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The identity of the generator in the problem of social cost

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Abstract

One of Coase's central insights is that distinguishing between the generator and recipient of an externality is of limited value because externality problems are reciprocal. We reconsider the relevance of the identity of the generator in a model with non-contractible investment ex ante but frictionless bargaining over the externality ex post. In this framework, a party may distort its investment to worsen the other's threat point in bargaining. We demonstrate that the presence of this distortion depends, among other factors, on whether the investing party is a generator. Social efficiency can sometimes be improved by conditioning property rights on the identity of the generator: for example, assigning damage rights if the rights holder is a generator and injunction rights if the rights holder is a recipient can be more efficient than either unconditional damage or injunction rights.

JEL classification: K11; D23; C78; H23

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1. Introduction

One of the most important insights in Coase's classic ''Problem of Social Cost'' [4] is his emphasis on the reciprocal nature of externality problems:

> The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm. ([4], p. 2)

In this paper we seek to understand whether the reciprocal nature of the externality problem obviates the need to distinguish between the generator and recipient of an externality or whether there is still some value in the distinction.

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We present a model with frictionless bargaining over the externality and other variables ex post but with transactions cost in the form of non-contractible investment ex ante. (The logic behind the model is that the initial investment decisions made by the first party to locate in an area will be non-contractible if the second party with whom the first will eventually negotiate has not yet shown up.) In this model, there is an asymmetry between the generator and the recipient that makes the distinction between them economically meaningful. The asymmetry arises because the generator's preferred level of the externality is an interior solution which may depend on its ex ante investment. By contrast, in the case of a purely negative externality, the recipient's preferred level is always zero, a corner solution that is independent of its investment.[1]

For example, consider the case of an airport generating noise harming a nearby homeowner raised in *Thrasher v. Atlanta*.[2] Allocating an injunction right to the airport essentially gives it a right to produce as much noise as it likes. The airport will have an incentive to distort its ex ante investment—for example expanding the runway to accommodate larger (and noisier) planes or building the runway closer to the homeowner—to increase its interior solution for its preferred externality level. In this way the airport can credibly threaten more harm to the homeowner in the event that bargaining over the noise level breaks down, thus allowing the airport to extract more bargaining surplus. Allocating an injunction right to the homeowner gives him the right to stop any noise from the airport. He can credibly threaten this same outcome whether his residence is a hovel or a mansion. Regardless of the size of the homeowner's investment in his residence, forbidding the airport to emit noise causes the same harm to it, forcing it to either shut down operations or pay for a device to muffle the noise. Hence an injunction right would not induce the homeowner to distort his ex ante investment as it would the airport.

In order to mitigate the distortion from the identified strategic effect, it may be efficient to weaken rights if the holder is a generator. For example if the airport is the rights holder (say by virtue of its having been in operation before the construction of the nearby residence in a ''coming to the nuisance'' regime in which the first party to locate obtains the rights), there are cases in which it would have been socially more efficient to have allocated it a damage right rather than an injunction right, that is, the right to collect damages for reducing its noise level to suit the homeowner rather than the right to set the noise level directly. There is no analogous benefit to weakening rights for a recipient since the identified strategic effect does not arise for a recipient. Consequently, we find that allocating an injunction right to the first mover is always more efficient than a damage right if the first mover is a recipient; but whether an injunction or a damage right is more efficient if the first mover is a generator depends on the parameters.

Is the identified strategic effect a real-world phenomenon or just a theoretical nicety? One of the more egregious cases of a party's investing to harm another's bargaining threat point is the ''spite fence'' built by millionaire Charles Crocker in San Francisco during the 1870s, described in Tamony [24]. Crocker offered to buy Nicholas Yung's property, which was surrounded by Crocker's estate. After Yung refused to sell, Crocker built a 40-feet-high wall surrounding Yung's house on three sides, blocking the light and air circulation. Although Yung refused to sell to his dying day (becoming a cause célèbre for the common man struggling against the establishment), the wall succeeded in convincing Yung's heirs to sell out.

The present paper builds on our earlier work in Pitchford and Snyder [20], which focused on sequential location as a source of transactions costs and on the question of whether it is more efficient for the court to assign property rights to the first mover or the second mover into an area. The efficiency of property rights did not depend on the identity of the generator in our earlier work because of assumptions in the model ensuring the generator's ideal externality level was not a function of its ex ante investment. In particular, we assumed

[1]The ''generator'' label for the party that prefers an interior solution for the externality level and ''recipient'' for the party that prefers a corner solution are appealing because they continue to be well defined if the negative-externality problem is translated into the equivalent positive-externality one (mapping, say, pollution into pollution abatement). There is a natural maximum that would be preferred by the recipient, namely abating pollution until the environment is returned to the state without any of the generator's pollution (it may prefer an even cleaner environment but the government typically would not enforce such a demand); there is no natural corner for the (negative) amount of abatement that the generator would prefer. Appendix C shows that calling the party that prefers a corner solution for the externality the ''generator'' and the party preferring a corner solution the ''recipient'' is consistent with a first-principles definition of the generator as the party that chooses an action affecting the recipient's utility.

[2]*Thrasher v. City of Atlanta*, 178 Ga. 514, 173 S.E. 817 (1934). This and the subsequent legal cases we cite were originally cited in Coase [4].

that the externality was constrained to lie in a bounded set $[0, \bar{e}]$; the recipient's ideal externality level was at one corner, zero, and the generator's ideal externality level was at the other corner, $\bar{e}$. In the present paper, we adopt the more natural assumption that the externality level is unbounded above, though it continues to be bounded below by zero. Hence, the generator's ideal externality level is an interior solution that in general depends on its ex ante investment. The assumptions in our earlier work simplified the analysis, but forced us to abstract from the strategic effects that are the focus of the present paper. The model in both Pitchford and Snyder [20] and the present paper is related to the incomplete-contracts literature begun in Grossman and Hart [9] and Hart and Moore [11] to explain ownership in a theory of the firm. As is standard in this literature, in our model there is non-contractible investment ex ante but efficient bargaining ex post. The possibility that injunction rights may lead a party to distort its investment to increase the harm it can threaten another party in externality problems was noted informally by Mumey [17], and is related to the extensive literature on blackmail [15,7,16,12,21,8]. As a by-product of our analysis, we provide a fresh view of the difference between damages versus injunctions, adding to the large literature including Calabresi and Melamed [3], Ayers and Talley [1], Kaplow and Shavell [13,14], and Sherwin [22].

We close the introduction by previewing the structure of the paper and results. The core of the paper is contained in Sections 2–4. These sections provide the model and analysis for the canonical case in which the externality is purely negative and property rights are allocated to the first mover into the area. We view this model as canonical on practical grounds—it encompasses many if not most practical settings—and on pedagogical grounds—it most clearly manifests the asymmetry between the generator and recipient.

Section 5 gauges the robustness of the basic results by analyzing alternatives to the canonical assumptions. In Section 5.1, we analyze the case in which property rights are allocated to the second rather than the first mover. With second-mover rights, the court's key problem is to prevent the second mover from holding up of the first's investment. The best way for the court to address this hold-up problem is to weaken the second-mover's rights as much as possible, regardless of whether it is a generator or recipient. Hence the identity of the generator is immaterial for the design of efficient second-mover rights.[3] Though we point out possibly severe inefficiencies with second-mover rights which may prevent their widespread allocation in practice, Section 5.1 is still of theoretical interest because the results highlight the importance of timing for the asymmetry between generators and recipients we identify in this paper.

In Section 5.2, we extend the analysis to the case of a mixed externality, providing benefits to the recipient at low levels but generating harm at higher levels. In this case, the recipient has an interior solution for its preferred externality level, blurring the strategic asymmetry between the generator and recipient. Still, we show that there are certain rights regimes under which the distinction between generator and recipient continues to be economically meaningful even for a mixed externality.

Appendix A details the regularity conditions on surplus functions to ensure that the social optimum is an interior solution. These conditions are not central, but allow us to state the propositions more elegantly with strict inequalities. Appendix B contains the proofs of the propositions. Appendix C shows how the mathematical difference between a generator and recipient—the generator's preferred externality level is an interior solution and the recipient's is a corner—can be traced back to the first-principles notion of the generator as the party choosing an action affecting the recipient's utility. The Appendix provides the necessary assumptions and propositions to make the connection. Appendix C is available through *JEEM*s online archive of supplementary material, which can be accessed at http://www.aere.org/journal/index.html.

2. Model

The variety of definitions in the economics literature (see [5]) suggest how difficult it is to define the concept of an externality, let alone the concepts of generator and recipient that underlie Coase's [4] debate with Pigou [19]. Thus, we devote considerable attention in Appendix C to a rigorous discussion of general definitions of the generator and recipient. To simplify discussion in the body of the paper, however, most of the analysis will

[3] The identity of the generator matters with first-mover rights because the investment distortion is more subtle than a simple hold-up problem. The problem is to prevent the first mover/rights holder from distorting its investment to extract more bargaining surplus from the second mover.

focus on the simple characterization of a unidirectional, purely negative externality provided in this section. Appendix C traces the connection between the general definitions and the simple characterization provided here. Extensions to other cases besides purely negative externalities will be analyzed in Section 5.2.

The model has two periods, an ex ante and an ex post period, two players $i = 1, 2$, and a court, which specifies and enforces a property-rights rule. In the ex ante period, the court specifies a property-rights regime. Then player 1 becomes aware of an opportunity to sink investment expenditure $x_1 \in [0, \infty)$ in a specific location. The land on which player 1 invests is assumed to have been purchased in a competitive market at a price of zero.[4] Player 2 arrives in the ex post period. It has the opportunity to invest $x_2 \in [0, \infty)$ at a location near player 1.[5] Location in the nearby area leads to a negative externality $e \in [0, \infty)$ between the players. We assume the players can engage in frictionless bargaining over x_2 and e, so that they end up choosing the levels which maximize their joint payoff. The sole transaction cost in the model is that players cannot bargain over x_1; this follows directly from our assumption of *ex ante anonymity*, i.e., that the identity of player 2 is unknown to player 1 when 1 makes its ex ante investment decision.[6]

Let $u_i(x_i, e)$ be the gross surplus function for player $i = 1, 2$. We will maintain a number of assumptions on $u_i(x_i, e)$ throughout the paper (Assumptions 1–5) including differentiability, concavity, and a series of Inada-type conditions ensuring interior solutions for the privately and socially optimal level of x_i and for the generator's optimal stand-alone externality level. Since these assumptions are standard regularity conditions not of central importance to the arguments of the paper, they are relegated to Appendix A. Under these maintained assumptions, the definition of a purely negative externality, which will be the focus of much of the subsequent analysis in the paper, can be made precise.

Definition 1. Let player i be the recipient of a unilateral externality e. Then e is a *purely negative externality* if and only if $\partial u_i(x_i, e)/\partial e < 0$ for all $x_i, e \in [0, \infty)$.

Definition 1 implies that the recipient suffers increasing harm from higher levels of a purely negative externality. Its preferred externality level is thus zero. It is also straightforward to characterize the generator in the case of a purely negative externality. Starting from the general definition of the generator as the party that chooses an action leading to the externality, under maintained regularity conditions (Assumptions 2, 4 and 5 in Appendix A), the identity of the generator reduces to the identity of the party whose surplus is initially increasing in e, reaching an interior optimum, and then declining for larger e. Appendix C, available through *JEEM*s online archive of supplementary material, which can be accessed at http://wwww.aear.org/journal/index.html, provides the details of the argument.

[4]This simplifying assumption can be justified if the second-highest bidder in a second-price private-values auction for the plot is neither a generator or recipient of the externality. If so, the land price will be independent of the property-rights regime and can be netted out of player 1's surplus function without loss of generality. More generally, the land price may depend on the property-rights regime. Abstracting from this complication does not sacrifice much generality because the land price only affects the extensive margin of whether 1 shows up in the location (taken for granted in the model) and not its marginal investment incentives. See White and Wittman [25] for a model of externalities with endogenous location.

[5]Player 2's surplus function can be thought of as netting out the price of land following the logic of the previous footnote. For 2 to win the land auction against other bidders that may be less affected by the externality problem, 2 must obtain quasi rents from locating in the area. If not, all externality problems could be solved by the land market and would never be observed in practice. A number of interesting outcomes could emerge from explicitly modeling the land auction. Assuming there are bidders who are affected by the externality but still obtain sufficient quasi rents to outbid those that are less affected, and assuming these bidders are fairly homogeneous, then the land price would be bid up to player 2's equilibrium surplus. The main results of the model would go through unchanged (the sole minor change being that the land price paid by 2 would extract all of its equilibrium bargaining surplus). Assuming instead that bidders are fairly heterogeneous in both their quasi rents and the effect of the externality effect on them, an additional strategic effect would arise in that the player 1's investment would affect the identity of the second mover who wins the auction for the land. We abstract from this selection effect in this paper.

[6]The implicit assumption is that 1 has perfect foresight regarding 2's surplus function but not 2's identity. This assumption simplifies the presentation of the results but is not crucial. The assumption could be dropped by extending the model to allow for a distribution over the second-mover's preferences and having the first mover maximize an expectation over this distribution. There also might be other sources of contractual incompleteness besides ex ante anonymity. As in Grossman and Hart [9], there may be resolution of uncertainty over time that makes contracting easier ex post than ex ante. Alternatively, the externality may be expected to harm one of a large number of current neighbors but unknown exactly which, and a collective-action problem may prevent efficient ex ante bargaining.

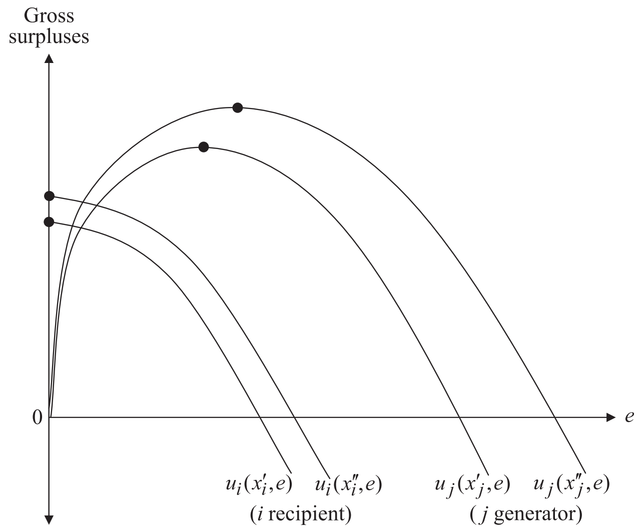

Fig. 1. Gross surplus functions in the case of a purely negative externality.

With a purely negative externality, the key distinction between the generator and recipient then is that the recipient's preferred externality level, zero, does not vary with its investment, while in general the generator's may. Referring to Fig. 1, taking i to the be recipient, an increase in its investment from x'_i to x''_i does not affect its preferred externality level, which is a corner solution at zero. In contrast, taking j to be the generator, an increase in its investment will affect its desired externality level. This strategic effect depends on the sign of the cross partial derivative $\partial^2 u_j/\partial x_j\,\partial e$. Fig. 1 depicts the case in which $\partial^2 u_j/\partial x_j\,\partial e > 0$, so that an increase in the generator's investment from x'_j to x''_j increases its marginal benefit from an additional unit of e, in turn implying that its preferred externality level increases. (This case would arise, for example, if investment increases the size of the generator's facility; the larger the facility, the more pollution generated when the facility is run at optimal capacity.) In this case, by investing more, the generator can increase the harm it can credibly threaten to inflict on the recipient. The absence of this effect with the recipient, and its presence with the generator, is the fundamental asymmetry that will lead to our main results in Section 4.

Let $v_1(x_1, e) = u_1(x_1, e) - x_1$ be 1's surplus net of investment. Let

$$v_2(e) = \max_{x_2 \in [0,\infty)} [u_2(x_2, e) - x_2]$$

be 2's. It turns out to be convenient to specify the second-mover's net surplus as the value function $v_2(e)$ because x_2 is chosen after players bargain and can be set at the private and social optimum, and so does not have an important bearing on the analysis. Define

$$e_1^*(x_1) = \operatorname*{argmax}_{e \in [0,\infty)} v_1(x_1, e),$$

$$e_2^* = \operatorname*{argmax}_{e \in [0,\infty)} v_2(e),$$

$$e^{**}(x_1) = \operatorname*{argmax}_{e \in [0,\infty)} [v_1(x_1, e) + v_2(e)].$$

In words, $e_1^*(x_1)$ and e_2^* are the privately optimal externality levels in the players' stand-alone problems, and $e^{**}(x_1)$ is the joint optimum.

The notation indexes the timing of players' moves independently of the identity of the generator/recipient and the identity of the rights holder. This will facilitate the analysis of a variety of cases including (a) the case in which rights are allocated to the first mover, and the first mover happens to be the recipient of an externality, (b) the case in which rights are allocated to the first mover, and the first mover happens to be the generator of the externality, and (c) analogous cases in which rights are allocated to the second mover. All of these cases will be analyzed below.

Player 1's ex ante choice of x_1 affects both players' equilibrium allocations through the bargain that takes place between players ex post. We assume efficient bargaining, in particular the version of Nash [18] bargaining in Binmore et al. [2] involving an exogenous probability of breakdown ex post. Let $\alpha \in (0,1)$ be player 1's share of the gains from Nash bargaining and $1-\alpha$ be 2's share. If bargaining breaks down, the default or threat-point outcome is determined by the property-rights regime specified by the court ex ante. That is, a breakdown in bargaining leaves the players to select e according to the property rights specified by the court. Let $t_i(x_1)$ be player i's threat-point payoff. Since the threat points typically involve an inefficient choice of e, players will bargain to the ex post efficient choice of e. Let $s(x_1)$ denote the resulting maximized joint surplus:

$$s(x_1) = \max_{e\in[0,\infty)} [v_1(x_1,e) + v_2(e)] \tag{1}$$

$$= v_1(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1)). \tag{2}$$

Player 1's equilibrium surplus from Nash bargaining is the sum of its threat point $t_1(x_1)$ and α times the gains from bargaining $s(x_1) - t_1(x_1) - t_2(x_1)$, which upon rearranging equals

$$s(x_1) - (1-\alpha)[s(x_1) - t_1(x_1)] - \alpha t_2(x_1). \tag{3}$$

The first term in (3) is social surplus; the remaining terms reflect the gap between social surplus and player 1's private surplus. Since it is based on net utility functions, expression (3) already nets out 1's investment expenditure x_1 and thus reflects player 1's surplus from an ex ante perspective. Eq. (3) is thus the relevant objective function player 1 maximizes when choosing x_1. We only need to specify player 1's ex ante payoff function because 1's choice of x_1 is the only welfare-relevant one in the model. All other variables (x_2 and e) are chosen optimally ex post conditional on x_1 due to efficient bargaining.

3. First-mover property rights

The court sets the property-rights regime ex ante. Property rights affect the equilibrium outcome through the following chain of logic. Property rights determine the threat points $t_i(x_1)$ that players would earn if bargaining were to break down. Although bargaining does not break down in equilibrium, the threat points still enter player 1's surplus function according to the Nash bargaining assumption reflected in (3). Player 1's bargaining surplus function (3) is the relevant objective function determining its equilibrium investment, x_1. Since x_1 is the only variable not subject to efficient negotiation, it completely determines the equilibrium outcome. In this section we will define various property-rights regimes; Section 4 will analyze their efficiency.

Property rights are multidimensional, specifying among other things the variables the holder is allowed to choose, the penalty for infringement, and rules for determining the identity of the holder. For example, property rights can be conditioned on the period in which the players show up. Rights are often allocated to the first mover into a location whether it is a generator or recipient, following the so-called "coming to the nuisance" doctrine. In theory, however, property rights could also be allocated to the second mover. Second-party property rights will be analyzed in Section 5.1. The present section and Section 4 will restrict attention to first-party rights because the asymmetry between generator and recipient is most apparent in this case.

Besides restricting attention to first-party rights, we restrict attention further to two commonly studied property-rights regimes, injunctions and damages. An injunction regime gives the holder the right to set e if bargaining breaks down. If player 1 is the injunction-rights holder, it would set e to maximize its stand-alone payoff, i.e., it would choose externality level $e_1^*(x_1)$. The threat-point payoffs corresponding to injunction rights are therefore $t_1(x_1) = v_1(x_1, e_1^*(x_1))$ and $t_2(x_1) = v_2(e_1^*(x_1))$.

We formalize damage rights in the following way. The holder does not have the right to set e—the other player does—but has the right to extract a payment equal to the difference between its surplus if the externality level were set at its preferred level less its realized surplus. More concretely, if player 1 is the damage-rights holder, player 2 has the right to set e but must pay player 1 $u_1(x_1, e_1^*(x_1)) - u_1(x_1, e)$. Player 1's threat-point payoff equals its realized surplus $u_1(x_1, e) - x_1$ plus the damage payment, which upon rearranging, equals $t_1(x_1) = v_1(x_1, e_1^*(x_1))$. To compute $t_2(x_1)$, we need to solve for 2's optimal choice of e if bargaining breaks down. This choice maximizes 2's surplus $v_2(e)$ minus the damage payment $u_1(x_1, e_1^*(x_1)) - u_1(x_1, e)$, which

Table 1
Threat-point payoffs for first-mover property-rights regimes

First-mover property-rights regime	Abbreviation	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Injunction rights	FIR	$v_1(x_1, e_1^*(x_1))$	$v_2(e_1^*(x_1))$
Damage rights	FDR	$v_1(x_1, e_1^*(x_1))$	$s(x_1) - v_1(x_1, e_1^*(x_1))$

upon rearranging equals

$$u_1(x_1, e) + v_2(e) - u_1(x_1, e_1^*(x_1)). \tag{4}$$

It is straightforward to see that expression (4) is maximized by setting e to the joint optimum $e^{**}(x_1)$. Substituting $e^{**}(x_1)$ for e in (4) and rearranging, we have $t_2(x_1) = s(x_1) - v_1(x_1, e_1^*(x_1))$.

Table 1 lists the threat points for reference. Note that $t_1(x_1)$ is the same in both injunctions and damages regimes; the rights regimes only differ in the specification of $t_2(x_1)$. Throughout the next section we will refer to the first-mover injunction regime simply as "injunctions" and a first-mover damages regime simply as "damages."

4. Analysis

4.1. Preliminaries

This section analyzes equilibrium investment and social welfare for the case of first-party rights and for the case of a unidirectional, purely negative externality. We will show that the generator of an externality differs in an economically meaningful way from the recipient in this case. To do this, we determine the social ranking of rights regimes in the case in which player 1 is the recipient (Proposition 1) and compare this ranking with the case in which player 1 is the generator (Proposition 2). Before turning to Propositions 1 and 2 in the next two subsections, we will provide some preliminary results in this subsection.

Lemma 1 verifies that the recipient's preferred level of the externality is zero and that the socially preferred level lies strictly between the recipient's and generator's preferred choices. The results will be used in the proofs of the subsequent propositions.

Lemma 1. *Suppose the externality is unidirectional and purely negative.*

(a) *If player* 1 *is the generator and player* 2 *is the recipient, then* $0 = e_2^* < e^{**}(x_1) < e_1^*(x_1)$.
(b) *If player* 1 *is the recipient and player* 2 *is the generator, then* $0 = e_1^*(x_1) < e^{**}(x_1) < e_2^*$.

The proofs of Lemma 1 and subsequent propositions are contained in Appendix B. Assumptions 1–5 are maintained in Lemma 1 and in all subsequent propositions, but for brevity have been omitted from the statement of these results.

As discussed in Section 2, the efficiency of a rights regime is completely determined in the model by how close ex ante investment x_1 is to the first best since x_1 is the only variable not set by frictionless bargaining. Let x_1^{FIR} be player 1's equilibrium ex ante investment if it holds injunction rights, x_1^{FDR} if it holds damage rights, and x_1^{1ST} first-best investment. (The F in the superscript designates that rights are allocated to the first mover.) To compare x_1^{FIR} and x_1^{FDR} to x_1^{1ST}, we will proceed by specifying player 1's surplus functions under injunctions and damages, taking the first-order condition characterizing equilibrium investment in each case, and nesting the first-order conditions so that we can easily see how equilibrium investment compares to the first best.

Player 1's surplus under injunctions is derived by substituting the relevant threat points from Table 1 into the expression for the Nash bargaining surplus (3), yielding

$$s(x_1) - (1-\alpha)[s(x_1) - v_1(x_1, e_1^*(x_1))] - \alpha v_2(e_1^*(x_1)). \tag{5}$$

Similarly, player 1's surplus under damages is

$$v_1(x_1, e_1^*(x_1)) = s(x_1) - [s(x_1) - v_1(x_1, e_1^*(x_1))]. \tag{6}$$

Before taking the first-order conditions associated with objective functions (5) and (6), we establish some useful facts. By the Envelope Theorem,

$$\frac{dv_1(x_1, e_1^*(x_1))}{dx_1} = \frac{\partial v_1(x_1, e_1^*(x_1))}{\partial x_1} \tag{7}$$

and

$$s'(x_1) = \frac{d}{dx_1}[v_1(x_1, e^{**}(x_1)) + v_2(e^{**}(x_1))] = \frac{\partial v_1(x_1, e^{**}(x_1))}{\partial x_1}. \tag{8}$$

By the Fundamental Theorem of Calculus,

$$\frac{\partial v_1(x_1, e^{**}(x_1))}{\partial x_1} - \frac{\partial v_1(x_1, e_1^*(x_1))}{\partial x_1} = \int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \, \partial e} de. \tag{9}$$

Differentiating (5) and (6) and substituting (7)–(9), we can nest the first-order condition determining player 1's investment as

$$s'(x_1) - \theta_1 \left[\int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 v_1(x_1, e)}{\partial x_1 \, \partial e} de \right] - \theta_2 \left[v_2'(e_1^*(x_1)) \frac{de_1^*(x_1)}{dx_1} \right], \tag{10}$$

where

$$\theta_1 = \begin{cases} 0 & \text{first best,} \\ 1-\alpha & \text{injunctions,} \\ 1 & \text{damages} \end{cases} \quad \text{and} \quad \theta_2 = \begin{cases} 0 & \text{first best,} \\ \alpha & \text{injunctions,} \\ 0 & \text{damages.} \end{cases} \tag{11}$$

Eq. (10) has three terms. The first, $s'(x_1)$, captures the first-best investment incentives. The remaining two terms capture distortions from the first best. The second term, premultiplied by θ_1, is the distortion arising because player 1's bargaining surplus depends in part on its threat point and not solely on the social surplus. This distortion is larger for damages than injunctions because, as Eq. (11) shows, θ_1 is greater for damages. Intuitively, players do not need to bargain with damage rights because the damage payment induces player 2 to set the externality efficiently. Thus, as Eq. (6) shows, player 1's bargaining surplus depends only on its threat point $v_1(x_1, e_1^*(x_1))$ and not at all on social surplus. An injunction right induces a more encompassing objective function since bargaining is required to set the externality efficiently, and 1 obtains a share of social surplus in the bargaining process in proportion to its bargaining power. On the basis of the second term in (10) alone, damages would be more distortionary than injunctions.

However, the third term in (10), premultiplied by θ_2, must still be accounted for. The third term reflects the distortion in 1's investment to gain a better bargaining position by worsening 2's threat point. We will argue in Section 4.2 that if 1 is the recipient, this third term disappears and injunctions are unambiguously more socially efficient than damages. We will show in Section 4.3 that if 1 is the generator, this third term has the same sign as the second and exacerbates the investment distortion. Injunctions and damages then cannot be unambiguously ranked.

4.2. *First-party recipient*

This section analyzes the case in which player 1 is the recipient of a purely negative externality. We will find that the recipient's equilibrium investment under injunctions (x_1^{FIR}) and damages (x_1^{FDR}) always differ from the first best (x_1^{1ST}). Whether the distortion is upwards or downwards depends on the interaction between the recipient's investment and the externality, which in formal terms depends on the sign of the cross partial $\partial^2 u_1(x_1, e)/\partial x_1 \, \partial e$. A series of definitions about this cross partial will allow us to state the main result of the section succinctly.

Definition 2. Investment *increases recipient i's vulnerability* if an increase in x_i increases i's marginal harm from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i\,\partial e < 0$.

For example, in the *Thrasher v. Atlanta* case, the homeowner, the recipient of the airport's noise externality, could invest in a big house with fine construction details, thus exposing more housing value to the noise externality. Such investments are referred to as increasing the homeowner's vulnerability. Alternatively, the homeowner could build the house farther from the property lines with soundproofed walls. As the following definition states, we refer to such investment as decreasing the homeowner's vulnerability.

Definition 3. Investment *reduces recipient i's vulnerability* if an increase in x_i reduces i's marginal harm from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i\,\partial e > 0$.

The next proposition states that x_1^{FIR} is closer to x_1^{1ST} than x_1^{FDR}—and so injunctions are more efficient than damages—if 1 is the recipient.

Proposition 1. *Suppose the following: the externality is unidirectional and purely negative, player* 1 *is the recipient and the rights holder, player* 2 *is the generator, investment either increases* 1*'s vulnerability for all* $x_1, e \in [0, \infty)$ *or decreases* 1*'s vulnerability for all* $x_1, e \in [0, \infty)$, *and* $\alpha \in (0, 1)$.

(a) *Social welfare is strictly less than in the first best with both injunctions and damages.*
(b) *Injunctions are strictly socially more efficient than damages.*
(c) *If player* 1*'s investment reduces its vulnerability, then there is underinvestment in both regimes relative to the first best, with* $x_1^{\text{FDR}} < x_1^{\text{FIR}} < x_1^{\text{1ST}}$.
(d) *If player* 1*'s investment increases its vulnerability, then there is overinvestment in both regimes relative to the first best, with* $x_1^{\text{1ST}} < x_1^{\text{FIR}} < x_1^{\text{FDR}}$.

Parts (a) and (b) of Proposition 1 follow from an examination of Eq. (10). When player 1 is the recipient, its preferred externality level is a corner at zero, $e_1^*(x_1) = 0$, implying $de_1^*(x_1)/dx_1 = 0$, in turn implying that the third term in (10) disappears. The only remaining distortion is the second term, premultiplied by θ_1. As Eq. (11) indicates, this distortion term is present for both injunctions and damages, but the distortion is larger for damages.

Parts (c) and (d) of Proposition 1 can be summarized together as saying that if player 1 is the recipient of a purely negative externality, its investment will always be distorted in the direction of making it more vulnerable whether it holds injunction or damage rights. (In particular, part (c) says that 1 will underinvest if investment reduces its vulnerability, and part (d) says 1 will overinvest if investment increases its vulnerability.) Player 1 does not fully internalize its vulnerability to the externality because, as the rights holder, it is insulated from harm from the externality in its threat point.

Inspection of Eqs. (10) and (11) allows us to characterize some knife-edged cases not covered by Proposition 1. First, if $\alpha = 0$, injunctions and damages are equally socially efficient, though both are strictly less efficient than the first best. Second, if $\alpha = 1$, injunctions yield the first best and are strictly more efficient than damages. This can be seen by substituting $\alpha = 1$ into the expression for θ_1 in (11) and because the third term in (10) disappears if 1 is the recipient. Third, if $\partial^2 u_1(x_1, e)/\partial x_1\,\partial e = 0$ for all $x_1, e \in [0, \infty)$, injunctions and damages both yield the first best. This result holds because the integrand in the second term of (10) equals zero if $\partial^2 u_1(x_1, e)/\partial x_1\,\partial e = 0$ and recalling the third term disappears if 1 is the recipient.

4.3. First-party generator

We next turn to the analysis of the case in which player 1 is the generator. When player 1 is the generator, the last term in Eq. (10) typically does not disappear because $e_1^*(x_1)$ is an interior solution with $de_1^*(x_1)/dx_1 \neq 0$. The second term, premultiplied by θ_1, is larger with damages than injunctions and the third, premultiplied by θ_2, is larger with injunctions than damages. Which distortion is larger depends on functional forms and parameters.

Proposition 2 provides some cases in which damages are socially more efficient than injunctions when player 1 is a generator. A series of definitions concerning the type of technology that the generator can adopt will allow us to state the proposition succinctly. An investment is said to be "dirty" if higher levels increase the generator's marginal benefit from the externality. For example, in the *Thrasher v. Atlanta* case, if the noise generated by the airport is in proportion to the number of takeoffs and landings, expanding the scale of the airport's operation will naturally increase its benefit from an extra interval of noise. Alternatively, a "clean" investment reduces the marginal benefit from pollution. This could occur, for example, if the airport buys new airplanes that are quieter than the old ones. Formally, we have the following definitions.

Definition 4. Generator i's investment is *clean* if an increase in x_i reduces i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i\, \partial e < 0$.

Definition 5. Generator i's investment is *dirty* if an increase in x_i increases i's marginal benefit from the externality; i.e., $\partial^2 u_i(x_i, e)/\partial x_i\, \partial e > 0$.

Proposition 2. *Suppose the following: the externality is unidirectional and purely negative, player* 1 *is the generator and rights holder, player* 2 *is the recipient,* $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ *for some* $\gamma > 0$, *and* $\alpha \in (0, 1)$.

(a) *Both injunctions and damages are strictly socially inefficient compared to the first best.*
(b) *If player* 1*'s investment is dirty, there exists* γ' *such that for all* $\gamma < \gamma'$, $x_1^{1ST} < x_1^{FDR} < x_1^{FIR}$, *and social welfare is higher with damages than with an injunction.*
(c) *If player* 1*'s investment is clean, there exists* γ'' *such that for all* $\gamma < \gamma''$, $x_1^{FIR} < x_1^{FDR} < x_1^{1ST}$, *and, again, social welfare is higher with damages than with an injunction.*

Given the functional form for player 1's surplus, $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$, the impact of the externality on its total and marginal payoff becomes negligible in the limit as $\gamma \to 0$, whereas the choice of externality by the generator is unaffected by γ since $e_1^*(x_1)$ solves $\partial h(x_1, e_1^*(x_1))/\partial e \equiv 0$. In other words, player 1 does not benefit much from polluting, but since the benefit is positive, its ideal pollution level can remain relatively high. Under an injunction it can continue credibly to threaten the other party with substantial harm from the externality. Thus, the strategic incentive that the generator has to harm the recipient through its choice of externality—the third term in Eq. (10)—does not vanish. The only source of distortion with damages—the second term in Eq. (10)—does vanish as $\gamma \to 0$. To see this, substituting $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$, the second term of (10) becomes

$$-\theta_1 \gamma \int_{e_1^*(x_1)}^{e^{**}(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1\, \partial e}\, de, \tag{12}$$

which obviously approaches zero as $\gamma \to 0$. Therefore, social welfare under damages approaches the first best as $\gamma \to 0$.

Inspection of Eqs. (10) and (11) allows us to characterize some knife-edged cases not covered by Proposition 2. If $\alpha = 0$, social welfare is the same whether the first-party generator is allocated an injunction or damage right, though both are strictly less efficient than the first best, as can be seen by substituting $\alpha = 0$ into (11). If $\partial^2 h(x_1, e)/\partial x_1\, \partial e = 0$ for all $x_1, e \in [0, \infty)$, injunctions and damages both yield the first best. This result holds because, in the second term in (10), the integrand equals zero and, in the third term in (10), $de_1^*(x_1)/dx_1 = 0$ if $\partial^2 h(x_1, e)/\partial x_1\, \partial e = 0$. Combined with the result from Section 4.2 that the first best is obtained if either injunction or damage rights are allocated to a first-party recipient if its surplus function satisfies $\partial^2 u_1(x_1, e)/\partial x_1\, \partial e = 0$ for all $x_1, e \in [0, \infty)$, we can conclude that the first best can be obtained regardless of the allocation of property rights and the identity of the generator if there is no interaction effect between x_1 and e in 1's surplus function.

4.4. Assessment of asymmetry

Under what conditions can the identified strategic effect, which leads to the asymmetry between the generator and recipient, be expected to lead to substantial social costs? One basic condition is that expenditures on x_1 be substantial, for this is the sole source of distortion in the model. Other conditions can be understood from an examination of the last term in (10), which is the mathematical expression for the asymmetric strategic effect. The distortion is larger if investment and the externality are mainly used for offensive rather than socially productive purposes, i.e., if $e_1^*(x_1)$ has a large effect on the threat point, and x_1 has a large effect on $e_1^*(x_1)$, but x_1 has little effect on social welfare given the efficient externality choice $e^{**}(x_1)$. The distortion is also larger the higher is α. If α is high, the only source of surplus for player 2 is its threat point, and so the only way for 1 to extract surplus from 2 is to distort its investment to harm 2's threat point. The law can target such cases by, for example, taking away property rights if they are abused solely to injure the other party.[7] In practice it may be difficult to condition rights on abuse/intent and simpler to condition rights on the identity of the generator/recipient.

5. Extensions

This section gauges the robustness of the results from the canonical model. As shown in Section 5.1, the asymmetry between the generator and recipient is only present with first-mover rights, disappearing with second-mover rights. While we argue first-mover rights may be of more practical relevance, the analysis of second-mover rights highlights the crucial role of the timing of moves for the asymmetry between generator and recipient. As shown in Section 5.2, the basic results continue to hold under some conditions if we generalize the model to allow for mixed externalities, i.e., those that are positive over an initial range and negative thereafter.

5.1. Second-mover property rights

The analysis has so far been restricted to property rights allocated to the first mover in the area. We restricted attention to this case for two reasons. One reason is that the asymmetry between generator and recipient only affects the efficiency of first-mover property-rights regimes, not second mover. The asymmetry between generator and recipient does not matter for the efficiency of second-mover rights because, as we will see from the main result proved in this section, the same second-mover rights regime will be efficient whether the rights holder is a generator or recipient.

Another reason for having focused on first-mover rights is that, while a particular second-mover rights regime will turn out to attain the first best in our simple model, in a richer model second-mover rights can be quite inefficient. In a fully specified dynamic model, second-mover rights would induce players to engage in a war of attrition, delaying until the other player moves in order to be second and win the property rights. The resulting delay may waste a considerable amount of social welfare. In addition, we have abstracted from player 1's decision to show up in the area. Player 2 may extract so much surplus from 1 if 2 holds the rights that 1 decides not to show up, again leading to a substantial loss of social welfare. The remainder of this section abstracts from these complexities, but they should be kept in mind as caveats to the result that the first best is attained by a particular second-mover rights regime.

Proposition 3 states that the first best is obtained under second-mover damage rights regardless of which player is the generator or recipient. To understand this result, note that the threat points under second-mover damages are $t_1(x_1) = s(x_1) - v_2(e_2^*)$ and $t_2(x_1) = v_2(e_2^*)$, by analogy to the entries for first-mover damages in Table 1. Substituting these threat points into the Nash bargaining formula (3) and rearranging yields $s(x_1) - v_2(e_2^*)$ for the objective function determining player 1's equilibrium investment. This objective function is identical to social welfare $s(x_1)$ except for the term $v_2(e_2^*)$, which is independent of x_1. Therefore, the first-best investment is obtained.

[7] See Hale [10] for additional relevant (and interesting) cases of malicious injury in tort law.

Proposition 3. *Suppose the externality is unidirectional and purely negative. The first best is obtained for all* $\alpha \in [0, 1]$ *if damage rights are allocated to player 2 regardless of whether it is the generator or recipient.*

5.2. Mixed externalities

The analysis through Section 4 was restricted to purely negative externalities. Another possibility is that the externality is mixed, providing a marginal benefit to the recipient at low levels but marginal harm at higher levels. For example, consider the *Thrasher v. Atlanta* case cited in the Introduction, in which the plaintiff was a homeowner harmed by the noise from the defendant's nearby municipal airport. Suppose for the sake of argument that the plaintiff obtained some benefits from the airport: increased local economic growth, better transportation, etc. The plaintiff then might prefer a small airport to none, although at higher air-traffic levels the harm from the noise might begin to outweigh the benefits. In this case, the municipal airport would be the generator and the homeowner the recipient of a mixed externality.

Fig. 2 depicts the mixed-externality case. The recipient is labeled i and the generator j. The fact that the recipient's surplus function $u_i(x_i, e)$ is initially increasing in e implies that the externality is positive at low levels. The fact that the recipient's optimal stand-alone externality level $e_i^*(x_i)$ is less than the generator's $e_j^*(x_j)$ implies that the externality is marginally harmful to the recipient in equilibrium, as will be seen.[8, 9]

With a purely negative externality, we saw that the generator and recipient were asymmetric because generator had an interior solution and the recipient a corner solution for their stand-alone optimal externality levels. With a mixed externality, the recipient's stand-alone optimum is also an interior solution, potentially eliminating the asymmetry between it and the generator. We will see that the asymmetry may or may not be eliminated, depending on the property-rights regimes that are feasible for the court to implement. To avoid a proliferation of subcases, we will return to the focus in Section 4 on first-party property-rights regimes, in particular, injunction and damage rights.

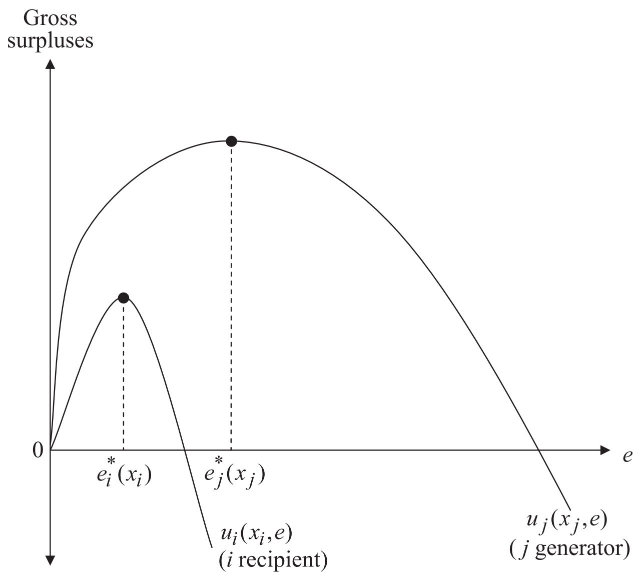

Fig. 2. Gross surplus functions in the case of a mixed externality.

[8]In most applications with mixed externalities, it is reasonable to suppose the recipient prefers lower levels of the externality than the generator, but it is theoretically possible that the reverse is true. In such cases, the analysis is similar to that for a purely positive externality and is omitted here since we are focusing on the problem of social harm.

[9]Note that $u_i(x_i, e)$ is drawn so that i is better off with $e = 0$ than $e = e_j^*(x_j)$. This implies that the recipient would rather do without the externality than allow the generator to pollute freely, an assumption that will be relied on to eliminate cases in the subsequent analysis.

Table 2
Threat-point payoffs for recipient of a mixed externality

First-mover property-rights regime	Abbreviation	Player 1's threat-point payoff $t_1(x_1)$	Player 2's threat-point payoff $t_2(x_1)$
Corner rights			
Injunction rights	FCIR	$v_1(x_1, 0)$	$v_2(0)$
Damage rights	FCDR	$v_1(x_1, 0)$	$s(x_1) - v_1(x_1, 0)$
Peak rights			
Injunction rights	FPIR	$v_1(x_1, e_1^*(x_1))$	$v_2(e_1^*(x_1))$
Damage rights	FPDR	$v_1(x_1, e_1^*(x_1))$	$s(x_1) - v_1(x_1, e_1^*(x_1))$

Mixed externalities introduce an additional degree of freedom into property-rights regimes not present with purely negative externalities. This additional degree of freedom must be specified for injunctions or damages not to be defined ambiguously. Consider the case of first-party injunction rights. One natural specification is that the injunction is all or nothing, requiring the externality to cease if the injunction is enforced ($e = 0$). Another natural specification is that the recipient can constrain the externality to be no greater than its stand-alone optimum ($e \leqslant e_1^*(x_1)$). We will call the first specification a "corner injunction," since e is forced to a corner at zero, and the second a "peak injunction," since e is constrained to the peak of 1's surplus function. (This ambiguity did not arise with a purely negative externality since the recipient's surplus function peaked at the corner of zero.) The same ambiguity arises with damage rights. One natural specification is that the recipient's compensation should make it as well off as it would be in the absence of the second party/generator, a regime we will call "corner damages," analogous to corner injunctions. Another is that the recipient's compensation make it as well off as it would be had e been set at its stand-alone optimum $e_1^*(x_1)$, a regime we will call "peak damages." Table 2 provides the threat-point payoffs for the corner and peak variants of first-party injunctions and damage rights.

There are two reasons for introducing corner and peak variants of the rights regimes. One is that it is unclear *a priori* which is socially more efficient. Another is that there may be technological barriers preventing the government from implementing a variant even if it would otherwise be more efficient. For example, in the *Thrasher* airport case, it may be prohibitively expensive for the government to monitor the frequency and decibel level of takeoffs and landings as would be required to implement a peak injunction, but straightforward for the government to shut the airport down, all that is required to implement a corner injunction. We expect that the corner variants of both injunctions and damages regimes would generally require less information and monitoring on the part of the government than their peak analogues in most applications and thus would be easier to implement.

The next proposition states that if the court is restricted to using corner rights, moving from a purely negative externality to a mixed externality preserves the results from Section 4. It is still the case that a court would never weaken rights offered to a recipient—allocating corner injunctions to a recipient is always more socially efficient than (weaker) corner damages—but it is sometimes still efficient to weaken the rights offered to a generator.

Proposition 4. *Suppose that the court is restricted to allocating corner rights to player* 1 *and that* $\alpha \in (0, 1)$.

(a) *Suppose further that player* 1 *is the recipient and* 2 *the generator of a mixed externality, that* $0 < e_1^*(x_1) < e_2^*$, *and that investment either increases* 1*'s vulnerability for all* $x_i, e \in [0, \infty)$ *or decreases* 1*'s vulnerability for all* $x_i, e \in [0, \infty)$. *Social welfare is strictly higher under corner injunctions than corner damages.*
(b) *Suppose instead that player* 1 *is the generator and* 2 *the recipient of a mixed externality and that* $0 < e_2^* < e_1^*(x_1)$. *There exist cases in which social welfare is higher under corner damages than corner injunctions.*

The proof of Proposition 4 is an immediate corollary of previous propositions and is omitted from Appendix B. In particular, part (a) of the proposition follows from the observation that the threat points in a corner-rights regime for the recipient of a mixed externality are identical to those for the recipient of a purely negative externality. (To see this, compare the entries in Table 2 to those from Table 1 after substituting the stand-alone optimum for the recipient of a purely negative externality, $e_1^*(x_1) = 0$.) Thus, part (b) of Proposition 1 immediately applies. Part (b) of Proposition 4 follows directly from parts (b) and (c) of Proposition 2. When player 1 is the generator, the nature of the externality—purely negative or mixed—is irrelevant for the analysis. Whether player 2's stand-alone optimum e_2^* is a corner or an interior solution does not affect the proofs because e_2^* does not show up directly in 1's bargaining surplus function, as can be seen by substituting the entries from Table 1 into Eq. (3).

The cases alluded to in part (b) of Proposition 4—in which social welfare is higher under corner damages than corner injunctions—are analogous to those identified in Proposition 2. Taking the functional form $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$, corner damages are more efficient than corner injunctions in the limit as $\gamma \to 0$. Whether investment is higher or lower than optimal depends on whether 1's investment is clean or dirty, as stated in Proposition 2.

With a mixed externality, the generator and recipient both have an interior solution for their preferred externality level and so are not asymmetric in this respect. The source of asymmetry with corner rights is that the court effectively imposes a corner solution in the recipient's threat point irrespective of the recipient's preferences. The court-imposed corner solution prevents the recipient from being able to alter the other party's payoff by distorting its own investment. The results therefore mirror the purely negative-externality case. With peak rights, this is no longer true. If player 1 is a recipient, it has an incentive to distort its investment in a way that reduces 2's threat-point payoff, for the same reason as a generator in the purely negative-externality case. Examples can be constructed in which this incentive is so strong for a recipient that the court wishes to weaken it by allocating damage rights to the recipient.

Proposition 5. *Suppose that player* 1 *is the recipient and* 2 *the generator of a mixed externality, that* $0 < e_1^*(x_1) < e_2^*$, *that investment either increases* 1'*s vulnerability for all* $x_i, e \in [0, \infty)$ *or decreases* 1'*s vulnerability for all* $x_i, e \in [0, \infty)$, *and that* $\alpha \in (0, 1)$. *There exist cases in which social welfare is higher under peak damages than under either peak injunctions or corner injunctions.*

The proof provides an example in which peak damages dominate injunctions (the example involves the functional form $v_1(x_1, e) = g(x_1) + \gamma h(x_1, e)$ and takes limits as $\gamma \to 0$ and $\alpha \to 0$. Combined with Proposition 2, Proposition 5 implies that if peak rights are available to the court, the asymmetry between recipients and generators is a matter of degree rather than kind.

6. Conclusions

We presented a model with frictionless bargaining over the externality and other variables ex post but with transactions cost in the form of non-contractible investment ex ante. We identified a strategic effect present for a generator of a purely negative externality but not a recipient. If a generator has an injunction right—an unrestricted right to emit the externality—it will distort its ex ante investment to make itself more harmful to the recipient, thus increasing the surplus it extracts from the recipient in ex post bargaining. If the recipient has an injunction right, it does not need to distort its ex ante investment to harm the generator: regardless of its investment level, the recipient can cause maximal harm to the generator by forbidding the generator to emit the externality. We showed that property rights over the externality, which to be socially efficient should minimize distortions in ex ante investment, can be made more efficient in some cases by conditioning rights on the identity of the generator and recipient in a way that takes account of the asymmetry in the identified strategic effect. Thus, the distinction between the generator and the recipient can be economically meaningful in externality problems.

The asymmetry between the generator and recipient is less marked in some of the extensions to the model discussed in Section 5. As shown in Section 5.1, the asymmetry does not show up in second-mover rights regimes. In second-mover regimes, the only distortion is the simple hold-up problem (the hold-up of the first-mover's investment by the second), which the court can solve by suitably weakening the second-mover's rights, whether the second mover is a generator or a recipient. The practical value of second-mover rights may be limited because, as argued, second-mover rights quickly become quite inefficient if certain dynamic elements of the model are fully specified. Still, the results in Section 5.1 are of theoretical value because they show what timing of moves is necessary for the strategic effect identified in this paper.

Section 5.2 shows that if the externality is mixed, the recipient's preferred externality level may be, like a generator's, an interior solution, raising the possibility that the recipient's investment may be distorted, like a generator's, if it is allocated an injunction right. However, even in this case, there are natural specifications of property rights—corner rights—under which the asymmetry between generator and recipient persists.

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Appendix A. Regularity conditions on surplus functions

The following regularity conditions on players' surplus functions are maintained throughout the paper.

Assumption 1. $u_i(x_i, e)$ is continuously differentiable in both arguments for all $x_i, e \in [0, \infty)$.

Assumption 2. $u_i(x_i, e)$ is strictly concave for all $x_i, e \in [0, \infty)$.

Assumption 3. $u_i(x_i, e)$ satisfies an Inada condition in x_i; i.e., $\partial u_i(0, e)/\partial x_i = \infty$ for all $e \in [0, \infty)$.

Assumption 4. The net utility function $u_i(x_i, e) - x_i$ is coercive; i.e.,

$$\lim_{\|x_i, e\| \to \infty} [u_i(x_i, e) - x_i] = -\infty,$$

where $\|x_i, e\| = \sqrt{x_i^2 + e^2}$ is the distance norm.

Assumption 5. If i is the generator, $u_i(x_i, e)$ satisfies an Inada condition in e: $\partial u_i(x_i, 0)/\partial e = \infty$ for all $x_i \in [0, \infty)$.

Assumptions 1 and 2 are standard. Assumptions 3 and 4 ensure that the privately and socially optimal investment levels are in the interior of $[0, \infty)$. This is not essential for the results, but allows us to state our propositions more elegantly with strict inequalities, eliminating a number of economically uninteresting cases. Note that the assumption of coerciveness implies that both players' net surpluses become very negative if either the investment or the externality grow without bound. Assumption 5 ensures that the generator's privately optimal externality level and the socially optimal one are both in the interior of $[0, \infty)$. Again, this assumption is not essential for the results, but allows us to state our propositions more elegantly with strict inequalities.

Appendix B. Proofs of propositions

Proof of Lemma 1. We will prove part (a); the proof of part (b) is similar and thus omitted. Suppose player 1 is the generator and 2 is the recipient. Then $\partial u_2(x_2, e)/\partial e < 0$ by definition of the recipient, implying $v_2'(e) < 0$. Thus $e_2^* = \text{argmax}_{e \in [0,\infty)} v_2(e) = 0$. Assumption 5 implies $e^{**}(x_1) > 0$. The proof is completed by showing $e^{**}(x_1) < e_1^*(x_1)$. Consider the nested objective function

$$u_1(x_1, e) + v_2(e) - \theta v_2(e), \tag{13}$$

where $\theta = 0$ yields the objective function for $e^{**}(x_1)$ and $\theta = 1$ yields that for $e_1^*(x_1)$. We proceed by verifying the conditions required for Strict Monotonicity Theorem 1 of Edlin and Shannon [6] hold for expression (13).

Expression (13) is continuously differentiable because the individual terms are continuously differentiable by Assumption 1. Assumptions 4 and 5 imply $e_1^*(x_1)$ is an interior solution. The second cross partial of expression (13) with respect to e and θ equals $-v_2'(e)>0$. Hence, (13) exhibits increasing marginal returns. Thus, Strict Monotonicity Theorem 1 of Edlin and Shannon [6] applies, implying $e^{**}(x_1)<e_1^*(x_1)$. □

Proof of Proposition 1. Consider the case in which investment by player 1 (the recipient) reduces its vulnerability. The case in which player 1's investment increases its vulnerability is analyzed similarly and thus omitted.

We will first prove $x_1^{\text{FDR}}<x_1^{\text{FIR}}$. By Lemma 1, since player 1 is the recipient, $e_1^*(x_1)=0$, implying $de_1^*(x_1)/dx_1=0$. Substituting into Eq. (10), the first-order condition for player 1's investment can be written

$$s'(x_1)-\theta_1\int_0^{e^{**}(x_1)}\frac{\partial^2 v_1(x_1,e)}{\partial x_1\,\partial e}\,de, \tag{14}$$

where θ_1 is given by Eq. (11). Since investment reduces 1's vulnerability, $\partial^2 u_1(x_1,e)/\partial x_1\partial e>0$, implying $\partial^2 v_1(x_1,e)/\partial x_1\partial e>0$. Further, since 1 is the recipient, $e^{**}(x_1)>0$ by Lemma 1. Hence the partial derivative of (14) with respect to θ_1 is negative, implying the objective function determining 1's investment exhibits decreasing marginal returns in x_1 and θ_1. Since θ_1 is higher under damages ($\theta_1=1$) than injunctions ($\theta_1=1-\alpha$), steps similar to the proof of Lemma 1 can be used to show that Strict Monotonicity Theorem 1 implies $x_1^{\text{FDR}}<x_1^{\text{FIR}}$.

Next, we will show $x_1^{\text{FIR}}<x_1^{\text{1ST}}$. Since the objective function determining 1's investment exhibits decreasing marginal returns in x_1 and θ_1 and since θ_1 is higher under injunctions ($\theta_1=1-\alpha$) than in the first best ($\theta_1=0$), steps similar to the preceding paragraph can be used to show $x_1^{\text{FIR}}<x_1^{\text{1ST}}$.

Finally, we need to translate the investment ranking into a social-welfare ranking. By Assumption 2, $u_2(x_2,e)-x_2$ is strictly concave. Furthermore, it is maximized over a convex set $x_2\in[0,\infty)$. By the Maximum Theorem under Convexity (see, e.g., [23, Theorem 9.17.3]), the associated value function $v_2(e)$ is also strictly concave. By Assumption 2, $u_1(x_1,e)$ is strictly concave, implying $v_1(x_1,e)$ is strictly concave. The sum of strictly concave functions $v_1(x_1,e)+v_2(e)$ is strictly concave. By the Maximum Theorem under Convexity, the associated value function $s(x_1)$ is strictly concave. Therefore, the ranking $x_1^{\text{FDR}}<x_1^{\text{FIR}}<x_1^{\text{1ST}}$ implies damages are strictly less efficient than an injunction, which in turn is less efficient than the first best. □

Proof of Proposition 2. Suppose $v_1(x_1,e)=g(x_1)+\gamma h(x_1,e)$ for some $g(x_1)$ satisfying Assumptions 1–4; for some $h(x_1,e)$ satisfying Assumptions 1–5; and for $\gamma>0$. We will prove the proposition for the case in which investment by player 1 (the generator) is dirty. The proof for the case in which its investment is clean is similar and thus omitted.

We will first show $x_1^{\text{1ST}}<x_1^{\text{FDR}}$ for all $\gamma>0$. Substituting the functional form for v_1 into Eq. (10) yields the following nested first-order condition for player 1's investment:

$$s'(x_1)-\theta_1\gamma\int_{e_1^*(x_1)}^{e^{**}(x_1)}\frac{\partial^2 h(x_1,e)}{\partial x_1\,\partial e}\,de, \tag{15}$$

where $\theta_1=0$ in the first best and $\theta_1=1$ under damages. Since 1's investment is dirty, we have both (a) that $\partial^2 h(x_1,e)/\partial x_1\,\partial e>0$ by definition and (b) that $e^{**}(x_1)<e_1^*(x_1)$ by Lemma 1. Hence the partial derivative of (15) with respect to θ_1 is positive, implying that the objective function determining 1's investment exhibits increasing marginal returns in x_1 and θ_1. Steps similar to the proof of Lemma 1 can be used to show that Strict Monotonicity Theorem 1 implies $x_1^{\text{1ST}}<x_1^{\text{FDR}}$.

Next, we shown $x_1^{\text{1ST}}<x_1^{\text{FIR}}$ for all $\gamma>0$. This fact, together with the fact from the previous paragraph that $x_1^{\text{1ST}}<x_1^{\text{FDR}}$, are sufficient to establish that both injunctions and damages are strictly socially inefficient. From Eq. (10), the first-order conditions determining 1's investment can be nested as follows:

$$s'(x_1)-\theta\left[\gamma\int_{e_1^*(x_1)}^{e^{**}(x_1)}\frac{\partial^2 h(x_1,e)}{\partial x_1\,\partial e}\,de+\left(\frac{\alpha}{1-\alpha}\right)v_2'(e_1^*(x_1))\frac{de_1^*(x_1)}{dx_1}\right], \tag{16}$$

where $\theta=0$ in the first best and $\theta=1-\alpha$ under injunctions. Both terms in the square brackets in (16) are negative. Calculations from the previous paragraph show that the first term in square brackets is positive. To see that the second term in square brackets is negative, note first that since player 2 is the recipient of a purely negative externality, $v_2'(e_1^*(x_1))<0$. Monotone comparative statics arguments similar to those used in the proof of Lemma 1 can be used to prove that, under the maintained assumption that player 1's investment is dirty, $de_1^*(x_1)/dx_1>0$. Since both terms in square brackets are negative, the objective function determining 1's investment exhibits increasing marginal returns in x_1 and θ. Steps similar to the preceding paragraph can be used to show $x_1^{1ST}<x_1^{FIR}$.

Next, we show that there exists $\gamma'>0$ such that $x_1^{FDR}<x_1^{FIR}$ for all $\gamma\in(0,\gamma')$. The first-order conditions for 1's investment under injunctions and damages, after substituting the functional form for v_1, can be nested as

$$s'(x_1)-\theta\gamma\int_{e_1^*(x_1,\gamma)}^{e^{**}(x_1,\gamma)}\frac{\partial^2 h(x_1,e)}{\partial x_1\,\partial e}de-(1-\theta)v_2'(e_1^*(x_1,\gamma))\frac{\partial e_1^*(x_1,\gamma)}{\partial x_1} \tag{17}$$

where $\theta=1-\alpha$ under injunctions and $\theta=1$ under damages. We have added an argument to $e_1^*(x_1,\gamma)$ and $e^{**}(x_1,\gamma)$ to reflect their dependence on γ, which we will vary in the comparative statics exercise to follow. In the limit as $\gamma\to 0$, the partial derivative of (17) with respect to θ equals

$$v_2'(e_1^*(x_1,\gamma))\frac{\partial e_1^*(x_1,\gamma)}{\partial x_1}, \tag{18}$$

which we argued in the previous paragraph is negative. Thus, the objective function determining 1's investment exhibits decreasing marginal returns in x_1 and θ for sufficiently small $\gamma>0$. Steps similar to the preceding paragraph can be used to show that $x_1^{FDR}<x_1^{FIR}$ for sufficiently small $\gamma>0$.

Using arguments paralleling those in the last paragraph of the proof of Proposition 1, we can show that the investment ranking translates into a social-welfare ranking, so that damages are socially more efficient than an injunction for sufficiently small $\gamma>0$. □

Proof of Proposition 5. The proof provides an example in which peak damages dominates both peak and corner injunctions. The example adopts the functional form assumption $v_1(x_1,e)=g(x_1)+\gamma h(x_1,e)$ and takes limits as $\gamma\to 0$ and $\alpha\to 0$.

Substituting the threat points from Table 2 into the equation for 1's Nash bargaining surplus (3), the objective function for 1's investment under peak damages is identical to (6), under peak injunctions is identical to (5), and under corner injunctions can be written

$$s(x_1)-(1-\alpha)[s(x_1)-v_1(x_1,0)]-\alpha v_2(0). \tag{19}$$

Differentiating these expressions with respect to x_1, substituting (7) through (9), and substituting the functional form $v_1(x_1,e)=g(x_1)+\gamma h(x_1,e)$, we can nest the first-order conditions for player 1's investment as

$$s'(x_1)-\theta_1\left[\int_{e_1^*(x_1)}^{e^{**}(x_1)}\frac{\partial^2 h(x_1,e)}{\partial x_1\,\partial e}de\right]-\theta_2\left[\int_0^{e_1^*(x_1)}\frac{\partial^2 h(x_1,e)}{\partial x_1\,\partial e}de\right]-\theta_3\left[v_2'(e_1^*(x_1))\frac{de_1^*(x_1)}{dx_1}\right], \tag{20}$$

where

$$\theta_1=\begin{cases}0 & \text{first best,}\\ 1 & \text{peak damages,}\\ 1-\alpha & \text{peak injunctions,}\\ 1-\alpha & \text{corner injunctions,}\end{cases}\qquad \theta_2=\begin{cases}0 & \text{first best,}\\ 0 & \text{peak damages,}\\ 0 & \text{peak injunctions,}\\ 1-\alpha & \text{corner injunctions,}\end{cases} \tag{21}$$

and

$$\theta_3=\begin{cases}0 & \text{first best,}\\ 0 & \text{peak damages,}\\ \alpha & \text{peak injunctions,}\\ 0 & \text{corner injunctions.}\end{cases} \tag{22}$$

We will first find a condition on α under which peak damages dominates corner injunctions and then find a condition on γ under which peak damages dominates peak injunctions. Since these conditions involve different parameters, we can investigate each independently.

Monotone comparative statics arguments used repeatedly in the previous proofs can be used to show that investment under peak damages, x^{FPDR}, is closer to $x_1^{1\text{ST}}$ than investment under corner injunctions, x^{FCIR}, if the magnitude of the last three distortion terms in (20) is smaller for peak damages than corner injunctions. The last distortion term drops out for these two regimes since $\theta_3 = 0$ for them. Some algebra shows that the absolute value of the remaining two distortion terms is larger for corner injunctions than peak damages if and only if

$$\alpha < \frac{\int_0^{e_1^*(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \, \partial e} de}{\int_0^{e^{**}(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \, \partial e} de}. \tag{23}$$

Since $\partial^2 h(x_1, e)/\partial x_1 \, \partial e$ is non-zero and does not change sign over the ranges of integration, and since $0 < e_1^*(x_1) < e^{**}(x_1)$ by part (b) of Lemma 1, the numerator and denominator on the right-hand side of (23) are non-zero and have the same sign, implying the right-hand side is positive. Hence, there is a non-empty subset of $\alpha \in (0, 1)$ satisfying (23). Peak damages dominate corner injunctions for these values of α.

Similarly, x^{FPDR} is closer to $x_1^{1\text{ST}}$ than investment under peak injunctions, x^{FPIR}, if the magnitude of the last three distortion terms in (20) is larger for peak injunctions than peak damages. After some algebra, this condition reduces to

$$\gamma < \frac{v_2'(e_1^*(x_1)) \dfrac{de_1^*(x_1)}{dx_1}}{\int_0^{e^{**}(x_1)} \frac{\partial^2 h(x_1, e)}{\partial x_1 \, \partial e} de}. \tag{24}$$

As shown in the proof of Proposition 2, the numerator and the denominator on the right-hand side of (24) have the same sign, so the right-hand side of (24) is positive. Condition (24) thus holds for sufficiently small $\gamma > 0$. □

References

[1] I. Ayers, E. Talley, Solomonic bargaining: dividing a legal entitlement to facilitate Coasean trade, Yale Law J. 104 (1995) 1027–1117.
[2] K. Binmore, A. Rubinstein, A. Wolinsky, The Nash bargaining solution in economic modelling, RAND J. Econ. 17 (1986) 176–188.
[3] G. Calebresi, A.D. Melamed, Property rules, liability rules, and inalienability: one view of the cathedral, Harvard Law Rev. 85 (1972) 1089–1128.
[4] R.H. Coase, The problem of social cost, J. Law Econ. 3 (1960) 1–44.
[5] R. Cornes, T. Sandler, The Theory of Externalities, Public Goods, and Club Goods, Cambridge University Press, Cambridge, 1996.
[6] A.S. Edlin, C. Shannon, Strict monotonicity in comparative statics, J. Econ. Theory 81 (1998) 201–219.
[7] R.A. Epstein, Blackmail, Inc., Univ. Chicago Law Rev. 50 (1983) 553–566.
[8] F. Gomez, J.-J. Ganuza, Civil and criminal sanctions against blackmail: an economic analysis, Int. Rev. Law Econ. 21 (2002) 475–498.
[9] S.J. Grossman, O.D. Hart, The costs and benefits of ownership: a theory of vertical and lateral integration, J. Polit. Econ. 94 (1986) 691–791.
[10] R.L. Hale, Prima facie torts, combination, and non-feasance, Columbia Law Rev. 46 (1946) 196–218.
[11] O.D. Hart, J. Moore, Property rights and the nature of the firm, J. Polit. Econ. 98 (1990) 1119–1158.
[12] R.H. Helmholz, The Roman law of blackmail, J. Legal Stud. 30 (2001) 33–52.
[13] L. Kaplow, S. Shavell, Do liability rules facilitate bargaining?, A reply, Yale Law J. 105 (1995) 221–233.
[14] L. Kaplow, S. Shavell, Property rights and liability rules: an economic analysis, Harvard Law Rev. 109 (1996) 713–790.
[15] W.M. Landes, R.A. Posner, The private enforcement of law, J. Legal Stud. 4 (1975) 1–46.
[16] J. Lindgren, Unraveling the paradox of blackmail, Columbia Law Rev. 84 (1984) 670–717.
[17] G.A. Mumey, The "Coase Theorem": a reexamination, Quart. J. Econ. 85 (1971) 718–723.
[18] J. Nash Jr., The bargaining problem, Econometrica 18 (1950) 155–162.
[19] A.C. Pigou, The Economics of Welfare, Macmillan and Company, London, 1912.

[20] R. Pitchford, C.M. Snyder, Coming to the nuisance: an economic analysis from an incomplete contracts perspective, J. Law, Econ. Organ. 19 (2003) 491–516.
[21] R.A. Posner, Blackmail, privacy, and freedom of contract, in: F. Parisi (Ed.), The Collected Essays of Richard A. Posner, vol. 2: The Economics of Private Law, Elgar, Northampton, MA, 2001, pp. 488–518.
[22] E. Sherwin, Introduction: property rules and remedies, Yale Law J. 106 (1997) 2083–2089.
[23] R.K. Sundaram, A First Course in Optimization Theory, Cambridge University Press, Cambridge, 1996.
[24] P. Tamony, "Spite fence": a mousetrap word, Amer. Speech 27 (1952) 233–235.
[25] M.J. White, D. Wittman, Optimal spatial location under pollution: liability rules and zoning, J. Legal Stud. 10 (1981) 249–268.