delay for these buyers by offering bids greater than $v$. This strategy is feasible because sellers can condition their bids on the auction decisions of all buyers; here their bids are conditioned on the auction decision of the deviating buyer, buyer $m$. Given these strategies, buyer $m$ earns no more than $U_{1}^{b m}$ if it delays purchase; thus it would not deviate.

Suppose second that buyer $m$ 's period-1 action in the original equilibrium was to conduct an auction in period 1. Then the players pursue the exact same strategies as in the off-path subgame that would arise in the original equilibrium following a deviation by buyer $m$. The existence of buyers that do not purchase in the first period in the original equilibrium is handled as in the previous paragraph.

Following arguments in Lemma 1, it can be shown that there exists an extremal equilibrium in which the price paid by each buyer is constant over time. Q.E.D.

Proof of Propositions 6 and 7. I shall prove Proposition 7 first and show that Proposition 6 is a straightforward consequence. The strategy of the proof will be to express equation (18) in a form to which Milgrom and Roberts' (1994) comparative-statics results can be applied.

Without loss of generality, I shall suppose that buyer 1 is larger in the second economy. Let $G \in \mathbf{R}$ be such that $G>\phi_{M}$. Define $x_{m} \equiv \phi_{m} S_{m} / G$ and $\zeta=\phi_{1}$. (Here $\zeta$ plays the role that $t$ plays in Milgrom and Roberts.) From (18) we have

$$
x_{1}=\min \left\{\frac{\zeta}{G}, \min _{k \in \dot{K}}\left[\frac{\zeta}{G} \alpha(k)+\beta(k) \sum_{i \neq 1} x_{i}\right]\right\}
$$

and, for all $m=2, \ldots, M$.

$$
x_{m}=\min \left\{\frac{\phi_{m}}{G}, \min _{k \in \hat{K}}\left[\frac{\phi_{m}}{G} \alpha(k)+\beta(k) \sum_{i \neq m} x_{i}\right]\right\} .
$$

Note that $G$ has been chosen to ensure $x_{m} \in[0,1]$. Note further that the right-hand sides of the above equations are nondecreasing in $x_{i}$ and $\zeta$. Hence, Theorem 4 of Milgrom and Roberts applies, implying that the value of $x_{m}$ associated with the extremal equilibrium (i.e., the highest fixed point) is nondecreasing in $\zeta$ for all $m=1, \ldots, M$.

Therefore, $\phi_{m} S_{m} / G$ is nondecreasing in $\phi_{1}$ for all $m=1, \ldots, M$. Thus $\sum_{m=1}^{m} \phi_{m} S_{m}$, total seller surplus,
is nondecreasing in $\phi_{1}$ (i.e., it is weakly higher in the second economy). For $m \neq 1$, this implies $S_{m}$ is nondecreasing in $\phi_{1}$, since $\phi_{m}$ and $G$ are constant. Thus buyer surplus is nonincreasing in $\phi_{1}$ (i.e., it is weakly lower in the second economy).

To determine the effect of an increase in $\phi_{1}$ on buyer 1 's surplus, redefine $x_{m} \equiv \phi_{m} S_{m} /\left(\phi_{1} G\right)$ and $\zeta \equiv 1 / \phi_{1}$. From (18) we have

$$
x_{m}=\min \left\{\frac{\zeta \phi_{m}}{G}, \quad \min _{k \in \hat{K}}\left[\frac{\zeta \phi_{m}}{G} \alpha(k)+\beta(k) \sum_{i \neq m} x_{i}\right]\right\}
$$

for all $m=1, \ldots, M$. Given the new definitions it is easily verified that the conditions of Theorem 4 of Milgrom and Roberts are satisfied, implying that $x_{m}$ is nondecreasing in $\zeta$, in turn implying that $x_{m}$ is nonincreasing in $\phi_{1}$. Taking $m=1$, we see that $\phi_{1} S_{1} /\left(\phi_{1} G\right)=S_{1} / G$ is nonincreasing in $\phi_{1}$, implying that $S_{1}$ is nonincreasing in $\phi_{1}$ (i.e., it is weakly lower in the second economy).

To prove Proposition 6, suppose buyers $m$ and $m+1$ are of equal size. If $S_{m}<S_{m+1}$, then another equilibrium exists in which $S_{m}>S_{m+1}$ (simply switch the labels of the two buyers), yiolating the uniqueness of the extremal equilibrium. Similarly, it cannot be the case that $S_{m}<S_{m+1}$. Thus $S_{m}=S_{m+1}$. To prove the proposition in the case $\phi_{m}<\phi_{m+1}$, consider the comparative-statics exercise of moving from an economy in which $\phi_{m}$ and $\phi_{m+1}$ are initially equal, to one in which $\phi_{m+1}$ is increased so that it exceeds $\phi_{m}$. As shown above, increasing $\phi_{m+1}$ increases $S_{m}$ and decreases $S_{m+1}$ at least weakly. Therefore, $S_{m} \geq S_{m+1}$. Q.E.D.
Proof of Proposition 9. Without loss of generality, suppose that buyers 1 and 2 are merged in the second economy. The two economies can be nested using the parameter $\gamma$ by writing the buyer sizes as $\tilde{\phi}_{1}=\phi_{1}+\gamma \phi_{2}, \tilde{\phi}_{2}=(1-\gamma) \phi_{2}$, and $\tilde{\phi}_{m}=\phi_{m}$ for all $m=3, \ldots, M$. In the first economy, $\gamma=0$, and in the second, $\gamma=1$. The effect of the merger can be computed from the comparative-statics exercise of increasing $\gamma$ from zero to one. The logic of the proof will follow that of the proof of Proposition 7.

Let $G \in \mathbf{R}$ be such that $G>\phi_{M}$. Define

$$
x_{m}=\frac{\tilde{\phi}_{m} S_{m}}{\left(\phi_{1}+\gamma \phi_{2}\right) G}
$$

and $\zeta \equiv 1-\gamma$. From (18) we have

$$
\begin{aligned}
& x_{1}=\min \left\{\frac{1}{G}, \min _{k \in \dot{K}}\left[\frac{\alpha(k)}{G}+\beta(k) \sum_{i \neq 1} x_{i}\right]\right\} \\
& x_{2}=\min \left\{\frac{\zeta \phi_{2}}{\left(\phi_{1}+(1-\zeta) \phi_{2}\right) G}, \min _{k \in \dot{K}}\left[\frac{\zeta \phi_{2} \alpha(k)}{\left(\phi_{1}+(1-\zeta) \phi_{2}\right) G}+\beta(k) \sum_{i \neq 2} x_{i}\right]\right\}
\end{aligned}
$$

and, for all $m \neq 1,2$,

$$
x_{m}=\min \left\{\frac{\phi_{m}}{\left(\phi_{1}+(1-\zeta) \phi_{2}\right) G}, \min _{k \in \dot{K}}\left[\frac{\phi_{m} \alpha(k)}{\left(\phi_{1}+(1-\zeta) \phi_{2}\right) G}+\beta(k) \sum_{i \neq m} x_{i}\right]\right\}
$$

The right-hand side of the above equations are nondecreasing in $x_{i}$ and $\zeta$. By Theorem 4 of Milgrom and Roberts, $x_{m}$ is nondecreasing in $\zeta$ for all $m=1, \ldots, M$, implying that $x_{m}$ is nonincreasing in $\gamma$ for all $m=1, \ldots . M$. In particular,

$$
x_{1}=\frac{\tilde{\phi}_{1} S_{1}}{\left(\phi_{1}+\gamma \phi_{2}\right) G}=\frac{S_{1}}{G}
$$

is nonincreasing in $\gamma$, implying $S_{1}$ is nonincreasing in $\gamma$. This exercise shows that the per-unit price for the merged buyer in the second economy is weakly less than the per-unit price for buyer 1 in the first economy.

Repeating the above analysis for $\tilde{\phi}_{1}=(1-\gamma) \phi_{1}, \tilde{\phi}_{2}=\phi_{2}+\gamma \phi_{1}$, and $\tilde{\phi}_{m}=\phi_{m}$ for all $m=3, \ldots, M$ shows that the per-unit price for the merged buyer is (weakly) less than the premerger per-unit price for buyer 2 as well.

To determine the effect of the merger on the other buyers, consider the set of $M-2$ equations derived from (18) by redefining $x_{m} \equiv S_{m}$ and $\zeta \equiv \bar{\phi}_{1} S_{1}+\tilde{\phi}_{2} S_{2}$ for $m=3, \ldots, M$. Note that by the above results, $\zeta$ is weakly higher in the first economy than in the second. For $m=3, \ldots M$ we have

$$
x_{m}=\min \left\{1, \min _{k \in \hat{K}}\left[\alpha(k)+\beta(k)\left(\zeta+\sum_{i \ngtr 1,2, m} x_{i}\right)\right]\right\}
$$

The right-hand side is nondecreasing in $\zeta$, so by Theorem 4 of Milgrom and Roberts, $x_{m}$ is nondecreasing in
$\zeta$ for $m=3, \ldots, M$. Therefore, since $\zeta$ is weakly lower in the second economy, $x_{m}$ is weakly lower in the second economy for $m=3, \ldots, M$, implying $S_{m}$ is weakly lower in the second economy for $m=3, \ldots, M$.

In sum, the per-unit price paid by all buyers is weakly lower in the second economy. Thus, seller surplus must be weakly lower in the second economy. Q.E.D.

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    ${ }^{1}$ For some recent examples of the conventional wisdom in the trade press, see Cox (1994), Strauss (1987), and Wolfe and Asch (1992).
    ${ }^{2}$ Brooks (1973). Buzzell, Gale, and Sultan (1975), Lustgarten (1975), McGuckin and Chen (1976), Clevenger and Campbell (1977), and, more recently, Boulding and Staelin (1990) provide interindustry studies. Adelman (1959) and McKie (1959) present case studies of the grocery and tin-plate industries. Chipty (1995) develops an econometric model of the cable television industry.
    ${ }^{3}$ In a recent Business Week article, Schiller and Zellner (1992) write, "Many manufacturers are drawn to the big retailers in the hopes that huge volumes will offset slender profit margins. 'Most suppliers would just do absolutely anything to sell to Wall-Mart,' says one manufacturers' representative."

[^1]:    ${ }^{4}$ Bagwell and Staiger (forthcoming), Haltiwanger and Harrington (1991), and Kandori (1991) also provide theoretical analyses of collusion over the business cycle.

[^2]:    ${ }^{5}$ Similar intuition can also be found in Klein and Leffler (1981), in which buyers use the threat of boycott to ensure that sellers produce a high-quality good. The threat is successful if the cost savings from producing low quality in the current period is outweighed by the loss in rents from repeat sales in the future.

[^3]:    ${ }^{6}$ A caveat: If the buyer is not constrained to accept the lowest bid, then the sellers may be able to collude perfectly unless it is assumed that the bids the buyer does not accept remain secret. For further discussion of collusion in auctions, see Hendricks and Porter (1989) and Robinson (1985).
    ${ }^{7}$ Given the assumptions about price-taking behavior on the part of the buyer made below, there is no additional loss of generality in assuming that players' strategies are a function of past observable actions only.

[^4]:    ${ }^{8}$ Supposing antitrust considerations force the sellers to randomize among themselves before the auction to pick the low bidder (rather than having the buyer randomize over equal bids), the worst-off seller in the continuation equilibrium would earn strictly less than the right-hand side of (6). The extremal level of collusion would thus be lower than the expression given in (10).

[^5]:    ${ }^{9}$ Suppose the sale is made in a later $l$-unit auction at price $q_{l}$, where $l>k$. The worst-off seller's continuation payoff (starting in the period with the unsuccessful $k$-unit auction) is bounded by

    $$
    \frac{\delta^{-k+1}}{N}\left(q_{i}+\frac{\delta q^{*}}{1-\delta}\right) \leq \frac{\delta q^{*}}{(1-\delta)(N-1)}
    $$

    where the inequality holds since $\delta^{1-k+1}<1$ and since $q_{l} \leq \delta q^{*} /[(1-\delta)(N-1)]$ or else $q_{i}$ would be undercut.
    ${ }^{10}$ True, reducing $q^{*}$ also reduces the gain from colluding, strengthening (7). However, the effect on sellers' incentives is less important than the effect on the buyer's. Technically, reducing $q^{*}$ increases the right-hand side of (5) at a faster rate than it reduces the right-hand side of (7) for $k$ in the relevant range ( $k \in \mathbb{R}$, where $R$ is defined in the text below).

[^6]:    "To specify the equilibrium fully, I need to characterize the strategies that the players would pursue if, off the equilibrium path, the sellers' bids were all greater than $q_{k}$. One possible construction is given in the discussion preceding Proposition 1.
    ${ }^{12}$ The curve is discontinuous because $S$ is a function of an integer-valued minimizer.
    ${ }^{13}$ Assume $\delta \in \Delta^{\text {imt }}$. Abusing notation slightly, we can write $S(\theta)=\min _{k \in \dot{K}} F(\theta, k)=F(\theta, k(\theta)$ ), where $F$ is the expression minimized in the right-hand side of (10). For all $\theta^{\prime}, \theta^{\prime \prime}$ such that $\delta \in \Delta^{\text {mit }}$, we have $S\left(\theta^{\prime}\right)=F\left(\theta^{\prime}, k\left(\theta^{\prime}\right)\right)>F\left(\theta^{\prime \prime}, k\left(\theta^{\prime}\right)\right) \geq F\left(\theta^{\prime \prime}, k\left(\theta^{\prime \prime}\right)\right)=S\left(\theta^{\prime \prime}\right)$. The first inequality holds because $\partial F / \partial \theta<0$ and the second because $k(\cdot)$ is a minimizer.
    ${ }^{14}$ This can be verified by calculations along the lines of footnote 13.

[^7]:    ${ }^{15}$ For example, suppose buyer $m$ accumulates a backlog of demand for $k$ periods and then conducts an auction to serve its entire order ( $\phi_{m} k$ units). A bid of $p_{t}^{n, m}$ would imply a per-unit price of $p_{t}^{n, m / k}$ and a total transfer from buyer to seller of $\phi_{m} p_{i}^{n, m}$.

[^8]:    ${ }^{16}$ Proposition 7 compares equilibria in two separate economies. Equivalently, the proposition could be phrased in terms of an unanticipated change in the size of buyer $m$ in one economy. To examine the effect of an anticipated event would involve complexities associated with nonstationarity (i.e., players' strategies would adjust as the date of the event approached).

[^9]:    ${ }^{17}$ If they are not bound by a profit-sharing agreement, a coalition of buyers will tend to be unstable, with one unit free-riding off the other's accumulation of a backlog (recall that while one buyer accumulates a backlog, the others obtain the good for free).
    ${ }^{18}$ Derivations are available from the author concerning two variants of the model. In the storage-cost variant, buyers choose the size of the bundle purchased in an auction and can store the good for an indefinite period by expending fixed cost $c$ per unit. The long-term-contracting variant is similar except that $c$ is interpreted as a fixed contracting cost.

