Supplemental Materials for McCabe, Mark J. and Snyder, Christopher M., 'Open Access as a Crude Solution to a Hold-Up Problem in the Two-Sided Market for Academic Journals,' *Journal of Industrial Economics*.

DESCRIPTION

This document contains two appendices not included in the published paper for space considerations. Online Appendix D provides further details omitted from the proofs appearing Appendix B of the published paper. Online Appendix E fills in technical details for the analysis of a hybrid journal omitted from Appendix C of the published paper. The figures are numbered starting from the last one in the published article.

ONLINE APPENDIX D:

FURTHER PROOF OF SELECTED PROPOSITIONS

This online appendix supplies additional details omitted for space considerations from the proofs included in Appendix B of the published paper.

Completing Proof of Proposition 1

Here we provide the counterexample, promised in the text following the proposition, having non-logconcave f_a in which $p_a^{mo} < p_a^{mt}$. The counterexample involves discrete distributions of author and reader values, but it is easy to construct continuous distributions approaching these discrete distributions in the limit. Suppose v_a takes on two values, 2 or 2.9, in equal measure. Suppose v_r also takes on two values, 0 for 30% of readers and a small, positive number, say 0.0001, for the remainder. Suppose $c_a = 1$ and $c_r = 0$. It follows that $\pi_r^{mt} \approx 0$ and $q_r^{mt} = 0.7$. Both types of journal have two possible pricing strategies: serving all authors with a low submission fee or serving just high-value ones with a high submission fee. One can show that a traditional journal earns approximately 0.4 from charging 1.4 and serving all authors and approximately 0.515 from charging 2.03 and serving just the high-value ones. Thus $p_a^{mt} = 2.03$. An open-access journal earns 1 from charging 2 and serving all authors and 0.95 from charging 2.9 and serving just the high-value ones. Thus $p_a^{mo} = 2 < 2.03 = p_a^{mt}$. $\square$

Completing Proof of Proposition 3

Here we set up and solve several subproblems that are components in the solution of the larger problem, MIN1, leading to the desired result. The first subproblem, labeled MAX1, involves the maximization of the integral I_2 defined in (B42) for given values of p_r^{mt} and q_r^{mt} subject to constraints (B37), (B39), and (B44). For p_r^{mt} and q_r^{mt} to be a well-defined price and quantity, they must be related by

(D1) $$\bar{F}_r(p_r^{mt}) = q_r^{mt}.$$

Imposing (D1) as an additional constraint, MAX1 can be represented by the following optimal-control problem:

(D2) $$\max_{x_1} \int_a^T t e^{-x_1} dt$$

(D3) subject to $\ddot{x}_1 = u$

(D4) $u \geq 0$

(D5) $x_1 \geq 0$

(D6) $$x_1(a) = \ln\left(\frac{p_r^r - c_r}{q_r^{mt}}\right)$$

(D7) $$\int_a^T e^{-x_1} dt = q_r^{mt},$$

where $a = p_r^{mt}$ and where the rest of the index, state, and control variables are defined as in MIN2. Rather than directly equating state variable x_1 with the density f_r in objective (D2), we have here specified $f_r(v_r) = e^{-x_1(t)}$. A logconcave density can always be written in this way for some convex $x_1(t)$, called the potential function; see, e.g., Definition 2.1 in Saumard and Wellner [2014]. Conditions (D3) and (D4) embody the needed convexity constraint on $x_1(t)$, and (D5) embodies the nonincreasingness of f_r.

MAX1 has some nonstandard features. Constraint (D3) on the state variable involves a second derivative rather than the usual first derivative. This can be handled by letting the first derivative of x_1 be a new state variable and controlling the derivative of this derivative. Further, (D7) is an integral rather than the usual derivative condition, leading the optimal-control problem to be what is called isoparametric. This can be handled by introducing a third state variable $x_3(t) = \int_a^t e^{-x_1(s)} dx$, leading to the conditions $\dot{x}_3 = x_1$, $x_3(a) = 0$, and $x_3(T) = q_r^{mt}$. Incorporating these new state variables and conditions into the problem yields the following equivalent optimal-control problem, labeled MAX2:

(D8) $$\max \int_a^T t e^{-x_1} dt$$

(D9) subject to $\dot{x}_1 = x_2$

(D10) $\dot{x}_2 = u$

(D11) $u \geq 0$

(D12) $$x_1(a) = \ln\left(\frac{p_r^r - c_r}{q_r^{mt}}\right)$$

(D13) $x_2 \geq 0$

(D14) $\dot{x}_3 = e^{-x_1}$

(D15) $x_3(a) = 0$

(D16) $x_3(T) = q_r^{mt}$

We will simplify the problem by ignoring constraints (D12), (D13), (D15), and (D16) for now. The associated Hamiltonian and Lagrangians then are

(D17) $$\mathscr{H} = t e^{-x_1} + \lambda_1 x_2 + \lambda_2 u + \lambda_3 e^{-x_1}$$

(D18) $$\mathscr{L} = \mathscr{H} + \mu u.$$

"Textbook" necessary conditions for an optimum are

(D19) $$0 = \frac{\partial \mathscr{L}}{\partial u} = \lambda_2 + \mu$$

$$\dot{\lambda}_1 = -\frac{\partial \mathscr{L}}{\partial x_1} = (t+\lambda_3)e^{-x_1} \tag{D20}$$

$$\dot{\lambda}_2 = -\frac{\partial \mathscr{L}}{\partial x_2} = -\lambda_1 \tag{D21}$$

$$\dot{\lambda}_3 = -\frac{\partial \mathscr{L}}{\partial x_3} = 0 \tag{D22}$$

$$\mu u = 0 \tag{D23}$$

$$\mu \geq 0. \tag{D24}$$

A series of arguments will show that these conditions entail (D11) binds for $t > a$. By (D19), we have $-\lambda_2 = \mu$, implying

$$\lambda_2 \leq 0 \tag{D25}$$

by (D24). Condition (D22) implies $\lambda_3(t)$ is a constant for all t, call it $\bar{\lambda}_3$. Hence $\ddot{\lambda}_2 = -\dot{\lambda}_1 = -(t+\bar{\lambda}_3)e^{-x_1}$, where the first equality holds by (D21) and the second by (D20). Thus $\lambda_2(t)$ is strictly convex for $t < -\bar{\lambda}_3$, strictly concave for $t > -\bar{\lambda}_3$, with an inflection point at $t = -\bar{\lambda}_3$. We will show $\lambda_2(t) = 0$ for at most one $t \in (a,T)$. Suppose for the sake of contradiction that there exist $t', t'' \in (a,T)$ with $t' < t''$ and $\lambda_2(t') = \lambda_2(t'') = 0$. There are two cases to consider. If $t' < -\bar{\lambda}_3$, then λ_2 is strictly convex and thus strictly quasiconvex in an ϵ-neighborhood around t'. Then $0 = \lambda_2(t') < \max[\lambda_2(t'-\epsilon), \lambda_2(t'+\epsilon)]$, contradicting (D25). If $t' \geq -\bar{\lambda}_3$, then the strict concavity of λ_2 for $t > -\bar{\lambda}_3$ implies the strict quasiconcavity of λ_2 for all $t \in (t', t'')$, in turn implying $\lambda_2(t) > \min[\lambda_2(t'), \lambda_2(t'')] = 0$ for all $t \in (t', t'')$, contradicting (D25).

We have thus shown that $\lambda_2(t) = 0$ for at most one $t \in (a,T)$, implying $\lambda_2 < 0$ almost everywhere (a.e.) by (D25), implying $\mu > 0$ a.e. by (D19), implying $u = 0$ a.e. by (D23), implying $x_2 = 0$ a.e. by (D10), implying x_1 is affine by (D9), i.e., $x_1(t) = \alpha t + \beta$ for some α, β. Substituting this affine $x_1(t)$ back into MAX2 and substituting $a = p_r^{mt}$ gives the equivalent problem, which we will label MAX3:

$$\max_{\alpha,\beta} \left[e^{-\beta} \int_{p_r^{mt}}^{T} t e^{-\alpha t} dt \right] \tag{D26}$$

$$\text{subject to} \quad \alpha \geq 0 \tag{D27}$$

$$e^{-(\alpha p_r^{mt}+\beta)} = \frac{q_r^r}{p_r^{mt} - c_r} \tag{D28}$$

$$e^{-\beta} \int_{p_r^{mt}}^{T} e^{-\alpha t} dt = q_r^{mt}, \tag{D29}$$

where condition (D13) has become (D27), (D6) has become (D28), and (D16) has become (D29). The rest of the constraints in MAX2 can be eliminated because they are satisfied by affine $x_1(t)$ by construction (as they were explicitly considered in the optimal-control solution of MAX2).

MAX3 is a standard constrained optimization problem involving the choice of variables α, β, and T, which can be solved using the Kuhn-Tucker method. Using (D28) to solve for β, substituting this β into the rest of the problem and rearranging leads to the Lagrangian

$$\mathscr{L} = \frac{q_r^{mt}}{p_r^{mt} - c_r} \int_{p_r^{mt}}^{T} t e^{-\alpha(t - p_r^{mt})} dt + \lambda \left[p_r^{mt} - c_r - \int_{p_r^{mt}}^{T} e^{-\alpha(t-p_r^{mt})} \right] + \mu\alpha, \tag{D30}$$

where λ and μ are the Lagrange multipliers on constraints (D29) and (D27). The following are necessary conditions for an optimum:

(D31) $$0 = \frac{\partial \mathscr{L}}{\partial \alpha} = \left(\frac{q_r^{mt}}{p_r^{mt} - c_r} - \lambda \right) \int_{p_r^{mt}}^{T} (t - p_r^{mt}) e^{-\alpha(t - p_r^{mt})} dt + \mu$$

(D32) $$0 = \frac{\partial \mathscr{L}}{\partial T} = \left(\frac{T q_r^{mt}}{p_r^{mt} - c_r} - \lambda \right) e^{-\alpha(T - p_r^{mt})}$$

(D33) $$0 = \mu\alpha.$$

Now (D32) implies $\lambda = T q_r^{mt}/(p_r^{mt} - c_r)$. Substituting into (D31) and rearranging,

(D34) $$\mu = \int_{p_r^{mt}}^{T} (T - t)(t - p_r^{mt}) e^{-\alpha(t - p_r^{mt})} dt > 0,$$

implying $\alpha = 0$ by (D33). Hence f_r must be the uniform distribution on $[p_r^{mt}, \bar{v}_r]$.

Using these results, the proof in Appendix B shows reduces the worst-case analysis down to the following minimization problem, labeled MIN3:

(D35) $$\min_{f_a} \Bigg[\underbrace{\int_{z_a^{mo}}^{\bar{v}_a} \ell_1(v_a) f_a(v_a) dv_a}_{I_7} - \underbrace{\int_{z_a^{mt}}^{z_a^{mo}} \ell_2(v_a) f_a(v_a) dv_a}_{I_8} \Bigg]$$

(D36) subject to $(z_a^{mo} - c_a - c_r)\bar{F}_a(z_a^{mo}) \geq (q_r^{mt} z_a^{mt} - c_a + \pi_r^{mt})\bar{F}_a(z_a^{mt})$

(D37) $f_a'(v_a) \leq 0$

(D38) f_a logconcave

(D39) $\bar{F}_a(0) \leq 1,$

where (D36) is the same as (B35) with profits written out and where the generic conditions on f_r have been omitted because we have found its functional form. We can progress toward solving MIN3 by considering two subproblems necessary for its solution. Consider fixing the four constants z_a^{mo}, z_a^{mt}, $q_a^{mo} = \bar{F}_a(z_a^{mo})$, and $q_a^{mt} = \bar{F}_a(z_a^{mt})$. The remaining terms in (D36) are constants independent of f_a by (B59) and (B60). Hence (D36) is automatically satisfied if these constants constitute a feasible solution to MIN3. Integrals I_7 and I_8 can then be optimized independently for the given values z_a^{mo}, z_a^{mt}, q_a^{mo}, and q_a^{mt} subject to the remaining constraints (D37), (D38), and (D39).

Since it enters positively in (D35), I_7 must be minimized subject to given values z_a^{mo} and q_a^{mo} that appear in it and subject to the nonincreasingness constraint (D37). The constraint on logconcavity and the survivior function, (D38) and (D39), turn out not to bind as will be verified later. Expressed as an optimal-control problem, labeled MIN4, we have

(D40) $$\min \left[\int_a^T \ell_1(t) x_1 dt \right]$$

(D41) subject to $\dot{x} = u$

(D42) $-u \geq 0$

(D43) $\dot{x}_2 = x_1$

(D44) $x_2(a) = 0$

(D45) $x_2(T) = q_a^{mo}.$

where $a = z_a^{mo}$, $T = \bar{v}_a$, $t = v_a$, $x_1(t) = f_a(v_a)$, and $x_2(t) = \int_a^t x_1(s) ds$. See MAX2 above for more details on the setup of a similar problem. The associated Hamiltonian and Lagrangian are

$$\mathscr{H} = \ell_1(t)x_1 + \lambda_1 u + \lambda_2 x_1 \tag{D46}$$

$$\mathscr{L} = \mathscr{H} - \mu u, \tag{D47}$$

yielding necessary conditions

$$0 = \frac{\partial \mathscr{L}}{\partial u} = \lambda_1 - \mu \tag{D48}$$

$$\dot{\lambda}_1 = -\frac{\partial \mathscr{L}}{\partial x_1} = -\ell_1(t) - \lambda_2 \tag{D49}$$

$$\dot{\lambda}_2 = -\frac{\partial \mathscr{L}}{\partial x_2} = 0 \tag{D50}$$

$$\mu(-u) = 0 \tag{D51}$$

$$\mu \geq 0. \tag{D52}$$

Equation (D50) implies λ_2 is a constant for all t, i.e., $\lambda_2(t) = \bar{\lambda}_2$. Then (D49) implies that λ_1 is strictly quasiconcave, increasing in t up to its maximum $\ell_1^{-1}(\bar{\lambda}_2)$ and decreasing above this. Now $0 \leq \mu = \lambda_1$ by (D52) and (D48). But then $\lambda_1(t) > \min[\lambda_1(a), \lambda_1(T)] \geq 0$ for all $t \in (a, T)$, where the first inequality follows from strict quasiconcavity and the second by $\lambda_1 \geq 0$ as just shown. Hence $\mu = \lambda_1 > 0$ for all $t \in (a, T)$, implying $0 = u = \dot{x}_1$ by (D51) and (D41). Therefore $f_a(v_a)$ is the uniform distribution on $[z_a^{mo}, \bar{v}_a]$.

Since it enters negatively in (D35), I_8 must be maximized subject to given values z_a^{mo}, z_a^{mt}, and $q_a^{mt} - q_a^{mo} = \bar{F}_a(z_a^{mt}) - \bar{F}_a(z_a^{mo})$ and constraints (D37), (D38), and (D39). Sparing the details, the setup is similar to MIN4 and yields the uniform distribution as the solution. □

Completing Proof Proposition 6

Here we fill in some omitted details from the proof in Appendix B of the published paper. We first need to verify that the conditions behind subcase G.2b entail $\Pi^{mo} \geq \Pi^{mt}$. Recall that case G.2 involves moving from structure $(T^*, O^*) = (0,1)$ to structure $(T', O') = (1,0)$ and that subcase G.2b involves the further condition $\Pi^{Ft} \leq 0 < \Pi^{Fo}$. Let p_a^{Ft} and p_a^{Fo} be the equilibrium prices in the duopoly structure with one of each journal mode indicated by point F in Figure 2. Since $\Pi^{Fo} > 0$ in the present subcase, the duopoly margin must be positive for the open-access journal:

$$p_a^{Fo} > c_a + c_r. \tag{D53}$$

Another condition behind the subcase is $\Pi^{Ft} \leq 0$. Now it cannot be that $\Pi^{Ft} < 0$ because the traditional journal could guarantee zero quantity and thus zero profit by deviating to a submission fee of $\bar{v}_a$. Hence Π^{Ft} is identically 0, which could follow from two possibilities: either the traditional journal serves no authors or serves a positive measure of authors at a zero margin. The latter possibility cannot be an equilibrium as demand is continuous, so the traditional journal would gain by slightly increasing price, earning a positive margin, and still have a positive author quantity. Hence, the traditional journal must serve no authors. For this to be true, it must be the case that for all $v_a \in [0, \bar{v}_a]$, either $q_r^{mt} v_a - p_a^{Ft} \leq 0$ (i.e., an author with that value would rather not submit than submit to the traditional journal) or $q_r^{mt} v_a - p_a^{Ft} \leq v_a - p_a^{Fo}$ (i.e., an author with that value prefers the competitor).

Consolidating these conditions, for the traditional journal to serve no authors,

$$q_r^{mt} v_a - p_a^{Ft} \leq \max(0, v_a - p_a^{Fo}) \leq \max(0, v_a - c_a - c_r) \quad \text{for all } v_a \in [0, \bar{v}_a], \tag{D54}$$

where the second inequality follows from (D53). Condition (D54) must hold not just for p_a^{Ft} but for any $p_a > c_a + \pi_r^{mt}$ or else the traditional journal could profitably deviate to this p_a and earn a positive margin on a positive measure of authors. Replacing p_a^{Ft} with $c_a + \pi_r^{mt}$ in (D54) yields the necessary equilibrium condition

$$q_r^{mt} v_a - c_a + \pi_r^{mt} \leq \max(0, v_a - c_a - c_r) \quad \text{for all } v_a \in [0, \bar{v}_a]. \tag{D55}$$

In particular, (D55) must hold for $v_a = p_a^{mt}/q_r^{mt}$, which upon substituting into (D55) gives

$$p_a^{mt} - c_a + \pi_r^{mt} \leq \max(0, p_a^{mt}/q_r^{mt} - c_a - c_r). \tag{D56}$$

We then have

$$\begin{aligned}
\Pi^{mt} &= (p_a^{mt} - c_a + \pi_r^{mt}) \bar{F}_(p_a^{mt}/q_r^{mt}) && \text{(D57)}\\
&\leq \max(0, p_a^{mt}/q_r^{mt} - c_a - c_r) \bar{F}_a(p_a^{mt}/q_r^{mt}) && \text{(D58)}\\
&\leq \max\left\{0, \underset{p_a \geq 0}{\operatorname{argmax}} [(p_a - c_a - c_r)\bar{F}_a(p_a)]\right\} && \text{(D59)}\\
&= \max(0, \Pi^{mo}) && \text{(D60)}\\
&= \Pi^{mo}, && \text{(D61)}
\end{aligned}$$

where (D57) follows from (8), (D58) follows from (D56), (D59) follows from the fact that the maximizer will generate a weakly higher value of the objective function than $p_a^{mt}/q_r mt$, and (D60) follows from (10). The last equality holds since $0 < \Pi^{Fo} \leq \Pi^{mo}$, where the first inequality is one of the conditions defining the subcase and the second inequality follows from monopoly being at least as profitable as duopoly. We have succeeded in showing $\Pi^{mo} \geq \Pi^{mt})$ in subcase G.2b, implying $SW^{mo} \geq SW^{mt}$ by Proposition 3.

The remaining detail to fill in is analysis of case D.1 Let SW^D be equilibrium social welfare in the initial market structure in this case, i.e., structure D, in which $(T^*, O^*) = (\infty, 1)$. Let SW^E be equilibrium social welfare in the market structure arising after the reduction in open access, i.e., structure E, in which $(T', O') = (\infty, 0)$. After some rearranging, one can show

$$SW^D - SW^E = \int_{z_a^{Do}}^{\bar{v}_a} \int_0^{p_r^{mt}} (v_a + v_r - c_r) f_r(v_r) f_a(v_a) dv_r dv_a, \tag{D62}$$

where z_a^{Do} is the lowest type served by the open-access journal in market structure D. Marginal type z_a^{Do} must weakly prefer submitting to the open-access over traditional journals in structure D: $z_a^{Do} - p_a^{Do} \geq q_r^{mt} z_a^{Dt} - p_a^{Dt} = q_r^{mt} z_a^{Dt}$, where the equality follows because competition among traditional journals drives p_a^{Dt} to 0. Rearranging, $(1 - q_r^{mt}) z_a^{Do} \geq p_a^{Do}$, implying $z_a^{Do} \geq p_a^{Do} > c_a + c_r \geq c_r$. Hence for all v_a above the lower limit of integration in (D62), $v_a + v_r - c_r \geq v_a - c_r \geq z_a^{Do} - c_r \geq 0$. □

Additional Reference

Saumard, A.and Wellner, J. A., 2014, 'Log-Concavity and Strong Log-Concavity: A Review,' *Statistics Surveys*, 8, pp. 45–114.

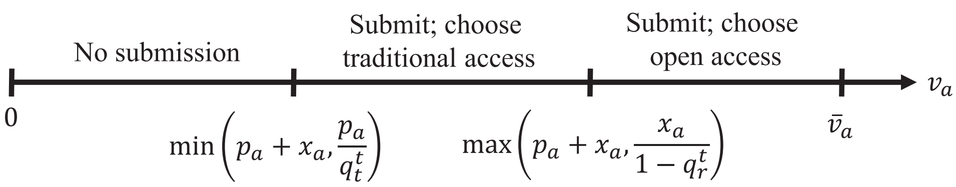

Figure 6
General Characterization of Author's Continuation Strategy Facing Hybrid Journal

Notes: Author value per reader partitioned into three subintervals. Author does not submit article for lowest values, submits under traditional access for intermediate values, and pays premium for open access for highest values. Depending on the prices (p^a, x^a) set by the hybrid journal, up to two subintervals may be empty.

ONLINE APPENDIX E

COMPLETING ANALYSIS OF HYBRID PRICING STRATEGY

This online appendix fills in the technical details of the of the hybrid-pricing strategy provided in Appendix C of the published paper.

As discussed there, the analyses of the reader-pricing stage and the subscription stage are identical to those already given. Thus the game can be folded back to the author submission stage. The author has three options: not submitting, providing him with surplus 0; submitting under traditional access, providing him with surplus $v_a q_r^{mt} - p_a$; and submitting under open access, providing him with surplus $v_a - p_a - x_a$. The author chooses the option providing him with the highest surplus. Not submitting provides strictly more surplus than the other options if $0 > v_a q_r^{mt} - p_a$ and $0 > v_a - p_a - x_a$, which upon combining conditions yields

$$v_a < \min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right). \tag{E1}$$

Submitting under open access provides strictly more surplus than the other options if $v_a - p_a - x_a > 0$ and $v_a - p_a - x_a > v_a q_r^{mt} - p_a$, which upon combining conditions yields

$$v_a > \max\left(p_a + x_a, \frac{x_a}{1 - q_r^{mt}}\right). \tag{E2}$$

Because

$$\min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right) \leq p_a + x_a \leq \max\left(p_a + x_a, \frac{x_a}{1 - q_r^{mt}}\right),$$

it is immediate that the interval of author values is partitioned into three subintervals as shown in Figure 6, with no submission for the lowest values, submission under traditional access for intermediate values, and submission under open access for the highest values.

Zero, one, or two of the subintervals in Figure 6 can be empty in specific cases. There are seven ways this can happen, leading to the seven cases. The seven cases are detailed in Figure 7. The necessary and sufficient conditions are mutually exclusive and exhaustive. Which case to place the boundaries between them is somewhat arbitrary. We adopted the convention of setting the inequalities (strict or weak) such that a partition is displayed only if it contains a positive measure of types.

We will establish the necessary and sufficient conditions behind each case in Figure 7 in turn,

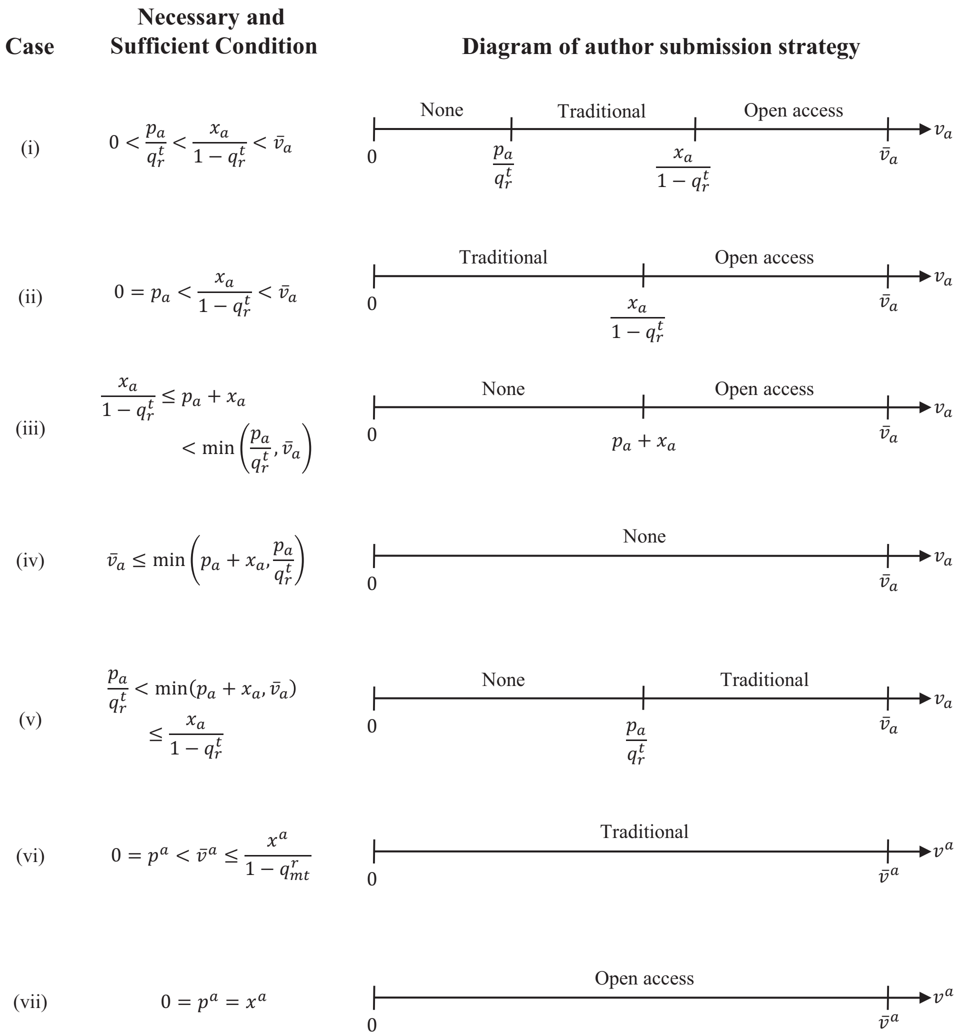

Figure 7
Detailed Subcases for Author's Continuation Strategy Facing Hybrid Journal

Notes: General characterization leads to seven cases depending on which partitions from the previous figure are empty.

starting with case (i). For there to be a positive measure of non-submitting authors,

$$0 < \min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right). \tag{E3}$$

But (E3) requires $p_a > 0$, implying

$$\frac{p_a}{q_r^{mt}} > 0. \tag{E4}$$

Next, for there to be a positive measure of authors submitting under traditional access,

$$\min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right) < \max\left(p_a + x_a, \frac{x_a}{1 - q_r^{mt}}\right). \tag{E5}$$

For (E5) to hold, $x_a > 0$. Furthermore, one of the following two conditions must hold:

$$p_a + x_a > \frac{p_a}{q_r^{mt}} \tag{E6}$$

$$p_a + x_a < \frac{x_a}{1 - q_r^{mt}}. \tag{E7}$$

Some algebra shows (E6) and (E7) are equivalent to each other and furthermore are both equivalent to

$$\frac{p_a}{q_r^{mt}} < \frac{x_a}{1 - q_r^{mt}}. \tag{E8}$$

Condition (E6) implies that the cutoff between types who do not submit and types who submit under traditional access is $v_a = p_a/q_r^{mt}$. Condition (E7) implies that the cutoff between types who submit under traditional access and types who submit under open access is $v_a = x_a/(1 - q_r^{mt})$. These simple expression for the boundaries are reflected in case (i) of Figure 7. Finally, for there to be a positive measure of authors submitting under open access, the boundary between traditional and open access must be strictly below $\bar{v}_a$:

$$\frac{p_a}{q_r^{mt}} < \bar{v}_a. \tag{E9}$$

Collecting conditions (E3), (E8), and (E9) yields the necessary and sufficient condition shown in case (i) of Figure 7.

Case (ii) can be analyzed using similar arguments maintaining the assumption $p_a = 0$. Turn therefore to case (iii). For there to be a zero measure of types submitting under traditional access,

$$\max\left(p_a + x_a, \frac{x_a}{1 - q_r^{mt}}\right) \le \min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right). \tag{E10}$$

Condition (E10) implies

$$\max\left(p_a + x_a, \frac{x_a}{1 - q_r^{mt}}\right) = \min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right) = p_a + x_a. \tag{E11}$$

Hence the cutoff between types who do not submit and types who submit under open access is $v_a = p_a + x_a$. For there to be a positive measure of authors submitting under open access, this cutoff type must be strictly below $\bar{v}_a$:

$$p_a + x_a < \bar{v}_a. \tag{E12}$$

The final step in deriving the necessary and sufficient condition for case (iii) in Figure 7 is to note that (E11) is equivalent to

$$\frac{x_a}{1-q_r^{mt}} \leq p_a + x_a \leq \frac{p_a}{q_r^{mt}}. \tag{E13}$$

Case (v) is analyzed similarly to case (iii). Case (vi) can then be analyzed using arguments similar to case (v), but maintaining the assumption $p_a = 0$. The details of these cases are omitted for brevity. This leaves cases (iv) and (vii). Consider each case in turn. For there to be a zero measure of submitting types (whether under traditional or open access),

$$\bar{v}_a \leq \min\left(p_a + x_a, \frac{p_a}{q_r^{mt}}\right), \tag{E14}$$

the necessary and sufficient condition shown in case (iv) of Figure 7. For there to be a zero measure of both types who either do not submit or submit under traditional access,

$$\max\left(p_a + x_a, \frac{x_a}{1-q_r^{mt}}\right) \leq 0, \tag{E15}$$

which is equivalent to $p_a = x_a = 0$, the indicated condition in case (vii) of Figure 7.

The algebraic conditions for the seven cases in Figure 7 are difficult to envision. Figure 8 graphs them in the price space for a hybrid journal, (p_a, x_a). The cases form four regions, two segments, and one point.

We next provide sufficient conditions under which equilibrium falls into the cases—(i) and (ii)—analyzed in Appendix C. The first step is to restrict attention to the case in which a monopoly journal, if it were prevented from choosing a hybrid strategy and forced to be either a purely traditional or purely open-access journal, would find the traditional strategy more profitable. Proposition 2 provided a sufficient condition for this outcome, and we will maintain that sufficient condition here. This rules out cases (iv), (v), and (vi) from Figure 7 as possible equilibria. The next step is to provide a further condition under which a traditional journal would find it profitable to move to a hybrid model if it could. This rules out cases (iii) and (vii) from Figure 7 as possible equilibria, leaving cases (i) and (ii) as the only possibilities, as desired. We have the following proposition.

Proposition 8. *A profitable monopoly traditional journal would strictly profit from moving to hybrid pricing if $\bar{v}_a$ is high enough, a sufficient condition being*

$$\bar{v}_a > \frac{c_r + \pi_r^{mt}}{1-q_r^{mt}}. \tag{E16}$$

Proof. Suppose a traditional journal currently charging author price p_a maintains that author price but adds the option of open access for a premium of $x^a = \pi_r^{mt} + c_r + \epsilon$ for some $\epsilon > 0$. For each author type who now chooses open access, the journal earns continuation profit

$$p_a + x_a - c_a - c_r = p_a - c_a + \pi_r^{mt} + \epsilon,$$

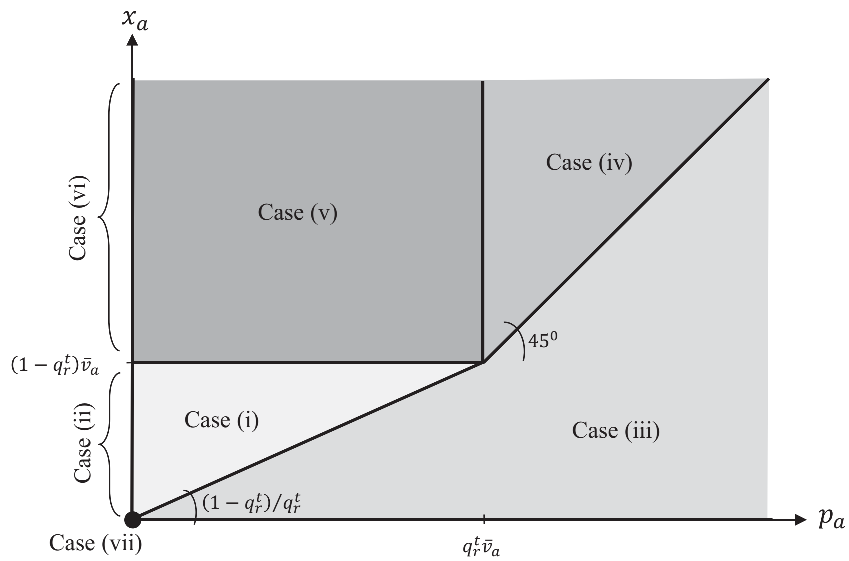

Figure 8
Diagramming Algebraic Conditions from Previous Figure

Notes: Cases along vertical axis involve zero submission fee: cases (ii) and (vi) are line segments and (vii) is a point.

$\epsilon > 0$ more than it earned under the original traditional strategy. We need to check that a positive measure of author types choose open access. An author chooses open access if his benefit v_a satisfies $v_a - p_a - x_a > q_r^{mt} v_a - p_a$, or upon substituting for x_a and rearranging,

(E17) $$v_a > \frac{c_r + \pi_r^{mt} + \epsilon}{1 - q_r^{mt}}.$$

But (E16) ensures that (E17) holds for some $\epsilon > 0$. □

The proof works by having the journal add an open access option, priced according to what in the context of telecommunications regulation is called the Efficient Components Pricing Rule (ECPR) (Baumol and Sidak 1994). To see this, consider the open-access premium for small ϵ:

$$\lim_{\epsilon \to 0} x_a = c_r + \pi_r^{mt} = (1 - q_r^{mt}) c_r + p_r^{mt} q_r^{mt}.$$

As specified by the ECPR, the open-access premium reflects two terms, a standard marginal cost term—in the present case the cost per reader c_r of serving the $1 - q_r^{mt}$ additional readers attracted by open access—and a second, opportunity cost term—in this case the lost revenue from these readers $p_r^{mt} q_r^{mt}$.

It remains to check that the sufficient conditions allowing us to focus on cases (i) and (ii) in equilibrium are not mutually inconsistent. This is easy to verify. We can ensure that hybrid pricing is more efficient than traditional by taking a distribution of author values with $\bar{v}_a = \infty$. Fixing this distribution and all other parameters, by Proposition 2 we can then ensure that a traditional journal is more profitable than an open-access journal by using α to scale the benefit distributions as in the statement of the proposition and considering a sufficiently low value of α.