A Simple Method for Bounding the Elasticity of Growing Demand with Applications to the Analysis of Historic Antitrust Cases

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We propose a simple method, requiring only minimal data, for bounding demand elasticities in growing, homogeneous-product markets. Since growing demand curves cannot cross, shifts in market equilibrium over time can be used to "funnel" the demand curve into a narrow region, bounding its slope. Our featured application assesses the antitrust remedy in the 1952 DuPont *decision, ordering incumbents to license patents for commercial plastics. We bound the demand elasticity significantly below 1 in many post-remedy years, inconsistent with monopoly, supporting the remedy's effectiveness. A second application investigates whether the 1911 dissolution of American Tobacco fostered competition in the cigarette market.*
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Though motivated by disparate questions, industrial-organization studies often boil down to the common exercise of estimating a demand elasticity. The exercise is sometimes hindered by a paucity of data, especially in historical contexts, and by the complexity of the methods required. We propose a method for bounding the demand elasticity in a growing, homogeneous product market. Based on the idea that growing demand curves do not cross, shifts in market equilibrium over time can be used to "funnel" the demand curve into a narrow region, thus bounding its slope. The method requires only minimal data—market price and quantity over a time span as short as two periods—and is simple to operationalize.

Figure 1 provides intuition for how the method works. In this example, the researcher has price and quantity data for two years. The equilibrium point in the initial year is e_0 and in the second is e_1. The researcher wants to bound the slope the inverse demand

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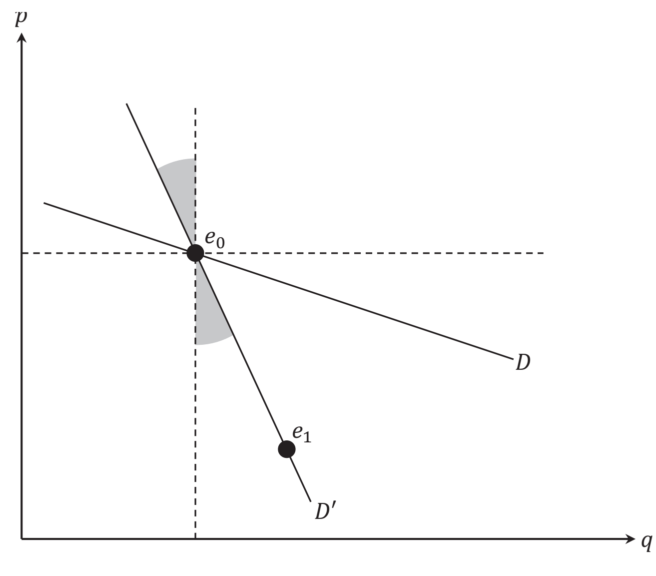

FIGURE 1. INTUITION FOR THE METHOD BOUNDING THE DEMAND ELASTICITY

curve through e_0. With so little data, there may be little hope to say anything more than inverse demand lies somewhere between the dotted horizontal line, corresponding to an infinitely elastic demand curve, and the dotted vertical line, corresponding to an infinitely inelastic one. In fact we can say more. Positing a functional form for demand, say linear, and assuming demand is growing over time, a curve like D can be ruled out because that would put the later equilibrium point e_1 on a lower demand curve. The demand curve through e_0 must be at least as steep as the line D' connecting e_0 and e_1. Comparing the two equilibrium points leads to a lower bound on the steepness of inverse demand, which translates into an upper bound on the elasticity of demand through e_0. A researcher who finds that demand is as inelastic as D' may be able to rule out monopoly. Intuitively, a drop from e_0 to e_1 may be so steep that a monopolist would never have decreased price this much for such a small increase in quantity, even if marginal costs fell to zero. A more plausible conclusion may be that competitive pressure drove firms onto the inelastic portion of demand.

Just two equilibrium points are being compared in the example in Figure 1. It is straightforward to see that with more data, the elasticity bound in the reference period could be improved by comparing e_0 to all other equilibrium points, taking the tightest of the results for the bound. Less straightforward is the computation of standard errors around the bounds, as standard bootstrapping methods are invalid for extreme order statistics. We show how bootstrapping method for extreme order statistics proposed by Zelterman (1993) can be adapted to our setting.

The example in Figure 1 implicitly assumes demand is linear, at least locally in the reference period. For pedagogical purposes, we work with linear demand throughout much of the paper; but the methods are quite general. We extend the bounds to whatever demand curve in a broad class the researcher imposes. We also provide bounds that only require a weak concavity or convexity assumption rather than an explicit functional form. Moving in the opposite direction of imposing stronger rather than weaker assumptions, we develop a variant of the methodology that can narrow the bounds on the demand elasticity considerably if the researcher is willing to assume that whatever functional form is imposed applies across all periods and over the whole range of demand rather than just locally around the equilibrium points.

Our featured application uses our method to assess the effectiveness of the antitrust remedy in the 1952 *DuPont* decision. The case involved two incumbent chemical manufacturers: the U.S. firm DuPont and the U.K. firm Imperial Chemical Industries (ICI). The two firms signed a Patents and Processes agreement in 1929, dividing the global market into exclusive territories between them. The U.S. government brought suit under the Sherman Act, alleging an illegal market division. The judge ruled in favor of the government, ordering the defendants to cancel their exclusive-territory arrangements, requiring them to license the patents behind polyethylene, a plastic widely used for commercial purposes.

Whether structural remedies for antitrust violations are effective in fostering competition has been an ongoing concern for scholars dating back to Adams (1951). As Areeda, Kaplow and Edlin (2013) note, remedies do not typically set the licensing terms, leaving them subject to commercial negotiation. This raises the opportunity for incumbent manufacturers to preserve the monopoly outcome by quoting exorbitant fees or withholding tacit knowledge necessary for entrants to compete effectively against them. The remedy in the *DuPont* case ostensibly had the desired procompetitive effect: eleven manufacturers entered by the end of the decade; prices steadily declined and output rose (Backman, 1964, p.71). However, the same price declines and output increases may have arisen in a monopoly market experiencing substantial cost declines, plausibly true for plastics in the 1950s and 60s. The entrants may merely have produced their share of the monopoly quantity, returning most of the rents to the incumbents.

Formal study that could cut through these criticisms using existing methods is hindered by a paucity of historical data for polyethylene, just yearly aggregate prices and quantities only for post-remedy years. Applying our method to this rather minimal dataset, we are able to bound the demand elasticity substantially and statistically significantly below 1 in many sample years. The bounds are robust to alternative products, functional forms, and method variants. Such inelastic demand is inconsistent with monopoly, suggesting the remedy may have been effective. The U.S. polyethylene market is a particularly opportune one in which to apply our methods because the assumption of growing demand needed for our method to work seems to be well founded for the product market during our 1958–72 sample period. Commercial applications that had only recently been invented were exploding, augmented by the baby boom and general macroeconomic growth.

To demonstrate the broader applicability of our method, we briefly explore a second application, investigating whether the breakup of the incumbent ordered by the remedy in the 1911 *American Tobacco* case led to competition in the cigarette market. Our method delivers extremely wide elasticity bounds—consistent with even perfect collusion—over the whole sample period except 1921–22, when the bound is 0.37, significantly less than 1 at the 5% level. Our method pinpoints the precise date of a breakdown in collusion identified anecdotally by Nicholls (1951) among other industry studies .

Our work is related to several literatures. One is the literature assessing the effectiveness of structural remedies for antitrust. In addition to the pioneering study by Adams (1951), a series of papers by U.S Federal Trade Commission staff—including Federal Trade Commission (1999, 2017) and Farrell, Pautler and Vita (2009)—have analyzed remedies accompanying a large sample of more modern mergers. We also contribute to the industrial-organization literature following Manski (1995) that seeks to bound rather than point-estimate elasticities and other parameters, including Haile and Tamer (2003); Ciliberto and Tamer (2009); Pakes (2010); and Pakes et al. (2015).

Our primary application is related to several literatures. The Patents and Processes agreement between DuPont and ICI was effectively a patent pool (Stocking and Watkins, 1946). Hence, that application relates to the larger literature on patent pools, notably Watzinger et al. (2017), who examine induced innovation in the aftermath of the 1956 Bell System consent decree. Our primary application also relates to commentaries on the *DuPont* remedy. Several leading commentaries criticized the remedy as overreaching; see, e.g., (Reader, 1975, p. 417) and (Hounshell and Smith, 1988, p. 206). We provide evidence supporting the effectiveness of the remedy in ending the monopoly and spurring competition in plastics.

Our secondary application contributes to the literature measuring conduct in the cigarette market. Leveraging the pass-through rate of excise taxes, Sumner (1981), Sullivan (1985), and Ashenfelter and Sullivan (1987) place firms' conduct strictly between the two extremes of perfect competition and perfect cartel. Our conclusions are consistent with theirs but for a more remote period using a bounds method does not rely on tax shifters. Our cigarette application also relates to historical commentaries on the 1911 *American Tobacco* case. Industry studies by Cox (1933), Tennant (1950), and Nicholls (1951) have been criticized for reaching tentative or unwarranted conclusions (see, e.g., Watkins, 1933; Vandermeulen, 1953) in part because of a paucity of data, a limitation which our study is designed to overcome.

I. Model

This section lays out a model of a growing market for a homogeneous product used in the analysis. Each period t, the interaction between producers and consumers on the market leads to an equilibrium $e_t \equiv (q_t, p_t)$, where $q_t \geq 0$ is quantity and $p_t \geq 0$ is price. The researcher observes the equilibrium over a set of periods $T \equiv \{\ldots, -2, -1, 0, 1, 2, \ldots\}$, where period $t = 0$ is the chosen reference period for which we will provide elasticity bounds. Let $E \equiv \{e_t \,|\, t \in T\}$ denote the set of time-series observations of equilibrium.

The following assumption streamlines the analysis by ruling out ties between price or quantity observations.

ASSUMPTION 1 (**Distinct Equilibria**): *For all $e_{t'}, e_{t''} \in E$ such that $t' \neq t''$, $q_{t'} \neq q_{t''}$ and $p_{t'} \neq p_{t''}$.*

This assumption entails little loss of generality supposing that two observations are never exactly equal when measured to arbitrary precision. We further assume the market is nontrivial in the sense of involving positive prices and quantities each period.

ASSUMPTION 2 (**Nontrivial Equilibria**): *For all $e_t \in E$, $q_t, p_t > 0$.*

We will characterize each side of the market in turn starting with producers. Since the goal of our applications is to determine whether the antitrust remedies were effective in changing producer conduct, it is natural to consider producer conduct as an unknown to be determined. Thus, we will be agnostic about producer behavior in the model, whether characterized by perfect competition, some form of oligopoly, or monopoly.

On the other side of the market, consumers are price takers whose behavior is captured by the demand curve $q = D_t(p)$. The index on the demand curve allows it to shift over time. Assume this function obeys the law of demand, formally that $D_t(p)$ is nonincreasing in p.

ASSUMPTION 3 (**Law of Demand**): *For all $t \in T$, $D_t(p') \geq D_t(p'')$ for all $p'' > p' \geq 0$.*

Assume that market demand is growing over time, formally that $D_t(p)$ is nondecreasing in t.

ASSUMPTION 4 (**Growing Demand**): *For all $t', t'' \in T$ such that $t' < t''$, $D_{t'}(p) \leq D_{t''}(p)$ for all $p \geq 0$.*

The assumption requires the demand curve to shift out at least weakly from one period to the next. The shift could be parallel, or it could involve some clockwise or counter-clockwise rotation as long as the rotation is not so acute that it leads demand curves in different periods to intersect.

Not all equilibrium configurations E are consistent with growing demand. To aid the discussion of which inconsistent configurations are ruled out, some additional notation is in order. Figure 2 depicts subsets of equilibrium determined by compass directions relative to the reference equilibrium point, e_0. Algebraically, $NW \equiv \{(q_t, p_t) \in E \mid q_t < q_0,\ p_t > p_0\}$ and analogously for the sets *NE*, *SE*, and *SW*. The fact that the compass-set definitions involve strict inequalities leaves points on the dotted lines through e_0 in the figure unclassified, but this is without loss of generality under Assumption 1, which precludes equilibrium points from sharing coordinates. The compass sets can be further partitioned depending on when the equilibria occur. For example, define $NW^- \equiv \{e_t \in NW \mid t < 0\}$, the subset of equilibrium points in *NW* that occur before e_0, and $NW^+ \equiv \{e_t \in NW \mid t > 0\}$, the subset that occur after. Define the other six subsets (NE^-, NE^+, SE^-, SE^+, SW^-, and SW^+) analogously. Using this notation, a necessary condition for Assumption 4 is that $SW^+ = NE^- = \emptyset$.

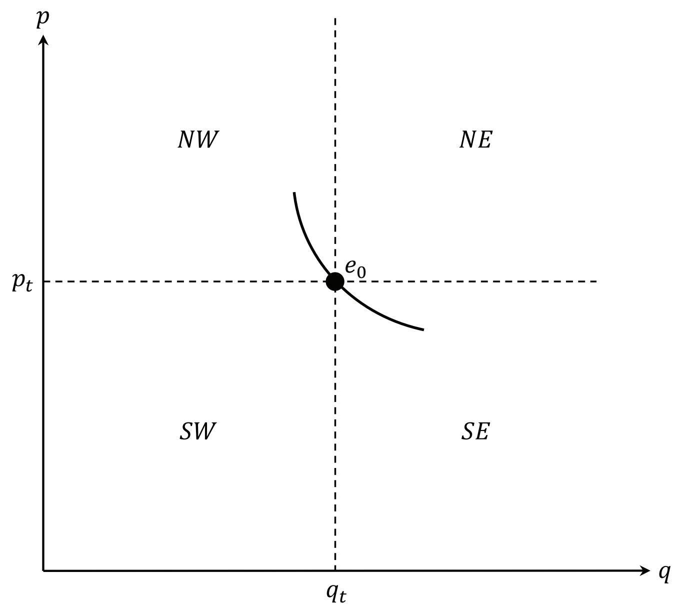

FIGURE 2. SETS DETERMINED BY COMPASS POSITION

II. Bounds Assuming Linear Demand

Our bounds methods require the researcher to impose an assumption on the functional form or curvature of demand. To build intuition, we start by assuming demand is linear. Section III extends the methods to general functional forms.

A. Linear Specification

Assume demand in the reference period $t = 0$ is given by

$$D_0(p) = a_0 - b_0 p. \tag{1}$$

Demand in other periods can be different from (1) and need not even be linear.

The demand specification in (1) involves two parameters, but further requiring the line to pass through the equilibrium point e_0 pins it down to a single-parameter family. Focus for now on b_0 as the key parameter. Letting $\tilde{D}(p, b_0, e_0)$ be the linear demand with parameter b_0 passing through equilibrium point $e_0 = (q_0, p_0)$, we have

$$D_0(p) = \tilde{D}(p, b_0, e_0) = q_0 + b_0(p_0 - p). \tag{2}$$

Assumption 3 (law of demand) holds if and only if $b_0 \geq 0$.

The absolute value of the demand elasticity is

$$\varepsilon_0 \equiv |\tilde{D}_p(p_0, b_0, e_0)| \frac{p_0}{q_0} = \frac{b_0 p_0}{q_0}, \tag{3}$$

where the subscript on $\tilde{D}_p$ denotes a partial derivative. Note that $b_0 \geq 0$ implies $\varepsilon_0 \geq 0$. Hereafter, we drop the "absolute value" modifier and simply call ε_0 the demand elasticity.

B. Method Incorporating Local Information

We refer to our basic method for bounding demand elasticities as *incorporating local information* since it uses no information beyond the relative location of the equilibrium points. This contrasts the method *incorporating limiting information*, which introduced in Section VI, that projects demand intercepts and constraints them not to cross.

The method incorporating local information derives bounds on demand steepness b_0 via pairwise comparisons of that period's equilibrium point e_0 with the location of other equilibrium points, e_t. Using equation (3), bounds on b_0 can then be translated into the desired bounds on ε_0.

Start by comparing e_0 to a later equilibrium point, e_t, with $t > 0$. We have

$$\tilde{D}(p, b_0, e_0) = D_0(p) \leq D_t(p), \tag{4}$$

where the equality follows from (2) and the inequality from the assumption of growing demand. Since (4) holds for all $p \geq 0$, it must hold in particular for p_t. Substituting p_t into (4) yields $\tilde{D}(p_t, b_0, e_0) = q_0 + b_0(p_0 - p_t) \leq D_t(p_t) = q_t$, or rearranging,

$$b_0(p_0 - p_t) \leq q_t - q_0. \tag{5}$$

The bounds on b_0 that can be derived from equation (5) depend on the compass position of e_t relative to e_0. There are three subcases to consider depending on whether e_t is in NW^+, NE^+, or SE^+ (as discussed in Section I, the remaining set SW^+ is empty). To streamline the subsequent analysis, introduce the function $B(e_{t'}, e_{t''})$, denoting the absolute value of the slope of the linear demand through points $e_{t'}$ and $e_{t''}$:

$$B(e_{t'}, e_{t''}) \equiv \left| \frac{q_{t''} - q_{t'}}{p_{t''} - p_{t'}} \right|. \tag{6}$$

Suppose $e_t \in NW^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \geq \frac{q_t - q_0}{p_0 - p_t} = B(e_0, e_t). \tag{7}$$

The first step follows from $e_t \in NW^+$, which implies $p_0 < p_t$ by definition of the compass set. The second step follows because the numerator and denominator of the middle fraction are both negative for $e_t \in NW^+$. Condition (7) provides a lower bound on the demand slope.

Next, suppose $e_t \in NE^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \geq \frac{q_t - q_0}{p_0 - p_t}. \tag{8}$$

The right-hand is negative since the numerator is positive and the denominator is negative for $e_t \in NE^+$. Hence, (8) is a weaker condition than the maintained assumption $b_0 \geq 0$. Thus, this case contributes no useful information to bound b_0.

Next, suppose $e_t \in SE^+$. Cross multiplying (5) by $p_0 - p_t$ yields

$$b_0 \leq \frac{q_t - q_0}{p_0 - p_t} = B(e_0, e_t). \tag{9}$$

Cross multiplying did not change the direction of the inequality because $p_0 > p_t$ for $e_t \in SE^+$. The second step follows because both numerator and denominator of the middle fraction are positive for $e_t \in SE^+$. Condition (9) provides an upper bound on the demand slope.

The pairwise comparison of e_0 and e_t can be repeated for $t < 0$. Sparing the details, analysis similar to the preceding can be used to show $b_0 \leq B(e_0, e_t)$ when $e_t \in NW^-$, $b_0 \geq B(e_0, e_t)$ when $e_t \in SE^-$, and no useful information is contributed when $e_t \in SW^-$. We have proved the following proposition.

PROPOSITION 1: *Suppose demands in all periods satisfy Assumptions 3 (law of demand) and 4 (growing demand). If demand in the reference period is linear, i.e., $D_0(p) = \tilde{D}(p, b_0, e_0)$, then $b_0 \in [\underline{b}_0^*, \bar{b}_0^*]$, where*

$$\underline{b}_0^* \equiv 0 \vee \sup_{e_t \in SE^- \cup NW^+} B(e_0, e_t) \tag{10}$$

$$\bar{b}_0^* \equiv \inf_{e_t \in NW^- \cup SE^+} B(e_0, e_t). \tag{11}$$

The $\vee$ operator denotes the join; i.e., $x \vee y \equiv \max\{x, y\}$. The use of this operator in (10) indicates the imposition of a floor of 0 on top of the supremum.[1] We will later use the related operator $\wedge$ for the meet; i.e., $x \wedge y \equiv \min\{x, y\}$.

Proposition 1 is more general than may first appear. The proposition only requires demand to be linear in the reference period in which we are deriving the elasticity bound. Functional forms in other periods can be arbitrary. Furthermore, the reference demand curve is only required to be locally linear, i.e., for quantities and prices on or inside

[1]The supremum is taken in (10) rather than the maximum and the infimum in (11) rather than the minimum even though the sets involved are discrete to accommodate the possibility that one of these sets is empty. An empty set does not have a maximum or minimum but does have a supremum and infimum; we use the conventional definitions $\inf \emptyset = \infty$ and $\sup \emptyset = -\infty$. If the set $SW^- \cup NW^+$ over which the supremum is taken in (10) is nonempty, then imposing a 0 floor on the supremum is superfluous because B is defined to be an absolute value. However, if the set over which the supremum is taken happens to be empty, then $\underline{b}_0^* = -\infty$ without the imposition of the 0 floor.

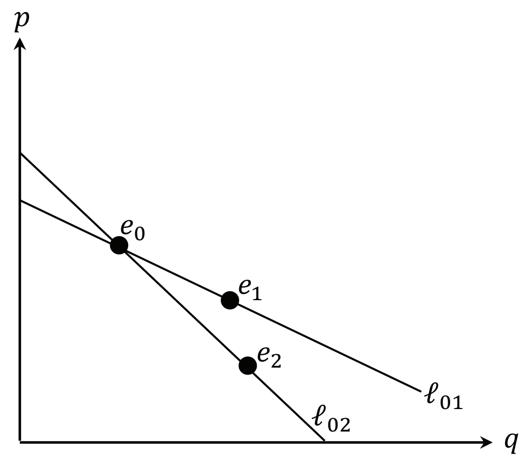

FIGURE 3. ILLUSTRATING METHOD INCORPORATING LOCAL INFORMATION

the minimum bounding box around E. For prices and quantities outside this box, the reference demand curve can have any functional form.

Bounds on b_0 can be translated into bounds on ε_0 using the elasticity formula (3). In particular, $\varepsilon_0 \in [\underline{\varepsilon}_0^*, \bar{\varepsilon}_0^*]$, where $\underline{\varepsilon}_0^* \equiv \underline{b}_0^* p_0/q_0$ and $\bar{\varepsilon}_0^* \equiv \bar{b}_0^* p_0/q_0$.

Proposition 1 prescribes a simple algorithm for bounding the elasticity of demand. Figure 3 provides an illustration with three equilibrium points. Taking the initial equilibrium as the reference point e_0, suppose we want to bound the steepness, b_0, and ultimately the elasticity, ε_0, of the demand curve through e_0. We first draw lines ℓ_{01} and ℓ_{02} connecting e_0 to the other equilibrium points. The steeper of the two, ℓ_{02}, provides the a lower bound on the steepness of the inverse demand curve through e_0, which translates into the upper bound $\bar{b}_0^*$ on the steepness demand stated in Proposition 1. Using (3), $\bar{b}_0^*$ can be translated into an upper bound on the demand elasticity: $\bar{\varepsilon}_0^* = \bar{b}_0^* p_0/q_0$. Because SE^- and NW^+ happen to be empty, pairwise comparisons do not yield a nontrivial lower bound $\underline{\varepsilon}_0^*$ in this illustration (apart from $\underline{\varepsilon}^* = 0$, implied by non-negativity).

C. *Assessment in Cournot Example*

This subsection takes stock of the performance of the bounding method under various market conditions in a simple Cournot model. Besides being a tractable representation of oligopoly, the Cournot model provides fairly robust insights because it nests both monopoly and perfect competition as special cases.

Consider a two-period model indexed by $t \in \{0,1\}$ in which n_t homogeneous firms with costant marginal and average cost equal to c_t engage in Cournot competition in a market with linear demand, $q_t = a_t - b_t p_t$. For concreteness, we will take the initial period as the reference period and determine the conditions under which the bounds $[\underline{\varepsilon}_0^*, \bar{\varepsilon}_0^*]$ are tight around the actual elasticity, ε_0. While the findings in this subsection do not rise to the status of general propositions—applying just to the Cournot example considered here—we still set them off as numbered results so that they can be referenced while relegating the technical details behind them to Appendix A.

Our first result clarifies what growing demand entails when demand is linear.

RESULT 1: *Assumption 4 that demand is growing holds in this subsection's model if and only if*

$$\frac{a_1}{a_0} \geq \max\left\{1, \frac{b_1}{b_0}\right\}. \tag{12}$$

The result indicates that the linear demand curve can grow in several ways. Linear demand can grow increasing a_t holding b_t constant, resulting in a parallel shift out from the origin. Demand can also grow holding a_t constant but reducing b_t, resulting in an rotation of the inverse demand curve outward from the origin, pivoting through the horizontal intercept. It is even possible for demand to grow when the inverse demand curve rotates the opposite way as long as the rotation does not overwhelm the parallel shift out, guaranteed by $a_1/a_0 \geq b_1/b_0$.

Solving for a firm's Nash equilibrium quantity, and using this to derive market quantity and price, we obtain equilibrium point $e_0 = (q_0, p_0)$, where

$$q_0 = \frac{n_0}{1+n_0}(a_0 - b_0 c_0), \quad p_0 = \frac{1}{1+n_0}\left(\frac{a_0}{b_0} + n_0 c_0\right) \tag{13}$$

and analogously for e_1.

The next result provides an example in which our bounds fail to contain the true elasticity if demand is shrinking in violation of condition (12).

RESULT 2: *Consider a market with a fixed structure ($n_0 = n_1$) that experiences a parallel, downward demand shift (i.e., $b_0 = b_1$ and $a_0 > a_1$) though $e_1 \in SE^+$. Then $(\varepsilon_0 - \bar{\varepsilon}_0^*)/\varepsilon_0 \geq (a_0 - a_1)/a_0$.*

The result not only provides an example in which the bound understates the true elasticity, it quantifies the understatement, showing that our bound understates the true elasticity by at least the percentage that demand shrinks.

While condition (12) ensures bounds $\underline{\varepsilon}_0^*$ and $\bar{\varepsilon}_0^*$ do not fail, a remaining problem is that they may be trivially uninformative, including all nonnegative values. By (10), $\underline{b}_0^*$ and $\underline{\varepsilon}_0^*$ are trivially equal to 0 unless $SE^- \cup NW^+$ is nonempty. Of course $SE^- = \emptyset$ because there is no equilibrium point prior to e_0 in this two-period model. Thus $\underline{\varepsilon}_0^* > 0$ only if NW^+ is nonempty, requiring $e_1 \in NW^+$, in turn requiring output to fall from period 0 to 1. In view of (13), one can show that the only way for output to fall when demand is increasing is for marginal cost or industry concentration to increase. One can similarly show that for $\bar{\varepsilon}_0^*$ to be nontrivial, marginal cost or industry concentration must decrease. We have the following result.

RESULT 3: *$\underline{\varepsilon}_0^*$ is nontrivial (i.e., $\underline{\varepsilon}_0^* > 0$) only if $c_1 > c_0$ or $n_1 < n_0$; $\bar{\varepsilon}_0^*$ is nontrivial (i.e., $\bar{\varepsilon}_0^* < \infty$) only if $c_1 < c_0$ or $n_1 > n_0$.*

Result 3 suggests that obtaining a nontrivial bound on ε_0 requires an appropriate market shock. As the conditions required for the lower bound to be nontrivial are the opposite of those required for the upper bound, we see that a single shock can inform the lower bound or the upper bound but not both. With many periods and equilibrium points, nontrivial values of both lower and upper bounds can be obtained if shocks in different directions are experienced, allowing some pairwise comparisons to contribute to the lower bound and some to the upper bound. In our *DuPont* application, there is never a shock that moves a later equilibrium point into NW^+, so we will only be able to identify nontrivial upper, not lower, bounds.

If luck has it that demand remains constant across the two periods, then one of our bounds will hit the true elasticity exactly, as the next result states.

RESULT 4: *Suppose $a_0 = a_1$ and $b_0 = b_1$. Then one of $\underline{\varepsilon}_0^*$ or $\bar{\varepsilon}_0^*$ equals ε_0. In particular, $\underline{\varepsilon}_0^* = \varepsilon_0$ if $e_1 \in NW^+$; and $\bar{\varepsilon}_0^* = \varepsilon_0$ if $e_1 \in SE^+$.*

Improbable luck is required for demand to remain constant while satisfying Assumption 4 because the assumption would then only be satisfied by the narrowest of margins. If, instead of remaining constant, demand increases from one period to the next, whether due to a parallel shift or rotation outward from the origin, $\bar{\varepsilon}_0^*$ will no longer hit ε_0. The greater the demand increase, the wider the gap between the two. A tension thus underlies our method: a sizable increase in demand provides assurance that Assumption 4 is satisfied, without which the researcher cannot be sure $\bar{\varepsilon}_0^*$ is a bound at all. If the increase in demand is too great, however, the bound ceases to be informative. Our bound works best if a market with moderately growing demand experiences a sharp enough shock (whether a drop in marginal cost or an increase in competitiveness) to induce firms to drop prices.

In the absence of identifying shocks, our bounds can perform poorly. Extreme cases can be constructed in which our upper elasticity bound is infinite despite the true elasticity being close to 0.[2] For our upper bound on the elasticity to be close to 0, the true elasticity must be close to 0 and the market must experience shocks during the sample period capable of revealing the low elasticity. Absent such shocks, our method will be uninformative.

Our underlying model assumes all consumers pay the same linear price on a common market. The assumption is violated if firms charge different prices on segmented markets, engage in nonlinear pricing, or bargain with individual consumers. An exhaustive study of the conditions under which our methods are robust under these alternatives is beyond the scope of this subsection. Indeed, the notion of a single elasticity that can be bounded may be ill-defined with multiple segmented markets. We will be content to demonstrate the robustness of our methods to one extreme departure from our model. Suppose the market is served by a monopolist ($n_0 = n_1 = 1$) who engages in perfect price discrimination. Assume that $e_1 \in SE^+$ and that the growing-demand condition (12)

[2]For example, consider a perfectly competitive market ($n_t \to \infty$) with growing demand that does not experience a cost shock ($c_0 = c_1$). Since neither marginal cost nor competitiveness change from one period to the next, the fact that demand is growing implies $p_1 \geq p_0$, in turn implying $\bar{b}_0^* = \bar{\varepsilon}_0^* = \infty$. The true elasticity is $\varepsilon_0 = b_0 c_0/(a_0 - b_0 c_0)$, which approaches 0 as $c_0 \to 0$.

holds. The next result shows that our upper bound continues to be valid in this case but is quite conservative—more than double the true elasticity.

RESULT 5: *Suppose a monopolist engages in perfect price discrimination. Then* $\bar{\varepsilon}_0^* \geq 2\varepsilon_0$.

III. Generalizations

This section describes several generalizations of the bounds obtained under the assumption of linear demand. Section III.A continues to require the researcher to assume a functional form for demand but allows the form to be chosen from a general class beyond linear. Section III.B requires the researcher to make an assumption on curvature (weak concavity or convexity) rather than functional form. Section III.C discusses application of the methods to markets with shrinking rather than growing demand.

A. General Functional Forms

Suppose the researcher specifies some (possibly nonlinear) form for demand, $q = D_0(p)$ in the reference period. While this demand curve may start out as a multiple-parameter family, assume that once it is required to pass through e_0, this pins it down to a single-parameter family indexed by θ_0:

$$D_0(p) = \tilde{D}(p, \theta_0, e_0). \tag{14}$$

The elasticity of demand in the general case is defined as

$$\varepsilon_0 \equiv |\tilde{D}_p(p_0, \theta_0, e_0)| \frac{p_0}{q_0}, \tag{15}$$

where the subscript on $\tilde{D}$ denotes the partial derivative with respect to the indicated argument.

Assume $\tilde{D}(p, \theta, e_0)$ is continuously differentiable of all orders in its first two arguments. Assume further that increases in θ cause demand to rotate. As an accounting convention, assume that the direction of the rotation is such that demand become steeper (and inverse demand less steep) when θ increases. Thus, for all $\theta \geq 0$,

$$\tilde{D}_\theta(p, \theta, e_0) < 0 \quad \text{if } p > p_0 \tag{16}$$

$$\tilde{D}_\theta(p, \theta, e_0) = 0 \quad \text{if } p = p_0 \tag{17}$$

$$\tilde{D}_\theta(p, \theta, e_0) > 0 \quad \text{if } p < p_0, \tag{18}$$

Further, impose the following Inada conditions:

$$\lim_{\theta \to 0} \tilde{D}(p, \theta, e_0) = q_0 \tag{19}$$

(20)
$$\lim_{\theta\to\infty} \tilde{D}(p,\theta,e_0) = \begin{cases} 0 & p > p_0 \\ \infty & p < p_0. \end{cases}$$

Equation (19) implies that $\tilde{D}(p,\theta,e_0)$ becomes perfectly inelastic for arbitrarily small θ, approaching a vertical line through quantity q_0. Equation (20) implies that $\tilde{D}(p,\theta,e_0)$ becomes perfectly elastic as θ becomes arbitrarily large. Together, (19) and (20) in effect say the domain of θ is rich enough to allow changes in θ to trace out all possible demand elasticities from perfectly elastic to perfectly inelastic.

Proceeding with the analysis, start by supposing $t > 0$. To respect Assumption 4 of growing demand, for all $p \geq 0$ we must have $\tilde{D}(p,\theta_0,e_0) = D_0(p) \leq D_t(p)$. The preceding inequality must hold in particular for $p = p_t$, implying

(21)
$$\tilde{D}(p_t,\theta_0,e_0) \leq D_t(p_t) = q_t.$$

Let $\Theta(e_0,e_t)$ be the solution to the equation formed by treating the preceding condition as an equality, i.e., the value of $\theta \geq 0$ solving

(22)
$$\tilde{D}(p_t,\theta,e_0) = q_t.$$

The proof of the next proposition, provided in Appendix A, shows that, for all $e_t \in SE^+$, $\Theta(e_0,e_t)$ exists, is unique, and provides an upper bound on the θ_0 satisfying (21). Similar analysis applies for other compass sets and for $t > 0$. We have the following proposition, which generalizes the bounds obtained by incorporating local information.

PROPOSITION 2: *Suppose demands in all periods satisfy Assumptions 3 and 4. If demand in the reference period has the general functional form specified in* (14)*, i.e.,* $D_0(p) = \tilde{D}(p,\theta_0,e_0)$*, and further satisfies* (16)–(20)*, then* $\theta_0 \in [\underline{\theta}_0^*, \bar{\theta}_0^*]$*, where*

(23)
$$\underline{\theta}_0^* \equiv 0 \vee \sup_{e_t \in SE^- \cup NW^+} \Theta(e_0,e_t)$$

(24)
$$\bar{\theta}_0^* \equiv \inf_{e_t \in NW^- \cup SE^+} \Theta(e_0,e_t).$$

The proposition prescribes the following algorithm for bounding θ_0. The lower bound, $\underline{\theta}_0^*$ is found by pairing e_0 with each of the equilibrium points e_t in SW^- and NW^+, computing $\Theta(e_0,e_t)$ by solving the nonlinear equation (22). Equation (22) is well-behaved. In particular—as shown in the proof of Proposition 2—the left-hand side of (22) is monotonic in θ for $e_t \in NW \cup SE$. Thus standard methods (including Newton-Raphson or even a straightforward grid search) can be used to rapidly solve (22) to derive $\Theta(e_0,e_t)$. The largest $\Theta(e_0,e_t)$ over all pairwise comparisons becomes $\underline{\theta}_0^*$. If both SW^- and NW^+ are empty, $\underline{\theta}_0^*$ is set to respect the non-negativity constraint on θ_0; i.e., $\underline{\theta}_0^* = 0$. The upper bound, $\bar{\theta}_0^*$, is found by pairing e_0 with each of the equilibrium points e_t in NW^- and SE^+, computing $\Theta(e_0,e_t)$ for each pair by solving the nonlinear equation (22). The smallest $\Theta(e_0,e_t)$ over these pairwise comparisons becomes $\bar{\theta}_0^*$. If both NW^- and SE^+ are empty, the formula (24) yields $\bar{\theta}_0^* = \inf \emptyset = \infty$, correctly implying no upper bound is obtained in this case.

Under maintained conditions, the right-hand side of (15) is nondecreasing in θ_t.[3] Thus, that elasticity formula can be used to translate bounds on θ_0 into bounds on the elasticity. In particular, defining $\underline{\varepsilon}_0^* \equiv |\tilde{D}_p(p_0, \underline{\theta}_0^*, e_0)| p_0/q_0$ and $\bar{\varepsilon}_0^* \equiv |\tilde{D}_p(p_0, \bar{\theta}_0^*, e_0)| p_0/q_0$, we have $\varepsilon_0 \in [\underline{\varepsilon}_0^*, \bar{\varepsilon}_0^*]$.

An important special case of general nonlinear demand is logit. Since its microfounding by McFadden (1973), logit has been a widely used functional form for demand in industrial organization, for example, in structural estimation of differentiated-product demand following Berry, Levinsohn and Pakes (1995). For reference, Appendix B provides a specification of logit demand in a homogeneous-product market and derives corollaries of the general-demand propositions in this special case. We draw on these corollaries in our empirical analysis, which provides a parallel set of results for logit alongside those for linear demand.

B. Curvature Assumptions

The bounds so far have required the researcher to posit a functional form for demand. This subsection explores bounds that obtain when a weaker curvature (concavity or convexity) assumption is imposed rather than a specific functional form. We obtain sharp results. The bounds have the exact same form as under the stronger linear-demand assumption. The penalty for imposing the weaker curvature assumption is that only a subset of the data contributes useful information to the bound.

Figure 4 provides an illustration. Assuming linear demand, the inverse demand through e_0 must be at least as steep as the dashed line ℓ or else it would pass above e_1, which is not allowed if e_1 is later than e_0. Consider the strictly convex curve u through e_0.[4] If u starts out less steep than ℓ as it passes through e_0, the curvature of u will bend it above e_1. To avoid this contradiction to growing demand, u must be steeper at e_0 than ℓ. By this logic, ℓ provides an upper bound on the steepness of all weakly convex inverse demands through e_0. On the other hand, a concave inverse demand curve like d can start out flatter than ℓ at e_0 yet bend so that does not pass above e_1. Hence pairwise comparison of e_0 and e_1 provides no useful bounding information under the assumption of weak concavity.

Analogous reasoning can be applied to the other compass sets—NW^+, SE^-, and NW^-—that were informative in the linear case. Relaxing the assumption on demand from linearity to weak concavity, one can show that pairwise comparisons of e_0 to later equilibrium points are uninformative, but pairwise comparisons to earlier equilibrium points are just as informative as under linear demand. Mirror-image results are obtained if the assumption on demand is relaxed from linearity to weak convexity rather than concavity.

[3]Conditions (16)–(18) together with the continuous differentiability of $\tilde{D}$ of all orders in all arguments imply $\tilde{D}_{p\theta}(p_t, \theta_t, e_t) \leq 0$. But then $\frac{\partial}{\partial \theta}[-\tilde{D}_p(p_t, \theta_t, e_t)] \geq 0$, implying that the right-hand side of (15) is nondecreasing in θ_t.

[4]Since convex and concave start with the same letter, we will distinguish between the two in the notation throughout this subsection using u for concave up (equivalent to convex) and d for concave down (equivalent to concave).

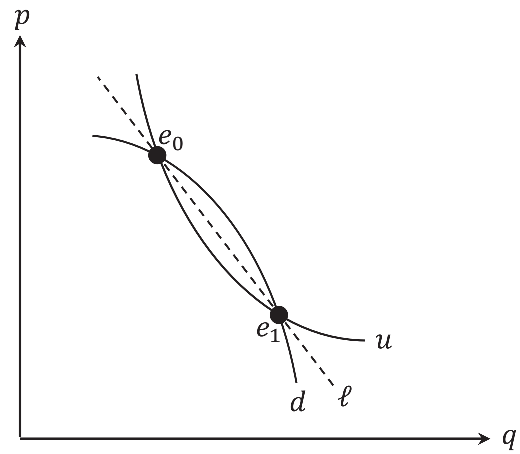

FIGURE 4. BOUNDS IMPOSING CURVATURE ASSUMPTIONS

PROPOSITION 3: *Suppose demands in all periods satisfy Assumptions 3 (law of demand) and 4 (growing demand). If demand in the reference period $D_0(p)$ is weakly concave, then $|D_0'(p_0)| \in [\underline{b}_0^d, \bar{b}_0^d]$, where*

$$\underline{b}_0^d \equiv 0 \vee \sup_{e_t \in SE^-} B(e_0, e_t) \tag{25}$$

$$\bar{b}_0^d \equiv \inf_{e_t \in NW^-} B(e_0, e_t). \tag{26}$$

If $D_0(p)$ is weakly convex, then $|D_0'(p_0)| \in [\underline{b}_0^u, \bar{b}_0^u]$, where

$$\underline{b}_0^u \equiv 0 \vee \sup_{e_t \in NW^+} B(e_0, e_t) \tag{27}$$

$$\bar{b}_0^u \equiv \inf_{e_t \in SE^+} B(e_0, e_t). \tag{28}$$

The formal proof in Appendix A is a consequence of the mean value theorem. It is worth emphasizing that the curvature assumptions need only be imposed on demand in the reference period 0; demand in other periods can have arbitrary curvature.

The bounds on the demand slope translate readily into elasticity bounds. If $D_0(p)$ is weakly concave, then $\varepsilon_0 \in [\underline{\varepsilon}_0^d, \bar{\varepsilon}_0^d]$, where $\underline{\varepsilon}_0^d \equiv \underline{b}_0^d p_0/q_0$ and $\bar{\varepsilon}_0^d \equiv \bar{b}_0^d p_0/q_0$. If $D_0(p)$ is weakly convex, then $\varepsilon_0 \in [\underline{\varepsilon}_0^u, \bar{\varepsilon}_0^u]$, where $\underline{\varepsilon}_0^u \equiv \underline{b}_0^u p_0/q_0$ and $\bar{\varepsilon}_0^u \equiv \bar{b}_0^u p_0/q_0$.

Focusing on the sets over which the suprema and infima in Proposition 3 are taken provides insight into when the bounds will be most informative. Taking the earliest sample period as the reference period, we have $SE^- = NW^- = \emptyset$, implying $\underline{b}_0^d = 0$ and $\bar{b}_0^d = \infty$, whereas $\underline{b}_0^u = \underline{b}_0$ and $\bar{b}_0^u = \bar{b}_0$. This shows that for initial sample years, the bounds for weakly concave demand are completely uninformative but the bounds for weakly convex demand are just as tight as those for linear demand. A similar argument can be used to show that for the end of the sample, the bounds for weakly convex demand are completely uninformative but the bounds for weakly concave demand are just as tight as those for linear demand.[5]

C. Shrinking Demand

In theory, one should be able to obtain analogous elasticity bounds under the opposite assumption of definitively declining demand simply by reversing all the signs and inequalities. While theoretically true, there are some practical difficulties. First, even for markets experiencing secular declines, it may be hard to contend that the demand curve is definitively shifting back each year. Consumers may be losing their taste for a product, but natural population growth may offset this taste change. Second, even if suppliers are not investing much, the spillover of technology from other industries may lower costs in the market, shifting supply out. Equilibrium points may end up shifting to the southwest over time, offering little useful information to bound elasticities.

IV. Application to *DuPont* Case

A. Institutional Background

After World War II, the U.S. Department of Justice (DOJ) sued U.S. chemical manufacturer, DuPont, and its U.K. co-conspirator, Imperial Chemical Industries (ICI), for violating the Sherman Act with their Patents and Processes agreement. Signed in 1929, the agreement granted each company exclusive licenses for the patents and secret processes controlled by the other and divided the global market into exclusive territories between them. The DOJ won the case;[6] and a remedy was ordered in 1952.[7] The remedy cancelled the exclusive-territory arrangements between the firms and required them to license the technologies behind some their most important products to all applicants at reasonable royalty rates.

The incumbents' most significant patents covered three products: polyethylene, nylon, and neoprene. The order did not compel licensing of neoprene. Of the remaining products, commentators have contended that only the polyethylene patents garnered substantial commercial interest (Whitney, 1958, p. 217). Thus, we focus on the structural

[5]The results for the two curvatures in Proposition 3 can be combined to provide bounds allowing one to be agnostic about demand curvature. One approach continues in the spirit of imposing no assumptions on demand outside of reference period 0. If demand in the reference period $D_0(p)$ is either weakly convex or weakly concave, then Proposition 3 implies $|D_0'(p_0)| \in [\underline{b}_0^{ud}, \bar{b}_0^{ud}]$, where $\underline{b}_0^{ud} \equiv \underline{b}_0^d \wedge \underline{b}_0^u$ and $\bar{b}_0^{ud} \equiv \bar{b}_0^d \vee \bar{b}_0^u$. Translating into elasticities, $\varepsilon_0 \in [\underline{\varepsilon}_0^{ud}, \bar{\varepsilon}_0^{ud}]$, where $\underline{\varepsilon}_0^{ud} \equiv \underline{b}_0^{ud} p_0/q_0$ and $\bar{\varepsilon}_0^{ud} \equiv \bar{b}_0^{ud} p_0/q_0$. These bounds for unknown curvature will tend to be most informative toward the middle of the sample since the bounds are uninformative in initial years for weakly concave demand and in final years for weakly convex demand; these bounds also require identifying market shocks to occur both before and after the reference period. If such data requirements prove too stringent in an application, another approach is to assume that demand in all periods has each curvature in turn. If concavity produces tight bounds in some sample years and convexity in others, one may be able to rule out monopoly in some years under either curvature.

[6]"United States v. Imperial Chemical Industries," 100 *F. Supp.* 504 (Southern District of New York, 1951).

[7]"United States v. Imperial Chemical Industries," 105 *F. Supp.* 215 (Southern District of New York, 1952).

remedy's effect on the polyethylene market. Polyethylene was among the first commercially developed plastics and continues to top global sales. We have data on the two main types: low-density polyethylene, used for example to make Tupperware (Clarke, 1999, p. 2) and high-density polyethylene, used for example to make Hula Hoops (Fenichell, 1966, p. 264). Although polyethylene can be differentiated across producers, Lieberman characterizes the differentiation as slight in a series of papers (Lieberman, 1984, 1987; Gilbert and Lieberman, 1987). We follow these papers in modeling polyethylene as a homogeneous, commodity-type chemical.

The remedy in the *DuPont* case ostensibly had a dramatic effect on the polyethylene market (Backman, 1964, p. 71). Eleven manufacturers entered by 1959. Prices steadily declined and output rose. On the face of these facts, one might be tempted to conclude that the remedy achieved its purpose of increasing market competitiveness. However, the same price declines and output increases may have arisen in a monopoly market experiencing substantial cost declines, plausibly realistic for plastics in the 1950s and 60s. The fact of entry seems to disprove monopoly unless it is thought that the entrants are merely producing their share of the monopoly quantity, returning most of the rents to the licensor. Our study will try to produce evidence for the effectiveness of the remedy cutting through these criticisms.

A key assumption for our bounds methods to work is that demand was growing over the sample period. The explosive quantity growth documented in next section does not by itself prove that demand was growing; a supply shift (decrease in cost or market concentration) could have accounted for the quantity increase. Independent historical sources bolster the case of growing demand. Prior to World War II, polyethylene mainly had military rather than commercial applications. (Reader, 1975, p. 356) writes that downstream commercial innovation after the war sparked polyethylene demand: "By the mid-fifties packaging film and household goods, made by injection or extrusion moulding, would carry the world consumption of low-density polythene into six figure tonnages, but in 1939 none of that had been thought of. . . ." Backman (1970) notes that consumer acceptance of the materials was unusually rapid. Gordon (2016) classifies plastics as a drastic innovation for many commercial users, dramatically reducing their materials costs, making a noticeable contribution to U.S. growth during the period. The baby boom, adding 65 million children to the United States population between 1944 and 1961, was an important demand driver (Hodges, 2016). In addition to the direct channel of increased demand for plastic toys and household goods, another channel was household formation. House construction boomed, and these new houses were increasingly built with polyethylene pipes, wire insulation, and other inputs containing polyethylene because of its unique combination of strength with low weight. New households also bought more cars, increasingly using polyethylene in their manufacture. More broadly, U.S. GDP grew strongly over our sample period. The NBER identifies three recessions during our sample; but each lasted only a quarter, and none led to an fall in annual GDP, which grew every year in our sample (in nominal and real terms). A deeper recession started in 1973, but our sample is cut off before then.

TABLE 1—DESCRIPTIVE STATISTICS FOR POLYETHYLENE DATA

		Low-density polyethylene		High-density polyethylene	
Variable	Units	Mean	Std. dev.	Mean	Std. dev.
Price	$ per pound	0.19	0.07	0.22	0.09
Quantity (sales)	Billion pounds	2.37	1.27	0.78	0.61
Quantity (production)	Billion pounds	2.57	1.41	0.92	0.69

Source: U.S. Tariff Commission (various years).
Note: Annual data over 1958–72. Price in nominal terms.

B. Data

Our dataset consists of annual price and quantity data, aggregated across all firms in the U.S. market, for low- and high-density polyethylene. We hand collected data from annual reports issued by the U.S. Tariff Commission (various years). While we would have liked to include the immediate window around the *DuPont* decision in 1952 in our sample, unfortunately the earliest year polyethylene data is available from that (or to our knowledge any) source is 1958. We cut the sample off after 1972 because the subsequent OPEC crisis created an oil shock, disrupting the plastics market, triggering a recession, challenging the assumption of growing demand.[8] The price series is an average wholesale price, computed by dividing total annual industry revenue by industry quantity sold. We use nominal prices in the analysis to avoid a mismeasured inflation index driving spurious price decreases; the online appendix reports qualitatively similar results using prices deflated by the Consumer Price Index.

Table 1 provides descriptive statistics. Prices are similar across products. The quantity of low-density polyethylene sold was about twice that of high-density polyethylene. To economize on space, we will focus the analysis on low-density polyethylene in the rest of the text. The results for high-density polyethylene, provided in the online appendix, are qualitatively similar.

Figure 5 displays the evolution of equilibrium over time in the market for low-density polyethylene. This is not a demand curve: each dot is an equilibrium point resulting from the interaction of demand and a supply relation in the given year. The figure shows how these equilibrium points shift over time. The predominant pattern is for equilibrium to shift to the southeast each year. With one exception, the equilibrium never shifts in the direction (southwest) that, as discussed in Section I, entails a violation of Assumption 4 that demand is growing over time.[9]

[8]Table 1 of Lieberman (1984), a canonical source of data on the chemical industry, also uses U.S. Tariff Commission reports as his primary source and also restricts the polyethylene sample to 1958–72. Rather than using Lieberman's data directly, we returned to the original source because, in addition to noticing typographical errors, we preferred using sales rather than production for quantity. The results using production for quantity, reported in the online appendix, are qualitatively similar.

[9]The lone exception is 1963. We exclude that year in the subsequent analysis. As a robustness check, we redo the analysis preserving the requirement that $SW^+ = \emptyset$ by dropping 1962 rather than 1963. The results, reported in the online appendix, are quantitatively similar. A likely cause of this violation of $SW^+ = \emptyset$ is that the market may have been

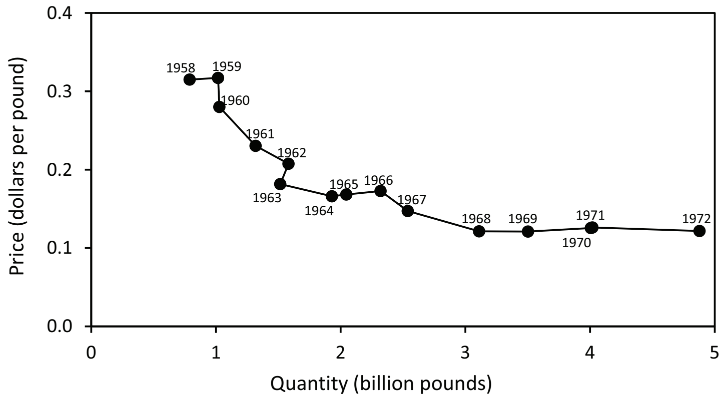

FIGURE 5. EVOLUTION OF EQUILIBRIUM IN THE LOW-DENSITY POLYETHYLENE MARKET

C. Empirical Results

This subsection presents empirical results using the method incorporating local information to bound the demand elasticity ε_t in the *DuPont* application. Our data only allow nontrivial upper bounds $\bar{\varepsilon}_t^*$ to be obtained; nontrivial lower bounds cannot be obtained because SE^- and NW^+ are empty for all $e_t \in E$ in our data. The trival lower bound $\underline{\varepsilon}_0^* = 0$ is obtained by default, as a consequence of Assumption 3 (law of demand). To gauge robustness to functional form, we carry out the analysis assuming both linear and logit demands.

Figure 6 presents the results.[10] The figure displays bootstrapped confidence intervals around $\bar{\varepsilon}_t^*$ as whiskers atop the bars. We chose to display a two-tailed 90% confidence interval because this allows easy visualization of the one-tailed test of whether the elasticity bound is less than 1 at the standard (5%) significance level.

The standard errors used to compute the confidence intervals—Zelterman (1993) bootstrapped standard errors—require further discussion. Standard bootstrap methods are invalid in our context because our upper elasticity bound involves an extreme order statistic, the minimum over pairwise comparisons between a given equilibrium point and others. Bickel and Freedman (1981) pointed out the impossibility of drawing a pseudosample generating an order statistic more extreme than the original one because any pseudosample is a subset of the original data. Hence the bootstrapped distribution will be bounded by—rather than centered on—the extreme order statistic estimated from the original data. Zelterman (1993) provides a way of circumventing this problem. Instead of sampling the data directly, to bootstrap the maximum order statistic, he proposes sampling the spacings between the highest k observations. After suitable normalization,

growing but not fast enough to offset a small observation error that made it look like demand shrunk that year. Section V discusses several approaches to robust estimation that accommodate such perturbations. High-density polyethylene does not exhibit a southwest shift, nor does low-density polyethylene when production rather than sales is used for quantity. Thus, all years are included in those analyses, reported in the online appendix.

[10]Precise numerical estimates and significance levels are provided in the online appendix for reference.

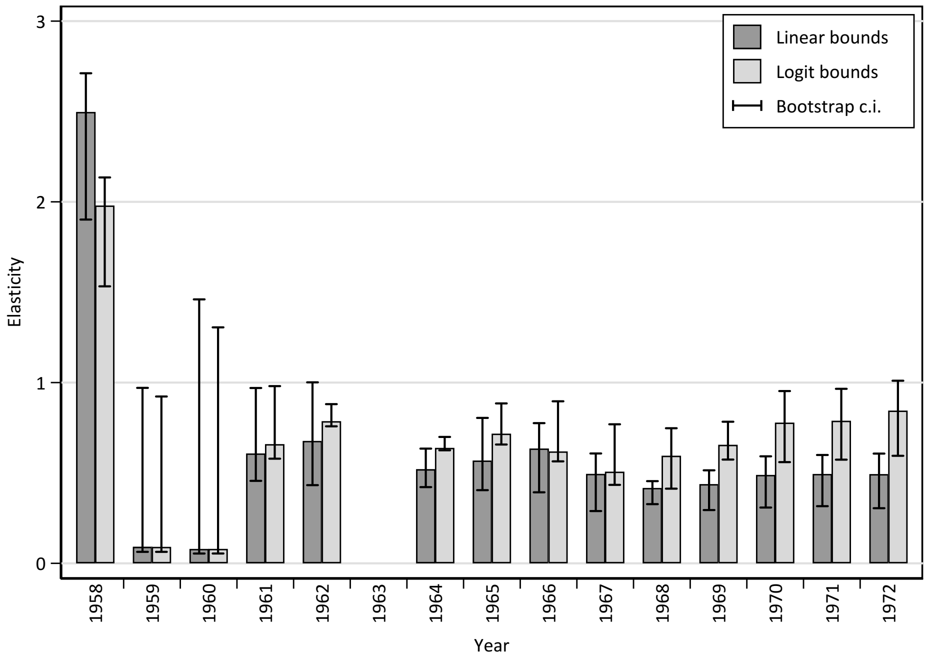

FIGURE 6. ELASTICITY BOUNDS FOR LOW-DENSITY POLYETHYLENE.

Note: Uses method incorporating local information. Shaded bars are bounds on the demand elasticity assuming linear demand (dark bars) or logit demand (light bars). Whiskers are 90% two-sided confidence intervals on the upper elasticity bound based on the Zelterman (1993) bootstrap. Upper elasticity bound is thus significantly less than the upper whisker in a one-sided test at the 5% level.

these spacings have an i.i.d. asymptotic distribution, a property inherited by pseudosamples drawn with replacement. The maximum order statistic can thus be bootstrapped by drawing pseudosamples of normalized spacings and adding them (after reversing the normalization) to the order statistic k positions away from the maximum. Appendix C illustrates the procedure in a numerical example and provides further technical details.

Focus first on the dark-shaded bars in Figure 6, representing the elasticity bounds estimated assuming linear demand in the reference period. The results are remarkable. Aside from the first year, in which the upper bound is quite high at 2.50, the upper bound on the elasticity is never higher than 0.68. The upper bound is extremely low, 0.09 or less, in two of the years. In 11 of the 15 years, the upper bound is significantly less than 1 in a one-sided test at the 1% level. Such low elasticities are inconsistent with a monopoly outcome, the equilibrium of which in theory lies in the elastic region of demand.

The light-shaded bars represent elasticity bounds assuming logit demand. The height of the bars is quite similar to that for linear demand, and the two sets move together over the sample. The quantitative similarity of the results for linear and logit demand provide confidence that the results are not dictated by functional form.[11]

[11]In the first sample year, the bounds assuming linear demand are slightly wider than that assuming logit demand,

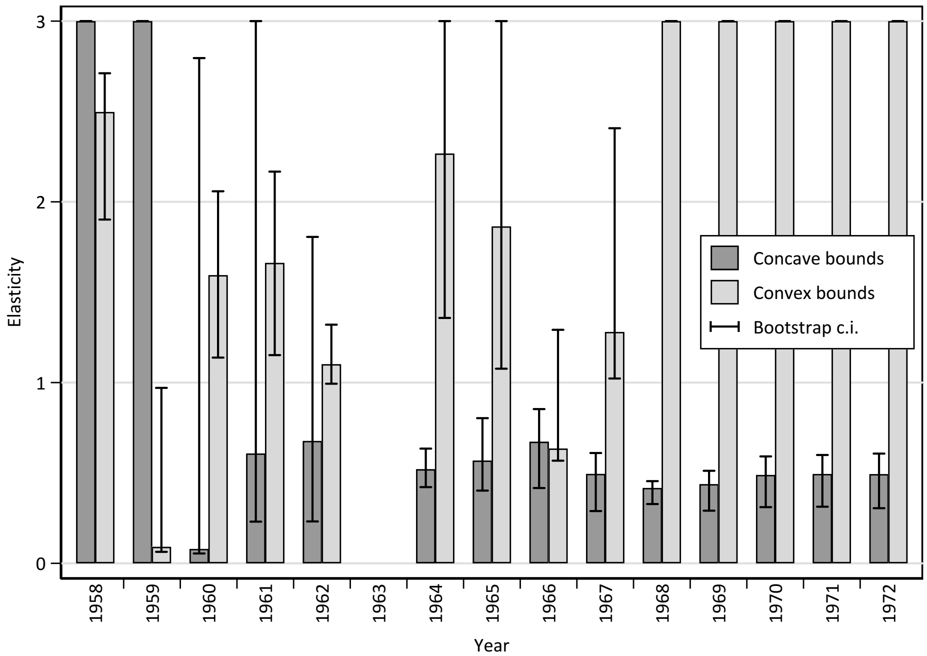

FIGURE 7. ELASTICITY BOUNDS IMPOSING CURVATURE ASSUMPTIONS FOR LOW-DENSITY POLYETHYLENE.

Note: Shaded bars are bounds on the demand elasticity assuming convex demand (dark bars) or concave demand (light bars). Vertical axis cropped at 3 for comparison to Figure 6. Upper elasticity bound is in fact infinite for those bars reaching the vertical-axis limit. Missing upper whisker indicates cropping of upper confidence threshold. Remaining notes from Figure 6 apply.

Figure 7 presents bounds imposing curvature rather than functional-form assumptions. Consider the dark-shaded bars, representing bounds $[\underline{\varepsilon}_t^d, \bar{\varepsilon}_t^d]$ assuming demand is weakly concave. As discussed in Section III.B, the upper bounds are infinite in initial sample years because of a lack of earlier equilibrium to make informative pairwise comparisons. Starting in 1964, however, the elasticity bounds and associated confidence intervals assuming weak concavity are just as tight as the bounds assuming linearity. The light-shaded bars represent bounds $[\underline{\varepsilon}_t^u, \bar{\varepsilon}_t^u]$ assuming demand is weakly convex. For these, upper bounds are infinite in the final sample years because later equilibrium points are lacking. The bounds assuming convexity perform poorly relative to those assuming linearity; in only one year (1959) is the upper bounds significantly less than 1 at the 5% level.[12]

confirming the theoretical result from Section III.B that the bounds assuming any convex demand curve including logit must be at least as tight as those assuming linear demand in the first sample year.

[12]The bound assuming convexity can perform well in other samples. In the application to the *American Tobacco* case, the bounds assuming convexity (not reported) are just as tight as those assuming linear demand in several years including the crucial year (1921) allowing the researcher to pinpoint a possible breakdown in collusion.

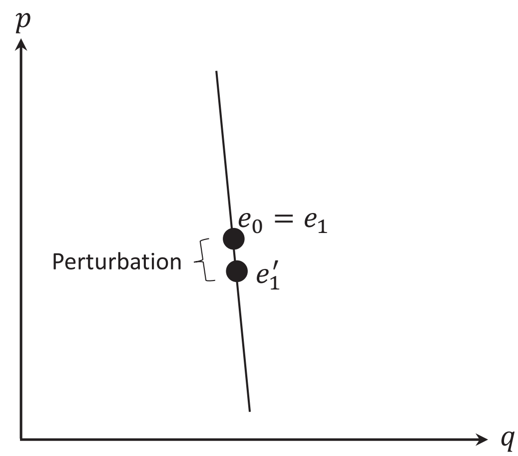

FIGURE 8. VULNERABILITY OF METHOD TO OBSERVATION ERRORS IN SLOW-GROWING MARKET

V. Robustness

Our method implicitly assumes price and quantity are perfectly observed. Observation errors can create potential problems for our method. These problems are likely to be relatively inconsequential if the market is growing rapidly over the sample but may be acute if the market experiences periods of only slow growth.

Figure 8 illustrates the potential vulnerability. To take an extreme example, suppose a monopoly serves a market that is unchanging over time. Suppose that the initial equilibrium point e_0 persists into the next period as e_1, but due to an observation error or other small perturbation, e_1' is instead recorded in the data. Using our method, say assuming linear demand, pairwise comparison of e_0 and e_1' implies that the inverse demand curve in both periods would have to be at least as steep as the line drawn. If the perturbation happened to nudge e_1' almost directly south of e_0, we would conclude that demand was almost perfectly inelastic, erroneously rejecting monopoly control of the market. Rapid market growth would drive e_0 and e_1 apart, attenuating the effect of a small perturbation. However, even in a growing market, growth may not be rapid every year. The concern is not just theoretical. In the *DuPont* application, one might be concerned that the extremely low elasticity bounds in 1959 and 1960 were generated by an isolated perturbation from 1959 to 1960 rather than being a robust consequence of several pairwise comparisons.

We take several approaches to addressing the robustness concern. One approach has already been discussed: bootstrapping confidence intervals. If the estimates are unduly influenced by the position of one or two equilibria, we will see higher estimates in bootstrapped subsamples from which they are omitted, widening the confidence intervals. This section takes a different approach. Rather than starting with an estimator that is potentially vulnerable to small perturbations and drawing a confidence interval around it to gauge its vulnerability, we propose a series of estimators that are robust to begin with. Section V.A introduces estimators that leave out the most influential equilibrium

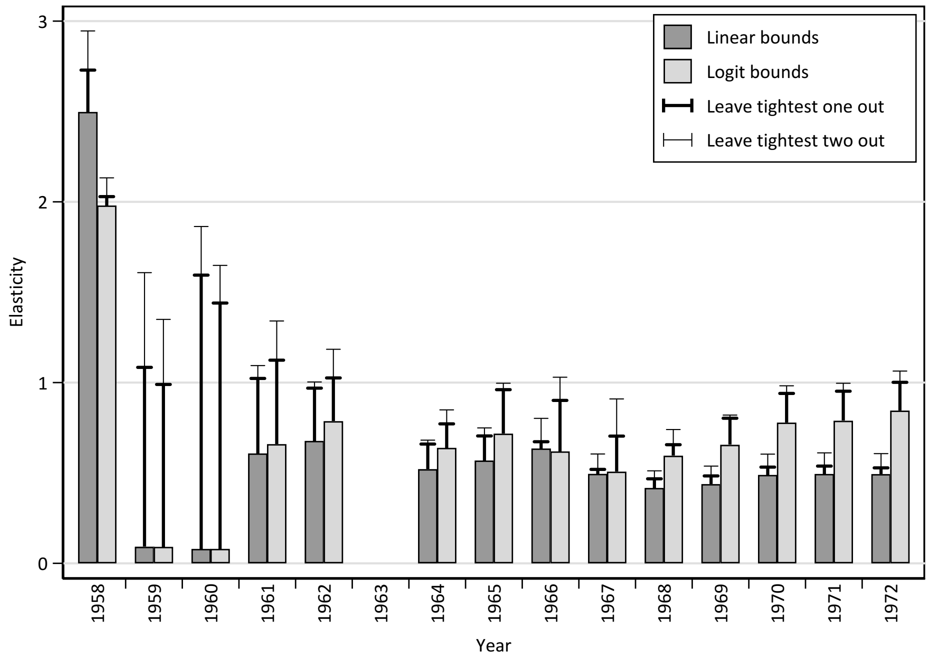

FIGURE 9. LEAVE-TIGHTEST-OUT ESTIMATORS FOR LOW-DENSITY POLYETHYLENE.

Note: Uses method incorporating local information. Shaded bars are the same elasticity bounds as in Figure 6 for linear demand (dark bars) and logit demand (light bars). Whiskers above bars represent leave-out estimators: the thick whisker represents the leave-tightest-one-out estimator, and the thin whisker the leave-tightest-two-out estimator.

points. Section V.B proposes biennial estimators that only require demand to grow over a two-year horizon rather than year to year.

A. Leave-Tightest-Out Estimators

Let $\bar{LT}_t^*(1)$ denote the the leave-tightest-one-out estimator, i.e., the estimator leaving the most influential observation out of the calculation of the upper bound on the demand elasticity. Formally, $\bar{LT}_t^*(1) \equiv \max_{t' \neq t} \bar{\varepsilon}^* \langle e_{t'} \rangle$, where $\bar{\varepsilon}^* \langle e_{t'} \rangle$ is the upper bound on the elasticity restricting the sample to $E \setminus \{e_{t'}\}$. Analogously, define the leave-tightest-two-out estimator $\bar{LT}_t^*(2) \equiv \max_{t', t'' \neq t} \bar{\varepsilon}^* \langle e_{t'}, e_{t''} \rangle$. Leave-out estimators with higher orders can also be defined.

Figure 9 presents the results from the leave-tightest-out estimators. The shaded bars repeat the elasticity bounds from Figure 6. The difference lies in the whiskers, which before represented bootstrapped confidence intervals but now represent the extension of the upper bound when the one or two most influential comparisons are left out. The thick whisker is the extension due to the leave-one-out estimator and the thin whisker to the leave-two-out estimator. The leave-tightest-out estimators causes some elasticity bounds to jump, in particular in years in which the original elasticity bounds were extremely low. These results validate the concern that the tight bounds for 1959–60 were due to the positioning of 1960 almost vertically below 1959. When one or the other is left out, this causes the upper bound in 1959–60 to jump up to a level on par with the other years.

Some reassurance can still be taken from Figure 9. First, while the bounds jump up in some early years, crossing the threshold of 1 ruling out monopoly, after 1964 the leave-tightest-out estimators remain below 1. Also reassuring is the stability of the leave-out estimators: the thin whisker often does not extend much beyond the thick one, implying that the leave-one-out and leave-two-out estimators are quite close to each other. Leaving one pairwise comparison out appears to be sufficient to limit vulnerability to small perturbations.

Panel A of Figure 10 compares the leave-tightest-one-out estimator against the 95% bootstrapped upper confidence threshold from Figure 6. The points lie almost directly on the 45-degree line, implying that the two approaches to robustness generate almost equivalent results. Most points lie inside the dashed box indicating the threshold below which monopoly behavior is ruled out.

Panel B plots the leave-tightest-two-out estimator versus the 95% bootstrapped upper confidence threshold. Given that the bootstrap threshold is nearly equivalent to the leave-tightest-one-out estimator, and the leave-tightest-one is by construction more conservative than the leave-tightest-two-out estimator, we expect the latter to be more conservative than the bootstrap threshold. Indeed we see this in the panel, as the dots lie above the 45-degree line. While the dots are above the 45-degree line, they are quite close to it, reinforcing our earlier conclusion that the bootstrap threshold (and by extension the leave-tightest-one-out) reflects much of the robustness of more conservative estimators.

B. Biennial Estimators

Assumption 4 that demand is growing is essential for the validity of our bounds. As we saw in Section II.C, the true elasticity need not be contained within our bounds if the assumption is violated. Few markets experienced the explosive growth evidenced by plastics during out sample period. Yet even with plastics, it may be hard to rule out a pause in growth over the decade and a half, perhaps due to a slowdown in the economy (the NBER identifies two brief recessions during our sample, from April 1960 to February 1961, and from December 1969 to November 1970), perhaps due to variation in when sales are booked relative to year end. A more conservative assumption is that demand is certain to grow biennially if not yearly. This subsection proposes two estimators leveraging this more conservative assumption.

The first estimator based on biennial growth leaves the year before and after t out of pairwise comparisons with e_t. Formally, this leave-neighbors-out estimator is defined as $\bar{LN}^*_t(1) \equiv \bar{\varepsilon}^*\langle e_{t-1}, e_{t+1}\rangle$. Estimators leaving out neighbors in wider windows around t can also be defined. It is not ex ante obvious how $\bar{LN}^*_t(1)$ compares to $\bar{LT}^*_t(1)$. On the one hand, for years other than the endpoints of the sample, $\bar{LN}^*_t(1)$ leaves out two comparison years, while $\bar{LT}^*_t(1)$ leaves out just one. On the other hand, the one comparison left out by $\bar{LT}^*_t(1)$ is by definition the tightest, while those left out by $\bar{LN}^*_t(1)$ may not be.

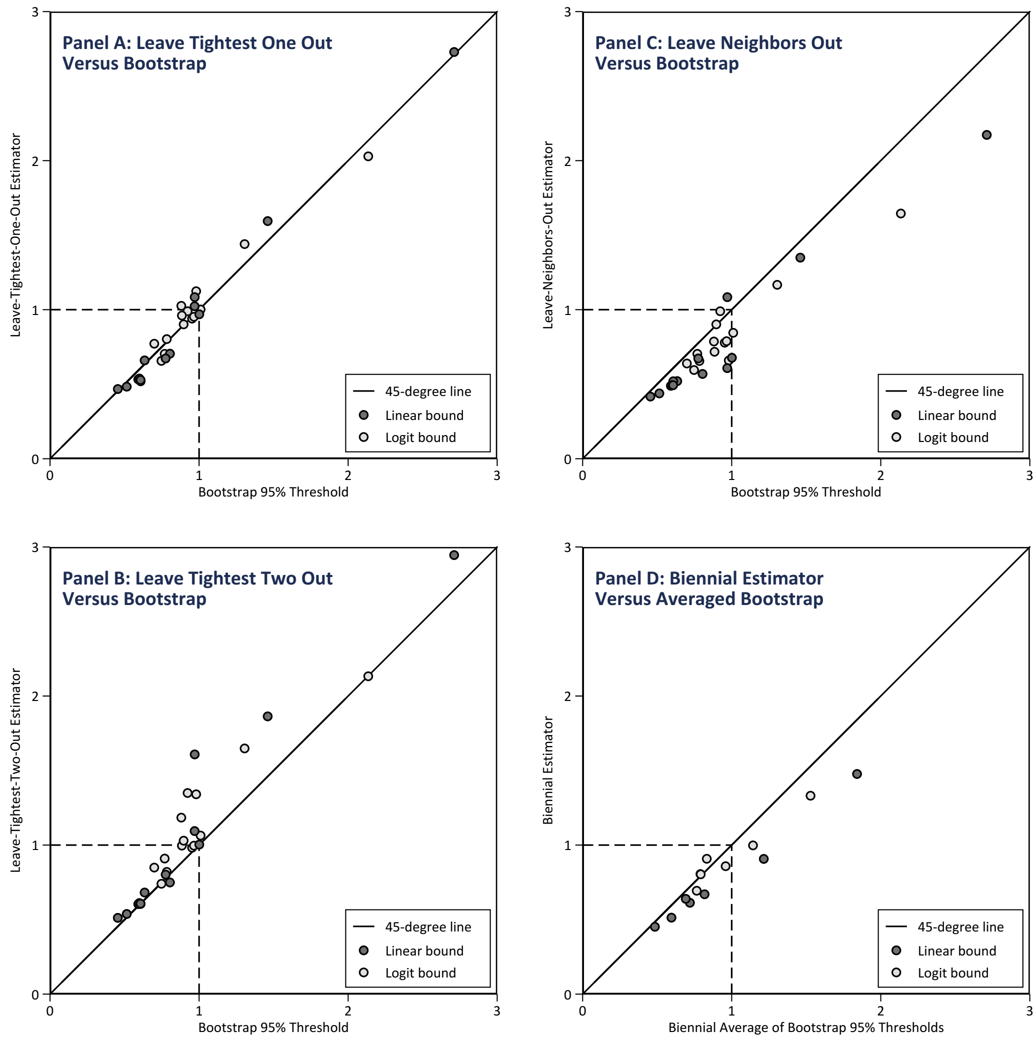

FIGURE 10. COMPARING VARIOUS ROBUST ESTIMATORS FOR LOW-DENSITY POLYETHYLENE.

Note: Uses method incorporating local information. Horizontal axis is the bootstrapped 95% upper confidence threshold from a one-tailed test, equivalent to the upper whisker for the 90% bootstrapped confidence interval from a two-tailed test in Figure 6, using the Zelterman (1993) bootstrap.

Panel C of Figure 10 shows that the bootstrapped 95% confidence threshold is more conservative than the leave-neighbors-out estimator as almost all the dots lie below the 45-degree line. Since the leave-tightest-one-out estimator is nearly equivalent to the 95% confidence threshold, this implies that leaving the tightest one out is as or more robust than leaving neighboring years out.

The second estimator based on biennial growth merges pairs of years together (1958–59, 1960–61, and so forth), treating each biennial pair as the unit of observation. Letting uppercase distinguishing variables related to the merged pair of years starting in t, quantity is simply the sum $Q_t = q_t + q_{t+1}$ and price is average revenue per unit: $P_t = (p_t q_t + p_{t+1} q_{t+1})/(q_t + q_{t+1})$. Obviously, this approach effectively cuts the dataset in half. Panel D of Figure 10 compares the bounds computed for this merged dataset to the bootstrapped 95% thresholds computed using the original data averaged over the years making up the pair. The dots are all quite close to the 45-degree line, suggesting that the two approaches provide similar levels of robustness. If anything, the average bootstrap threshold is slightly more conservative, as virtually all dots lie slightly below the 45-degree line. Most are inside the box, indicating that the bounds are below the elasticity value of 1 used to reject monopoly; some are well inside the box.

VI. Method Incorporating Limiting Information

The elasticity bounds can be tightened by exploiting additional information about the relative position of demand curves for limiting values of prices, $p \to 0$ and $p \to \infty$. The method delivers tighter bounds at the cost of requiring more demanding assumptions: whatever functional form the researcher posits for demand must extend over the whole domain of the demand curve rather than just locally and across all periods rather than just in the reference period.

A. *Intuition for Method*

Figure 11 provides an intuitive explanation of how limiting information can tighten the elasticity bounds. The figure revisits the simple example introduced in Figure 3, carrying over the features from that previous figure as the solid lines. Recall the goal of that simple example was to bound the elasticity ε_0 of the linear demand curve through the reference equilibrium point e_0. The method of incorporating local information from pairwise comparisons allowed us to conclude that the inverse demand through e_0 must be at least as steep as line ℓ_{02}, providing a lower bound of the steepness of inverse demand, translating into an upper bound $\bar{b}_0^*$ on the steepness of demand and an upper bound $\bar{\varepsilon}_0^*$ on the elasticity.

The new features of Figure 11, drawn as dotted lines and open circles, can be used to tighten the bounds. Assume the demand curves through all equilibrium points are linear. Pairwise comparison of points e_1 and e_2 shows that the inverse demand through e_2 must be as steep as line ℓ_{12} for e_1 not to lie on a higher demand curve. But notice that ℓ_{12} intersects ℓ_{02}. Unless the inverse demand curve through e_0 is steeper than ℓ_{12}, parts of the curve will lie above the curve through e_2, violating the assumption of growing demand. For the whole inverse demand through e_0 to be lower than the whole inverse demand through e_2, the demand curves cannot cross even for prices approaching 0, implying in the case of linear inverse demands that their horizontal intercepts cannot cross. To ensure their horizontal intercepts do not cross, the inverse demand through e_0 must be at least as steep as the dotted line that connects e_0 with the horizontal intercept of ℓ_{12}, drawn as the open circle. This new line through e_0 is even steeper than ℓ_{02}, tightening the lower

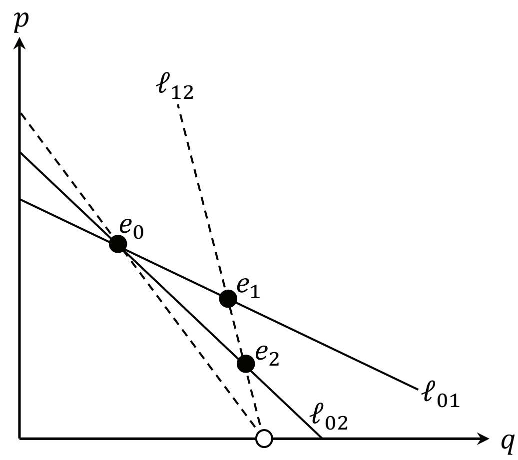

FIGURE 11. ILLUSTRATING METHOD INCORPORATING LIMITING INFORMATION

bound on the steepness of inverse demand, and thus tightening the upper bound on the demand elasticity.

B. Formal Bounds

The next proposition, proved in Appendix A, operationalizes the intuition from the preceding subsection.

PROPOSITION 4: *Suppose the demand curve in every period $t \in T$ is linear, i.e., $D_t(p) = \tilde{D}(p, b_t, e_t)$, and satisfies Assumptions 3 (law of demand) and 4 (growing demand). Whichever element $e_0 \in E$ the researcher chooses for the reference equilibrium point, we have $b_0 \in [\underline{b}_0^{**}, \bar{b}_0^{**}]$, where*

$$\underline{b}_0^{**} \equiv \underline{b}_0^* \vee \sup_{t<0} \left\{ \frac{1}{p_0}(q_t - q_0 + \underline{b}_t^* p_t) \right\} \vee \sup_{t>0} \left\{ \frac{q_0}{p_t - p_0 + q_t/\underline{b}_t^*} \right\} \tag{29}$$

$$\bar{b}_0^{**} \equiv \bar{b}_0^* \wedge \inf_{t<0} \left\{ \frac{q_0}{p_t - p_0 + q_t/\bar{b}_t^*} \,\middle|\, p_t + \frac{q_t}{\bar{b}_t^*} > p_0 \right\} \wedge \inf_{t>0} \left\{ \frac{1}{p_0}(q_t - q_0 + \bar{b}_t^* p_t) \right\}. \tag{30}$$

The proposition prescribes an iterative procedure for computing the bounds. In the first stage, the bounds incorporating local information, indicated with a single star—$\underline{b}_t^*$ and $\bar{b}_t^*$—are computed for every equilibrium point. In the second stage, the first-stage bounds are translated into demand intercepts. Ensuring none of these intercepts crosses the intercepts of $D_0(p)$ generates new bounds on the steepness of $D_0(p)$. If the tightest of these is tighter than the corresponding bound from the first stage incorporating local information, we update the second-stage bound, indicated with two stars in the superscript, to this tighter value. Otherwise, the second-stage bound is just set to first-stage bound.

The next proposition extends the method incorporating limiting information to general demand curves.

PROPOSITION 5: *Suppose the demand curve in every period $t \in T$ has the general functional form specified in* (14)*, i.e., $D_t(p) = \tilde{D}(p, \theta_t, e_t)$. Suppose these curves satisfy Assumptions 3 and 4 as well as conditions* (16)–(20)*. Whichever element $e_0 \in E$ the researcher chooses for the reference equilibrium point, we have $\theta_0 \in [\underline{\theta}_0^{**}, \bar{\theta}_0^{**}]$, where*

$$\underline{\theta}_0^{**} \equiv \underline{\theta}_0^{*} \vee \sup_{t<0} \left\{ \theta \,\middle|\, \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta, e_0)}{\tilde{D}(p, \underline{\theta}_t^{*}, e_t)} \right] = 1 \right\} \vee \sup_{t>0} \left\{ \theta \,\middle|\, \lim_{p \to \infty} \left[\frac{\tilde{D}(p, \theta, e_0)}{\tilde{D}(p, \underline{\theta}_t^{*}, e_t)} \right] = 1 \right\} \quad (31)$$

$$\bar{\theta}_0^{**} \equiv \bar{\theta}_0^{*} \wedge \inf_{t<0} \left\{ \theta \,\middle|\, \lim_{p \to \infty} \left[\frac{\tilde{D}(p, \theta, e_0)}{\tilde{D}(p, \bar{\theta}_t^{*}, e_t)} \right] = 1 \right\} \wedge \inf_{t>0} \left\{ \theta \,\middle|\, \lim_{p \to 0} \left[\frac{\tilde{D}(p, \theta, e_0)}{\tilde{D}(p, \bar{\theta}_t^{*}, e_t)} \right] = 1 \right\}. \quad (32)$$

The proof in Appendix A includes guidance on computing these somewhat complex formulas. Appendix B provides analytical expressions for (31) and (32) in the special case of logit demand.

The bounds on the demand parameters from the previous propositions can be translated into bounds on elasticities using the provided elasticity formulas. In the case of linear demand, substituting the bounds on b_0 from Proposition 4 into (3) yields $\varepsilon_0 \in [\underline{\varepsilon}_0^{**}, \bar{\varepsilon}_0^{**}]$, where $\underline{\varepsilon}_0^{**} \equiv \underline{b}_0^{**} p_0/q_0$ and $\bar{\varepsilon}_0^{**} \equiv \bar{b}_0^{**} p_0/q_0$. In the case of general demands, substituting the bounds on θ_0 from Proposition 5 into (15) and noting the right-hand side of (15) is nondecreasing in θ_t (see footnote 3) yields $\varepsilon_0 \in [\underline{\varepsilon}_0^{**}, \bar{\varepsilon}_0^{**}]$ where $\underline{\varepsilon}_0^{**} \equiv |\tilde{D}_p(p_0, \underline{\theta}_0^{**}, e_0)| p_0/q_0$ and $\bar{\varepsilon}_0^{**} \equiv |\tilde{D}_p(p_0, \bar{\theta}_0^{**}, e_0)| p_0/q_0$.

C. Application

Figure 12 presents elasticity bounds in the polyethylene market using the method incorporating limiting information. The bounds are again quite similar across linear and logit demand, and both are sharply tighter than in Figure 6. When limiting information is incorporated, the upper bounds on the elasticity fall below 0.4 in the first year and extremely close to 0 in all later years.

The confidence intervals suggest that the extremeness of these estimates may not be particularly robust: the whisker representing the upper confidence-interval threshold is an order of magnitude higher than the estimated elasticity bound in most years. Extremely low elasticity bounds obtained in isolated cases in the first stage propagate across the other years once the limiting information those extreme bounds imply is incorporated. Bootstrapping helps account for this since isolated cases will not be drawn in many second-stage pseudosamples, resulting in larger bootstrapped bounds and wider confidence intervals.

While the method incorporating limiting information can improve on the method incorporating local information, the former method carries some caveats. First, if out of a concern for robustness one focuses on the upper bootstrapped confidence thresholds rather than the bounds themselves, the method incorporating limiting information does

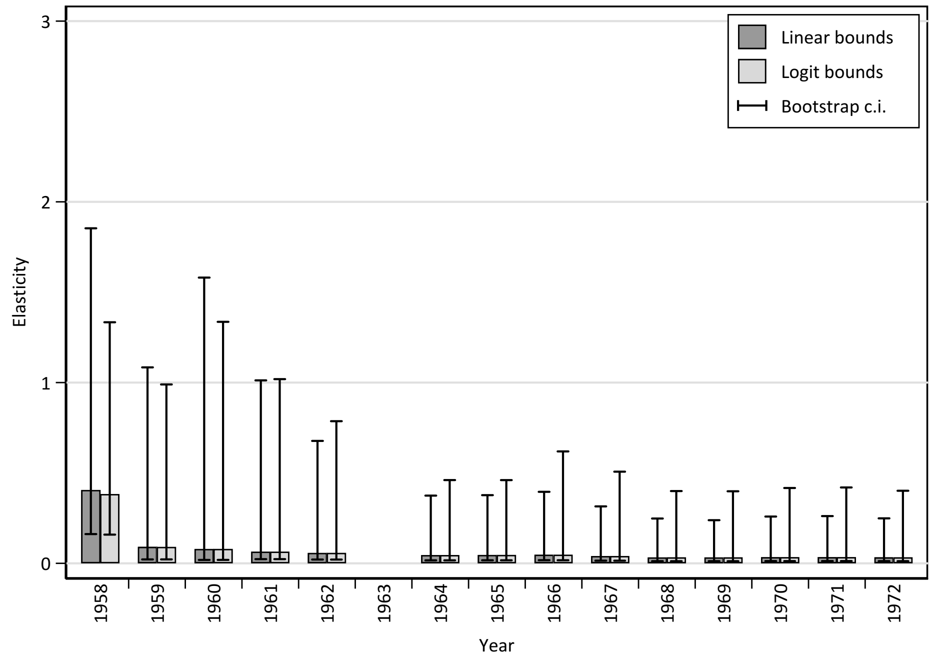

FIGURE 12. ELASTICITY BOUNDS FOR LOW-DENSITY POLYETHYLENE USING METHOD INCORPORATING LIMITING INFORMATION.

Note: Uses different method than Figure 6—incorporating limiting rather than local information—but remainder of notes from Figure 6 apply.

not always tighten the thresholds (as we see in the early sample years). Second, tighter thresholds come at the cost of more stringent assumptions: the assumed functional for demand must hold globally, not just locally, and across the whole sample, not just in the reference year.[13]

VII. Application to *American Tobacco* Case

To demonstrate the broader applicability of our bounding methods, this section provides an abbreviated application to the 1911 *American Tobacco* case. We seek to assess whether the structural remedy in the case successfully curtailed the monopoly in the cigarette market.

[13]An additional caveat is that the leave-tightest-out estimators for the method incorporating limiting information (denoted $\bar{LT}_t^{**}(1)$ and $\bar{LT}_t^{**}(2)$, not reported here for space considerations) have performance issues: $\bar{LT}_t^{**}(1)$ is often considerably higher than the upper bootstrapped confidence threshold; further, the jump from $\bar{LT}_t^{**}(1)$ to $\bar{LT}_t^{**}(2)$ can be higher than that from the original bound to $\bar{LT}_t^{**}(1)$, an instability not apparent with the method incorporating local information.

A. Institutional Background

In the 1911 *American Tobacco* case,[14] the DOJ alleged that the merger of the five largest firms in the U.S. market into the American Tobacco Company and its subsequent conduct violated the Sherman Act. The U.S. Supreme Court ruled in favor of the DOJ. The remedy broke American Tobacco up into over a dozen companies, three of which were cigarette manufacturers. Louis Brandeis, soon to become a Supreme Court justice, criticized the remedy as too weak, calling for the creation of at least seven cigarette manufacturers. The ambivalent conclusions reached in the industry studies by Cox (1933), Tennant (1950), and Nicholls (1951) embody the uncertainty surrounding how collusive the post-remedy market was.

We will study the effect of the remedy on subsequent performance of the tobacco market focusing on cigarettes, which became the most popular tobacco product during our sample and which form a well-defined product category. Cigarette brands were not perfect substitutes: brand advertising was important and consumers had expressed preferences (Johnson, 1984). Still, our methods may be usefully applied because quantity and price retain meaning in this market—quantity because of brands' physical similarity, price because the major brands adopted an identical price policy during much of our sample (Rostas, 1953).

Independent historical sources bolster the case of growing demand in this market, a precondition for our bounding methods to work. A relatively new tobacco product, cigarettes became a social fashion over the sample period (Tennant, 1950, p. 140). Cigarettes were included in soldiers' rations during World War I (Cox, 1933, p. 41) and marketed to women for weight control ("Reach for a Lucky instead of a sweet," quoted in Goodman, 1993). The new medium of radio gained popularity, cigarettes being primary advertisers (Johnson, 1984, p. 22). While the Great Depression led to a dip in cigarette consumption, we cut our sample off before then.

B. Data

Our dataset consists of annual quantity and price data for the U.S. cigarette market. For quantity, we use cigarette consumption, compiled in Table 2 of the American Lung Assocation (2011) from U.S. Department of Agriculture tobacco yearbooks. Price data are more scarce. Table 55 from Tennant (1950) provides a comprehensive list of price announcements for the two most popular brands, Camel and Lucky Strike, obtained from testimony in the sequel *American Tobacco* case in 1946.[15] Given that both brands' prices closely matched each other but Camel's series dated back further, we used Camel's to represent the market price. For years experiencing price changes, we took the average weighted by number of days spent at each price. We use net wholesale price, i.e., list price including taxes less wholesale discount—and nominal prices as in the polyethylene application. Our sample starts when price data becomes available in 1913 and ends in

[14] "United States v. American Tobacco Company," 221 U.S. 106 (1911).
[15] "American Tobacco Co. v. United States," 328 U.S. 781 (1946).

TABLE 2—DESCRIPTIVE STATISTICS FOR CIGARETTE DATA

Variable	Units	Mean	Std. dev.
Price (wholesale net)	$ per 1000	5.21	1.24
Quantity (consumption)	Billion cigarettes	53.8	28.8

Source: Price computed by authors using data from Table 55 of Tennant (1950). Quantity from Table 2 of American Lung Assocation (2011).
Note: Annual data over 1913–28. Price in nominal terms.

1928, before the Great Depression.

Table 2 provides descriptive statistics, and Figure 13 shows the evolution of market equilibrium over the sample. After initially shooting up, prices declined precipitously over 1921–22, leading to a period of virtually unchanging prices after 1923. Quantity increased by almost an order of magnitude over the sample, reaching over 100 billion cigarettes in 1928.

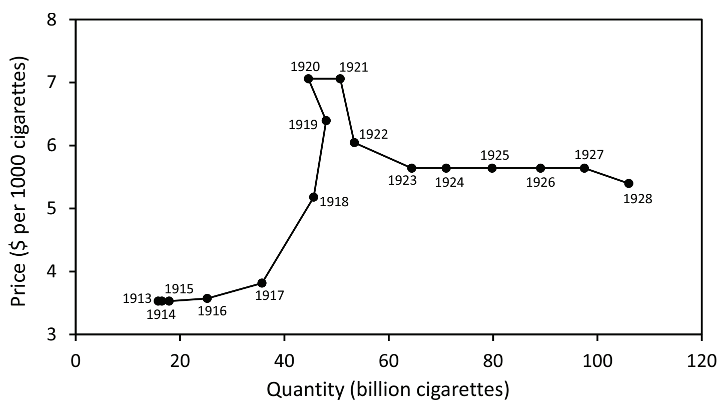

FIGURE 13. EVOLUTION OF EQUILIBRIUM IN THE CIGARETTE MARKET

C. Empirical Results

Figure 14 reports elasticity bounds using the method incorporating local information. Since the informative set $NW^- \cup SE^+$ is empty for each year in 1913–18, our method delivers an infinite upper bound in those years. After that, the upper bound is finite. Two of these years, 1921 and 1922, stand out as the only ones having upper bounds significantly less than the monopoly threshold of 1 at the 5% level. Our method picks out the exact years experiencing the precipitous declines we detected from visual inspection of Figure 13. Our method goes beyond visual inspection by gauging how precipitous the decline was—in this case, too precipitous to be consistent with a monopoly responding to a cost shock.

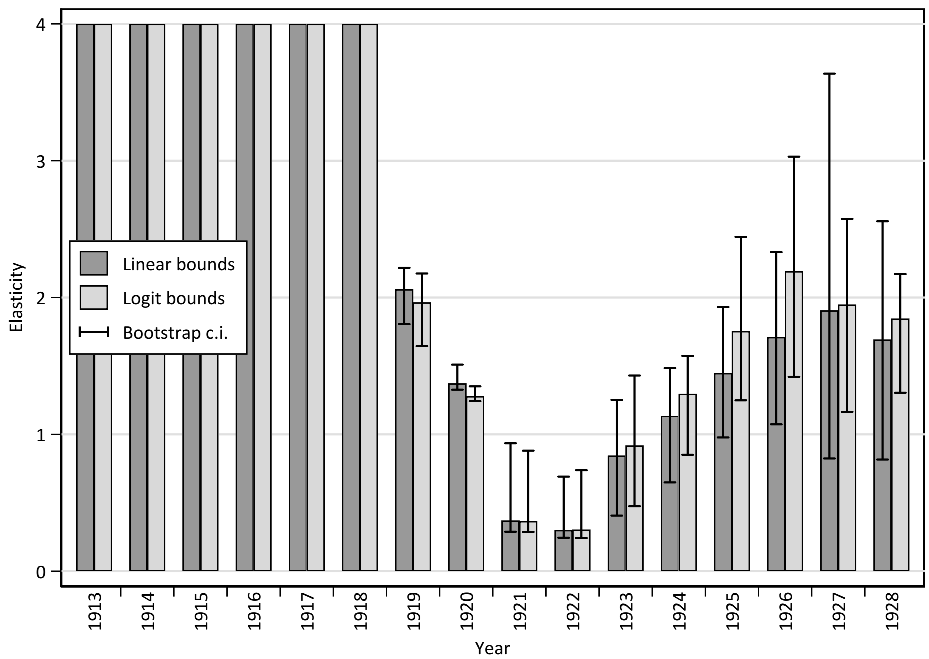

FIGURE 14. ELASTICITY BOUNDS FOR CIGARETTES.

Note: For 1913–18, upper elasticity bounds are infinite but are cropped at 4 in the graph to aid visualization. In addition, notes to Figure 6 apply.

We conclude that the firms created by the remedy were not operating as a perfect cartel every year; 1921–22 may represent a notable breakdown in collusion, supporting the contention in (Nicholls, 1951, pp. 174–175) that cigarette manufacturers used this period to battle for price leadership and test rival reactions. The painful lesson may have convinced manufacturers to maintain identical and unchanging prices over the subsequent half decade.[16]

VIII. Conclusion

This paper provided a methodology for bounding the elasticity of demand that works in growing markets for homogeneous products. The method requires minimal information, working with as few as two time-series observations on aggregate prices and quantities. The underlying idea is that the demand curve through a given equilibrium point cannot be either so steep or so flat that it passes below earlier equilibria or above later equilibria, violating the assumption of growing demand. The resulting inequalities bound the elasticity of demand in any given year.

[16]The unchanging prices after 1923 are themselves evidence of imperfect collusion. Competitive prices should fluctuate with costs shocks such as changing tobacco leaf prices. Unchanging prices are also inconsistent with the other extreme of a perfect cartel, whose industry-profit-maximizing price should vary with demand and cost conditions.

A potential drawback of any methodology delivering bounds rather than point estimates is that the resulting bounds may be so wide as to be uninformative. In the *DuPont* application, the bounds turned out to be quite informative, producing bounds significantly less than the threshold of 1 at which monopoly is ruled out at the 5% level in 11 of 15 years. The results are robust across products (low- versus high-density polyethylene), demand specifications (linear versus logit), methods (incorporating local or limiting information), quantity measures (sales versus production), and price measures (nominal versus real). In some cases, the functional-form assumption could be generalized to a curvature assumption without impairing the bounds; but in other cases bounds assuming only curvature performed poorly.

Based on our finding that the elasticity of demand was bounded in the inelastic region, we reject the contention that monopoly behavior effected by the Patents and Processes agreement between DuPont and ICI continued after the remedy in the *DuPont* case. An unconstrained monopolist would never have dropped its price by the amount observed unless that generated a more substantial gain in quantity, leaving competitive pressure as the leading explanation of the price drop.

The configuration of equilibrium points in 1959 and 1960 in the low-density polyethylene market, with the 1960 lying close to but almost vertically below 1959, raised a question of robustness. The vertical shift might be due to observation error or other small perturbation not the consequence of a perfectly inelastic demand curve. We dealt with the robustness issue in two ways. First, we provided bootstrapped confidence intervals around the estimates using a procedure due to Zelterman (1993) for extreme order statistics. Second, we proposed estimators leaving out the most influential equilibria for pairwise comparison. The leave-one-out estimator generated very similar results to the 95% bootstrapped upper confidence threshold across products, functional-forms, and methods. While the tight bounds for early years such as 1959 and 1960 did not prove to be robust, those for later years did. The method incorporating limiting information sometimes delivered tighter bounds, although the advantage was diminished in bootstrapped confidence intervals and leave-out estimators. Furthermore, the method incorporating limiting information requires additional functional-form assumptions that some empirical researchers may find too restrictive.

In a second application, to the 1911 *American Tobacco* case, we obtained bounds significantly below the threshold of 1 in 1921–22. Commentators singled out exactly these years as the period during which cigarette manufacturers battled for price leadership.

We hope researchers will find our methods valuable and easy to apply using the supplied Stata code. As demonstrated by the applications, our methods can be valuable in historical settings lacking detailed data (at a high frequency, for a cross section of markets, or including cost information) required by familiar structural methods outlined in Bresnahan (1989). Our methods appear to work even for products that are not homogeneous in the narrow sense of being perfect substitutes across manufacturers but in the broader sense of having a well-defined physical unit for quantity and limited price dispersion, leading to a meaningful price variable.

Crucial for our methods to work is that demand grow over time. This is a key obstacle in applying our methods because the very data limitations that motivated the researcher to use our methods may preclude formal tests of growing demand. In our applications, we dealt with this obstacle by citing factors from independent historical sources supporting growing demand. The most reassuring factors, common to both applications, were, first, that the products were not mature but relatively new ones exploding in popularity and, second, that the sample periods exhibited relatively stable macroeconomic conditions—by construction, as we cut the samples off before impending recessions. Researchers could look for the presence of these factors in other markets when seeking to apply our methods. Another approach would be to use our methods as an initial screen. Depending on the results from the screen, the researcher could either acquire more data to verify the growing-demand assumption or apply additional structural methods to pin down the elasticity more precisely. This might be an efficient approach not just when studying an isolated historical case but facing a broad cross section of modern markets.

Overall, we view of methods as a supplement rather than substitute for existing methods. The methods leverage the time-series evolution of a market, an underappreciated source of information, to assess market competitiveness, complementing narrative methods using historical anecdotes or structural methods using more detailed cost data.

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APPENDIX A: PROOFS

This appendix provides omitted proofs or omitted technical details for the results and propositions stated in the text.

A1. Proof of Result 1

To prove necessity, note that Assumption 4 requires $D_1(0) \geq D_0(0)$, implying

$$a_1 \geq a_0. \tag{A1}$$

Assumption 4 also requires $D_1(a_0/b_0) \geq D_0(a_0/b_0) = 0$, implying $a_1 - b_1 a_0/b_0 \geq 0$, in turn implying

$$\frac{a_1}{a_0} \geq \frac{b_1}{b_0}. \tag{A2}$$

Conditions (A1) and (A2) together are equivalent to (12).

To prove sufficiency, suppose (12) holds. Then

$$D_1(p) = a_1 - b_1 p = \frac{a_1}{a_0}\left(a_0 - \frac{b_1 a_0 p}{a_1}\right) \geq a_0 - b_0 p = D_0(p), \tag{A3}$$

where the inequality uses (A1) and (A2), just shown to be equivalent to (12). Hence, Assumption 4 holds. *Q.E.D.*

A2. Proof of Result 2

We have

$$\frac{\varepsilon_0 - \bar{\varepsilon}_0^*}{\varepsilon_0} = 1 - \frac{1 - \bar{b}_0^*}{b_0} = 1 - \frac{1}{b_0}\left(\frac{q_1 - q_0}{p_0 - p_1}\right) = \frac{(1 + n_0)(a_0 - a_1)}{a_0 - a_1 + b_0 n_0 (c_0 - c_1)}. \tag{A4}$$

The first equality follows from elasticity formula (3). The second equality follows from $e_1 \in SE^+$, implying $q_1 > q_0$ and $p_0 > p_1$. The third equality follows by substituting from (13) and further substituting $n_1 = n_0$ and $b_1 = b_0$. The derivative of the last expression in (A4) with respect to n_0 can be shown to equal

$$\text{(A5)} \qquad -\frac{(a_0-a_1)(1+n_0)(q_1-q_0)}{n_0[a_0-a_1+b_0n_0(c_0-c_1)]^2},$$

which is negative. Thus (A4) exceeds the limit

$$\text{(A6)} \qquad \lim_{n_0\to\infty}\frac{(1+n_0)(a_0-a_1)}{a_0-a_1+b_0n_0(c_0-c_1)}=\frac{a_0-a_1}{b_0(c_0-c_1)}$$

using l'Hôpital's rule. Combining (A4) and (A6) yields

$$\text{(A7)} \qquad \frac{\varepsilon_0-\bar{\varepsilon}_0^*}{\varepsilon_0}>\frac{a_0-a_1}{b_0(c_0-c_1)}>\frac{a_0-a_1}{a_0}.$$

To see the second inequality, $q>0$ implies $a_0>b_0c_0>b(c_0-c_1)$. *Q.E.D.*

A3. Proof of Result 3

Suppose $c_1\leq c_0$, $n_0\geq n_1$, and the maintained assumptions hold (in particular, Assumption 4). Then

$$\text{(A8)} \qquad q_1=\frac{n_1}{1+n_1}(a_1-b_1c_1)$$

$$\text{(A9)} \qquad \geq\frac{n_0}{1+n_0}(a_1-b_1c_1)$$

$$\text{(A10)} \qquad \geq\frac{n_0}{1+n_0}\left(a_1-\frac{a_1b_0c_0}{a_0}\right)$$

$$\text{(A11)} \qquad =\frac{a_1q_0}{q_0}.$$

Equations (A8) and (A11) follow from (13). Condition (A9) follows from $n_0\geq n_1$ and $c_1\leq c_0$. Condition (A10) follows from (A2), which the previous proof showed is entailed by (12). Result 1 showed that (12) is equivalent to Assumption 4.

Conditions (A8)–(A11) imply $q_1\geq a_1q_0/a_0$, implying $q_1\geq q_0$ by (A1), entailed by Assumption 4. But $q_1\geq q_0$ implies $e_1\notin NW^+$, implying $NW^+=\emptyset$. As argued in the text, this implies $\underline{\varepsilon}_0^*=0$.

Similarly, one can show that $c_1\geq c_0$ and $n_0\leq n_1$ imply $p_1\geq p_0$ under maintained assumptions. Thus $SE^+=\emptyset$, implying $\bar{\varepsilon}_0^*=\infty$ by (11). *Q.E.D.*

A4. Proof of Result 4

Suppose $a_0=a_1$ and $b_0=b_1$. Substituting these values into (13), and then substituting the resulting equilibrium points into (6), tedious calculations can be used to show

$B(e_0,e_1) = b_0$. By Assumption 1 that equilibrium points are distinct, $p_0 \neq p_1$, implying $p_0 < p_1$ or $p_0 > p_1$. First suppose $p_0 < p_1$. Then $e_1 \in NW^+$, implying $\underline{b}_0^* = b_0$ by (10), implying $\underline{\varepsilon}_0^* = \varepsilon_0$. Similar reasoning implies $\bar{\varepsilon}_0^* = \varepsilon_0$ if $p_0 > p_1$. *Q.E.D.*

A5. *Proof of Result 5*

Under perfect price discrimination, market quantity is the same as under marginal-cost pricing: $q_t = a_t - b_t c_t$. Revenue r_t equals the area of the trapezoid under inverse demand: $r_t = q_t(a_t/b_t + c_t)/2$. While a single price is ill-defined under perfect price discrimination in theory, price is measured in our data as average revenue: $p_t = r_t/q_t = (a_t/b_t + c_t)/2$. We have

$$B(e_0,e_1) = \frac{q_1 - q_0}{p_0 - p_1} \tag{A12}$$

$$= 2b_0 \left(\frac{a_1 - b_1 c_1 - a_0 + b_0 c_0}{a_0 + b_0 c_0 - a_1 b_0/b_1 - b_0 c_1} \right) \tag{A13}$$

$$\geq 2b_0 \left[\frac{a_1 - b_1 c_1 - a_0 + b_0 c_0}{b_0(c_0 - c_1)} \right] \tag{A14}$$

$$= 2b_0 + 2b_0 \left[\frac{a_1 - b_1 c_1 - (a_0 - b_0 c_1)}{b_0(c_0 - c_1)} \right] \tag{A15}$$

$$= 2b_0 + 2b_0 \left[\frac{D_1(c_1) - D_0(c_1)}{b_0(c_0 - c_1)} \right]. \tag{A16}$$

Equation (A12) follows from $e_1 \in SE^+$. Equation (A13) follows from substituting q_t and p_t and rearranging. Equation (A14) follows from $a_0 - a_1 b_0/b_1 \leq 0$ by (12). Equations (A15) and (A16) follow from rearranging. The factor in brackets in (A16) is nonnegative. The numerator is nonnegative by assumption of demand growth. To show that the denominator is positive, note $e_1 \in SE^+$ implies $p_1 < p_0$, implying $(a_1/b_1 + c_1)/2 < (a_0/b_0 + c_0)/2$, in turn implying $c_0 > c_1$ because $a_1/b_1 \geq a_0/b_0$ by (12).

Translating the results into elasticities, $\bar{\varepsilon}_0^* = \bar{b}_0^* q_0/p_0 = B(e_0,e_1)q_0/p_0 \geq 2b_0 q_0/p_0 = 2\varepsilon_0$, computing the relevant elasticity under perfect price discrimination at the marginal unit sold. *Q.E.D.*

A6. *Proof of Proposition 2*

Suppose $t > 0$. We proceed by showing that for all $e_t \in SE^+$, $\Theta(e_0,e_t)$ exists, is unique, and provides an upper bound on the θ_0 satisfying (21). Condition (19) implies $\lim_{\theta \to 0} \tilde{D}(p_t,\theta,e_0) = q_0 < q_t$, where the inequality follows from $e_t \in SE^+$. Condition (20) implies $\lim_{\theta \to \infty} \tilde{D}(p_t,\theta,e_0) = \infty$ since $p_t < p_0$ for $e_t \in SE^+$. Condition (18) implies that $\tilde{D}(p_t,\theta,e_0)$ is increasing in θ since $p_t < p_0$. Together, these results imply $\tilde{D}(p_t,\theta,e_0)$ is below q_t for low θ and monotonically increases in θ until it rises above q_t for high θ. Thus the solution $\Theta(e_0,e_t)$ to (22) exists and is unique. Since (21) is satisfied for all $\theta_0 \leq \Theta(e_0, e_t)$ and violated for all $\theta_0 > \Theta(e_0, e_t)$, we see that $\Theta(e_0, e_t)$ provides an upper bound on θ_0 satisfying (21).

One can similarly show that for all $e_t \in NW^+$, then $\Theta(e_0, e_t)$ exists, is unique, and provides a lower bound on the θ_0 satisfying (21). For all $e_t \in NE^+$, there exists no $\theta \geq 0$ solving (22) since (21) is a strict inequality for all $\theta_t \geq 0$. In this case, pairwise comparison of e_0 and e_t yields no bounding information. The case in which $e_t \in SW^+$ can be ignored because the set is empty under Assumption 4 of growing demand.

Pairwise comparison of e_0 and e_t can be repeated for $t < 0$. Growing demand then implies the same inequality as (21) with the direction reversed, leading to the same bound but for the opposite compass direction. *Q.E.D.*

A7. *Proof of Proposition 3*

Consider the comparison between the reference equilibrium point e_0 and some $e_t \in SE^+$. By the mean value theorem, there exists $\tilde{p} \in [p_t, p_0]$ such that

$$D_0'(\tilde{p}) = \frac{D_0(p_0) - D_0(p_t)}{p_0 - p_t} \geq \frac{q_0 - q_t}{p_0 - p_t}, \tag{A17}$$

where the second step follows from $q_t = D_t(p_t) \geq D_0(p_t)$ by Assumption 4 of growing demand. Taking absolute values,

$$|D_0'(\tilde{p})| \leq \left| \frac{q_0 - q_t}{p_0 - p_t} \right| = B(e_0, e_t). \tag{A18}$$

If $D_0(p)$ is weakly convex, then for $\tilde{p} \leq p_0$, we have $D_0'(\tilde{p}) \leq D_0'(p_0)$, implying $|D_0'(\tilde{p})| \geq |D_0'(p_0)|$. Combining this inequality with (A18) yields $|D_0'(p_0)| \geq B(e_0, e_t)$. On the other hand, if $D_0(p)$ is weakly concave, then $|D_0'(\tilde{p})| \leq |D_0'(p_0)|$, which does not generate conclusive information combined with (A18).

Pairwise comparisons between e_0 and e_t in the other compass sets relevant to the proposition (NW^+, SE^-, and NW^-) are similar and omitted for brevity. *Q.E.D.*

A8. *Proof of Proposition 4*

We will derive new bounds emerging from an examination of the limits $p \to 0$ and $p \to \infty$ in turn. Start by considering the limit $p \to 0$ and comparing e_0 to later equilibrium points e_t, $t > 0$. Assumption 4 of growing demand implies $\tilde{D}(p, b_0, e_0) \leq \tilde{D}(p, b_t, e_t)$ for all $p \geq 0$, implying in particular that $\tilde{D}(0, b_0, e_0) \leq \tilde{D}(0, b_t, e_t)$. Substituting for $\tilde{D}$ from (2) into this inequality, and further substituting $p = 0$, yields $q_0 + b_0 p_0 \leq q_t + b_t p_t$. Notice this is a condition on the relative position of the inverse demand curves' horizontal intercepts. Rearranging,

$$b_0 \leq \frac{1}{p_0}(q_t - q_0 + b_t p_t) \leq \frac{1}{p_0}(q_t - q_0 + \bar{b}_t^* p_t), \tag{A19}$$

where the second inequality follows from Proposition 1.

Next, continue considering the limit $p \to 0$ but now suppose $t < 0$. Using the arguments behind (A19), just reversing the inequalities, we obtain

$$b_0 \geq \frac{1}{p_0}(q_t - q_0 + b_t p_t) \geq \frac{1}{p_0}(q_t - q_0 + \underline{b}_t^* p_t). \tag{A20}$$

Next, consider the limit $p \to \infty$. Start by comparing e_0 to e_t for $t > 0$. The fact that $\tilde{D}(p, b_0, e_0) \leq \tilde{D}(p, b_t, e_t)$ for all $p \geq 0$ implies $\lim_{p \to \infty} \tilde{D}(p, b_0, e_0) \leq \lim_{p \to \infty} \tilde{D}(p, b_t, e_t)$. These limits are just the vertical intercepts of the respective inverse demands. Hence, the preceding inequality implies $p_0 + q_0/b_0 \leq p_t + q_t/b_t$, or after rearranging,

$$b_0 \left(p_t - p_0 + \frac{q_t}{b_t} \right) \geq q_0. \tag{A21}$$

We will show that the factor in parentheses is positive. It is immediate that it is positive when $p_t > p_0$ since $q_t > 0$ and $b_t \geq 0$. Suppose instead that $p_t < p_0$. Then $e_t \in SW^+ \cup SE^+ = SE^+$ since $SW^+ = \emptyset$ under Assumption 4 as discussed in Section I. Then

$$b_t \leq \bar{b}_t^* \leq B(e_0, e_t) = \frac{q_t - q_0}{p_0 - p_t} < \frac{q_t}{p_0 - p_t}. \tag{A22}$$

The first and second steps follow from Proposition 1. The next step follows from the definition of $B(e_0, e_t)$ from (6) and from $q_t > q_0$ and $p_0 > p_t$ for $e_t \in SE^+$. The last step follows from $q_0 > 0$, which holds by Assumption 2. Cross multiplying by $p_0 - p_t$, which is positive, and rearranging proves that the term in parentheses is positive. Cross multiplying (A21) by the positive factor in parentheses yields

$$b_0 \geq \frac{q_0}{p_t - p_0 + q_t/b_t} \geq \frac{q_0}{p_t - p_0 + q_t/\underline{b}_t^*}. \tag{A23}$$

Continue to consider the limit $p \to \infty$, but now suppose instead that $t < 0$. Applying the analysis behind (A21), just reversing inequalities, yields

$$b_0 \left(p_t - p_0 + \frac{q_t}{b_t} \right) \leq q_0. \tag{A24}$$

An important difference with (A21) is that the factor in parentheses in (A24) cannot be unambiguously signed when $t < 0$. When the factor in parentheses is non-positive, (A24) holds for all b_0. In that case, the condition provides no useful bounding information. When the factor in parentheses is positive, cross multiplying by it preserves the direction of the inequality, yielding

$$b_0 \leq \frac{q_0}{p_t - p_0 + q_t/b_t} \leq \frac{q_0}{p_t - p_0 + q_t/\bar{b}_t^*}. \tag{A25}$$

To emphasize, condition (A25) only provides a potentially useful upper bound if the denominator of the last fraction is positive, i.e.,

$$p_t + \frac{p_t}{\bar{b}_t^*} > p_0. \tag{A26}$$

If (A26) does not hold, we must ignore (A25) lest we conclude that b_0 is bounded above by the negative number on the right-hand side of (A25), violating the law of demand (Assumption 3), which implies $b_0 \geq 0$.

The second-stage lower bound $\underline{b}_0^{**}$ in (29) is set to the lower of the first-stage bound $\underline{b}_0^*$ and the supremum of the bounds (A20) and (A25) incorporating limiting information. Similarly, the second-stage upper bound $\bar{b}_0^{**}$ in (30) is set to the lower of the first-stage bound $\bar{b}_0^*$ and the infimum of the bounds incorporating limiting information (A19) and (A24). *Q.E.D.*

A9. Proof of Proposition 5

Begin the analysis by considering the limit $p \to 0$ and comparing equilibrium points e_0 and e_t for $t > 0$. We have

$$1 \geq \lim_{p\to 0} \left[\frac{\tilde{D}(p,\theta_0,e_0)}{\tilde{D}(p,\theta_t,e_t)}\right] \geq \lim_{p\to 0} \left[\frac{\tilde{D}(p,\theta_0,e_0)}{\tilde{D}(p,\bar{\theta}_t^*,e_t)}\right], \tag{A27}$$

To see the first inequality, Assumption 4 that demand is nondecreasing over time implies $\tilde{D}(p,\theta_t,e_t) \geq \tilde{D}(p,\theta_0,e_0)$. Dividing by $\tilde{D}(p,\theta_t,e_t)$ and taking limits gives the first inequality in (A27). To see the second inequality in (A27), note that in the limit as $p \to 0$, eventually $p \leq p_t$. But (18) then implies $\tilde{D}_\theta(p,\theta,e_t) \geq 0$ in the limit as $p \to 0$, in turn implying $\tilde{D}(p,\bar{\theta}_t^*,e_t) \geq \tilde{D}(p,\theta_t,e_t)$ in the limit as $p \to 0$ since $\bar{\theta}_t^* \geq \theta_t$ by Proposition 2. It can be further argued by (18) the numerator in the last expression in (A27) is increasing in θ_t, as is the expression itself. Treating the inequalities in (A27) as equalities,

$$\lim_{p\to 0} \left[\frac{\tilde{D}(p,\theta_0,e_0)}{\tilde{D}(p,\bar{\theta}_t^*,e_t)}\right] = 1, \tag{A28}$$

thus provides an equation in θ_0 that can be solved to provide a upper bound on θ_0.

Repeating the analysis for each $t > 0$, the lowest of the solutions to (A28) across $t > 0$ is an upper bound that may be tighter than $\bar{\theta}_0^*$ from the method incorporating local information. The upper bound may be further tightened by examining the opposite limit, $p \to \infty$, comparing the reference equilibrium point e_0 to e_t for $t < 0$. The analysis is similar to that above and omitted for brevity. Also similar and omitted for brevity are the analysis of the limit $p \to 0$ comparing the reference equilibrium point e_0 to e_t for $t > 0$ and the analysis of the limit $p \to \infty$ for $t < 0$. These analyses turn out to contribute to lower rather than upper bounds.

The proposition implicitly prescribe an iterative procedure for computing bounds incorporating limiting information for general demands. Suppose one wants to compute the upper bound $\bar{\theta}_0^{**}$. In the first stage, one computes the upper bounds $\bar{\theta}_t^*$ incorporating local information for all $e_t \in E$. In the second stage, one solves the equation for θ,

$$\lim_{p\to\infty}\left[\frac{\tilde{D}(p,\theta,e_0)}{\tilde{D}(p,\underline{\theta}_t^*,e_t)}\right]=1 \tag{A29}$$

for each $t<0$ and takes the smallest solution. One then solves a similar equation, just involving a different limit, for θ,

$$\lim_{p\to 0}\left[\frac{\tilde{D}(p,\theta,e_0)}{\tilde{D}(p,\underline{\theta}_t^*,e_t)}\right]=1 \tag{A30}$$

for each $t>0$ and takes the smallest solution. The new bound $\bar{\theta}_0^{**}$ is set to whichever is smallest of (a) the upper bound from the method incorporating local information, $\bar{\theta}_0^*$, (b) the smallest solution over $t<0$ to (A29), and (c) the smallest solution over $t>0$ to (A30). The lower bound $\underline{\theta}_0^{**}$ is computed analogously. *Q.E.D.*

APPENDIX B: LOGIT DEMAND

In the context of a homogeneous product market under study, logit demand can be specified as

$$D_t(p)=\frac{n_t\exp(-\theta_t p)}{1+\exp(-\theta_t p)}=\frac{n_t}{1+\exp(\theta_t p)}, \tag{B1}$$

where n_t is interpreted as a market-size parameter and θ_t as a price-sensitivity parameter. For demand to be nonnegative, $n_t\geq 0$; for the law of demand (Assumption 3) to hold, $\theta_t\geq 0$.

This specification of demand involves two independent parameters, but further requiring the curve to pass through the equilibrium point e_t pins it down to a single-parameter family. Focus for now on price sensitivity, θ_t, as this key parameter. Given θ_t and equilibrium point (q_t,p_t), equation (B1) can be solved for the market-size parameter:

$$n_t=q_t[1+\exp(\theta_t p_t)]. \tag{B2}$$

Substituting for n_t from equation (B2) into (B1) yields an expression for logit demand in terms of the single unknown parameter θ_t and known equilibrium point $e_t=(q_t,p_t)$:

$$\tilde{D}(p,\theta_t,e_t)=\frac{q_t[1+\exp(\theta_t p_t)]}{1+\exp(\theta_t p)}. \tag{B3}$$

The elasticity of logit demand is

$$\varepsilon_t = -\tilde{D}_p(p_t,\theta_t,e_t)\frac{p_t}{q_t} = \frac{\theta_t p_t}{1+\exp(-\theta_t p_t)}. \tag{B4}$$

To apply Propositions 2 and 5, covering general demand curves, to the logit special case, we need to verify that conditions (16)–(20) are satisfied by logit demand. Condition (16)–(18) can be verified by differentiating (B3):

$$\tilde{D}_\theta(p,\theta,e_t) = \frac{q_t[(p_t-p)\exp(\theta p_t)\exp(\theta p) + p_t\exp(\theta p_t) - p\exp(\theta p)]}{[1+\exp(\theta p)]^2}. \tag{B5}$$

It is easy to see that (B5) is negative if $p > p_t$, verifying (16); that (B5) equals 0 if $p = p_t$, verifying (17); and that (B5) is positive if $p < p_t$, verifying (18). To verify (19), substitute $\theta_t = 0$ into (B3), yielding $\tilde{D}(p,0,e_t) = q_t$. To verify (20),

$$\lim_{\theta\to\infty}\tilde{D}(p,\theta,e_t) = q_t\lim_{\theta\to\infty}\frac{\exp(\theta(p_t-p))+\exp(-\theta p)}{1+\exp(-\theta p)} = q_t\lim_{\theta\to\infty}\exp(\theta(p_t-p)). \tag{B6}$$

It is easy to see that (B6) equals 0 if $p > p_t$ and equals ∞ if $p < p_t$, verifying (20).

Applying Propositions 2 and 5, which cover general demand curves, to the special case of logit demand is fairly straightforward; but a few computational details deserve mention. In the general case, the solution of equation (22) for θ is the $\Theta(e_t,e_{t'})$ having a prominent role in the general propositions. In the special case of logit demand, (22) can be written after rearranging as

$$q_0[1+\exp(\theta p_0)] = q_t[1+\exp(\theta p_t)]. \tag{B7}$$

Proposition 5 bounds the price-sensitivity parameter for general demand curves using the method incorporating limiting information. To apply this proposition to a given demand curve requires the relevant limits to be computed and substituted into the the bounds formulas. For reference, we report the computations for logit demand in the following corollary.

PROPOSITION 6: *Suppose the demand curve in every period $t\in T$ is logit, i.e., $D_t(p) = \tilde{D}(p,\theta_t,e_t)$ defined in* (B3). *Suppose further these curves satisfy Assumptions 3 and 4. Whichever element $e_0 \in E$ the researcher chooses for the reference equilibrium point, we have $\theta_0 \in [\underline{\theta}_0^{**},\bar{\theta}_0^{**}]$, where*

$$\underline{\theta}_0^{**} \equiv \underline{\theta}_0^{*} \vee \sup_{t<0}\left\{\frac{1}{p_0}\ln\left(\frac{q_t}{q_0}[1+\exp(\underline{\theta}_t^{*}p_t)]-1\right)\right\} \vee \sup_{t>0}\underline{\theta}_t^{*}. \tag{B8}$$

$$\bar{\theta}_0^{**} \equiv \bar{\theta}_0^{*} \wedge \inf_{t<0}\bar{\theta}_t^{*} \wedge \inf_{t>0}\left\{\frac{1}{p_0}\ln\left(\frac{q_t}{q_0}[1+\exp(\bar{\theta}_t^{*}p_t)]-1\right)\right\}. \tag{B9}$$

APPENDIX C: ZELTERMAN BOOTSTRAP

As noted in the text, standard bootstrap methods are invalid in the context of extreme order statistics. Since any pseudosample is a subset of the original data, the bootstrapped distribution will be bounded by—rather than centered on—the extreme order statistic estimated from the original data. Zelterman (1993) provides a method for circumventing this problem. Instead of sampling the data directly, to bootstrap the maximum order statistic, he proposes sampling the spacings between the highest k observations. The maximum order statistic can be bootstrapped by adding the spacings from the pseudosample to the order statistic in the sample anchored k positions away from the maximum.

C1. *Semiparametric Method*

Zelterman classifies the technique as semiparametric since the key result behind the technique—that the normalized spacings are asymptotically i.i.d. exponential—holds for a wide range of distributions of the underlying data. The sole parameter is k, anchoring the order statistic to which the sampled spacings are appended when simulating the maximum order statistic.

Weissman (1978) provides a sufficient condition for the asymptotic result: there exists real numbers $a_t > 0, b_t, t = 1, 2, \ldots$, such that for all $x \in (-\infty, \infty)$, the distribution function $F(x)$ for the underlying data satisfies

$$\lim_{t\to\infty} F^t\left(\frac{x-b_t}{a_t}\right) = \exp(-\exp(-x)). \tag{C1}$$

In words, the highest order statistic—properly normalized by constants a_t and b_t—converges in distribution to a Gumbel extreme value as the sample size grows large. One can show this condition is satisfied by the normal, lognormal, logistic, gamma, Pareto, Gompertz, Weibull, and Gumbel, among others.

C2. *Implementation for Method Incorporating Local Information*

Consider applying the technique to bootstrap a confidence interval around $\bar{\varepsilon}_t^*$, the upper bound on the demand elasticity in period t using the method incorporating local information. Although it is an upper bound, $\bar{\varepsilon}_t^*$ is a minimum order statistic: the minimum of the upper bounds derived from pairwise comparisons between e_t and the other equilibrium points. Let $\bar{\varepsilon}_t^{[1]} \le \bar{\varepsilon}_t^{[2]} \le \cdots \le \bar{\varepsilon}_t^{[T-1]}$ be the elasticity bounds from these pairwise comparisons ordered from smallest to largest; superscripts on these terms thus denote the order of these order statistics. We have $\bar{\varepsilon}_t^* = \bar{\varepsilon}_t^{[1]}$.

As Zelterman's analysis is predicated on a maximum rather than a minimum order statistic but $\bar{\varepsilon}_t^*$ is a minimum order statistic, we will bootstrap the reciprocal of $\bar{\varepsilon}_t^*$ rather than $\bar{\varepsilon}_t^*$ itself, reversing the reciprocal in the final step to recover the elasticity bound.

Defining the reciprocal $\bar{r}_t^{[i]} = 1/\bar{\varepsilon}_t^{[i]}$, the inequalities between the order statistics on the reciprocals are the reverse of those between the elasticity bounds: $\bar{r}_t^{[1]} \geq \bar{r}_t^{[2]} \geq \cdots \geq \bar{r}_t^{[T-1]}$. Let $d_t^{[i]} \equiv i(\bar{r}_t^{[i]} - \bar{r}_t^{[i+1]})$ denote the normalized spacing between two of these reciprocals. The normalization needed to generate the exponential distribution asymptotically is simply multiplication by i, the degree of the order statistics involved in the spacing.

Let $\mathbf{d}_t \equiv \{d_t^{[i]} \,|\, i = 1,\ldots,k\}$ denote the set of the normalized spacings observed in the data between the k highest reciprocals. Pseudosamples $\tilde{\mathbf{d}}_t = \{\tilde{d}_{ti}\}$ of size k are drawn with replacement from $\mathbf{d}_t$. (In practice, we take 10,000 draws of bootstrap pseudosamples to provide the precision demanded by 99% confidence intervals.) For each pseudosample, the bootstrap maximum order statistic is computed as

$$\tilde{r}_t^* \equiv \bar{r}_t^{[k+1]} + \sum_{i=1}^{k} \tilde{d}_{ti}/i. \tag{C2}$$

Intuitively, k draws of normalized spacings are subtracted (after reversing the normalization by dividing by i) from the observed $(k+1)$st order statistic $\bar{r}_t^{[k+1]}$, which anchors the procedure, to arrive at the simulated maximum $\tilde{r}_t^*$. Following Zelterman's recommendation, we take $k = \lfloor (T-1)/3 \rfloor$. Taking the reciprocal of this bootstrapped maximum order statistic recovers the bootstrapped elasticity bound: $\tilde{\varepsilon}_t^* = 1/\tilde{r}_t^*$.

Zelterman notes that the asymptotic argument requires $k \to \infty$ as well as $T \to \infty$. Our sample of pairwise comparisons is small since it comes from a relatively short time series. We still proceed to apply the technique and the recommendation for k but note that we may be straining the asymptotic arguments behind the bootstrap. On a positive note, despite the small size of T and k in our application, the bootstrapped confidence intervals appear quite sensible and well behaved.

C3. Numerical Example

A numerical example serves to illustrate the Zelterman bootstrap. In Figure 6, the upper bound on the demand elasticity for low-density polyethylene derived under the assumption of linear demand is $\bar{\varepsilon}_t^* = 0.52$ in the year $t = 1964$. Suppose we want to bootstrap this estimate. This is the minimum of the elasticity bounds from pairwise comparisons, the five lowest of which are shown in the second column of Table C1. Setting the key parameter for the technique to $k = 4$, the five lowest pairwise bounds are all that are relevant for the computation. Taking reciprocals in the next column, the entry in the first row now becomes the maximum order statistic among them. The last column computes the normalized spacings. This is the set from which pseudosamples are drawn with replacement to compute bootstrapped estimates. Suppose, for example, that the pseudosample $\{0.40, 0.40, 0.64, 0.24\}$ was drawn. This would generated the bootstrapped reciprocal

$$\tilde{r}_t^* = 1.22 + \frac{0.40}{1} + \frac{0.40}{2} + \frac{0.64}{3} + \frac{0.24}{4} = 2.09, \tag{C3}$$

equal to the anchor reciprocal in the last row plus the four draws of normalized spacings with normalizations reversed. Reversing the reciprocal generates the bootstrapped value of the elasticity bound: $\tilde{\varepsilon}_t^* = 1/2.09 = 0.48$. This procedure can be repeated (we do this 10,000 times) to generate a bootstrapped sample of elasticity bounds from which standard errors and confidence intervals can be derived.

TABLE C1—NUMERICAL EXAMPLE OF ZELTERMAN BOOTSTRAPPING

$[i]$	$\bar{\varepsilon}_t^{[i]}$	$\bar{r}_t^{[i]}$	$d_t^{[i]}$
1	0.52	1.92	
			$(1.92 - 1.52) \times 1 = 0.40$
2	0.66	1.52	
			$(1.52 - 1.46) \times 2 = 0.12$
3	0.68	1.46	
			$(1.46 - 1.38) \times 3 = 0.24$
4	0.72	1.38	
			$(1.38 - 1.22) \times 4 = 0.64$
5	0.72	1.38	

Note: Entries in $\bar{\varepsilon}_t^{[i]}$ column are the five lowest of the actual elasticity bounds from pairwise comparison of the 1964 equilibrium point to other years' assuming linear demand.

C4. Implementation for Method Incorporating Limiting Information

Bootstrapping the bound $\bar{\varepsilon}^{**}$ incorporating limiting information is more involved because it is a two-stage estimator, and the outcome of the second stage depends in a complicated way on the first stage. We adopt what in our view is the most natural alternative: applying Zelterman's technique to simulate a bootstrapped estimate from the normalized spacings observed in each separate stage, and then taking the minimum over the result from each stage to generate the final bootstrapped bound $\tilde{\varepsilon}_t^{**}$.

An alternative would be to bootstrap the demand parameter rather than the elasticity directly. In the case of linear demand, b_t could be bootstrapped. Then the first stage bound ($\bar{b}^*$) could be treated equivalently to the pairwise comparisons between e_t and the other equilibrium points incorporating limiting information in the second stage (the expressions in braces in equation (30)). We tried this alternative, and the results were nearly identical.