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Bounding the benefits of stochastic auditing: The case of risk-neutral agents[★]

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Summary. In the context of a costly-state-verification model with a risk-neutral agent having limited liability, it has been postulated that allowing stochastic auditing reduces the asymmetric information problem to a trivial one: i.e., the first best can be approached arbitrarily closely with feasible contracts. This paper proves the postulate to be false: the surplus from feasible contracts is bounded strictly below the first-best surplus level. The bound is straightforward to compute in examples. The paper thus removes a justification for the restriction to deterministic auditing commonly made in the literature.

Keywords and Phrases: Stochastic auditing, Costly state verification model, Risk neutrality.

JEL Classification Numbers: D82, D20, G20, H26.

1 Introduction

There is an extensive literature on the use of auditing in investment finance, insurance, and tax collection.[1] The costly-state-verification model, originally proposed by Townsend (1979), is perhaps the most widely-used tool to analyze the issue of auditing in these contexts. It is often assumed in applications (e.g., Townsend, 1979; Gale and Hellwig, 1985) that auditing is a

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[1] In the investment context, see e.g. Bond and Crocker (1993), Boyd and Smith (1993, 1994), Diamond (1984), Gale and Hellwig (1985) and Townsend (1979). In the insurance context, see Mookherjee and Png (1989). In the context of tax collection, see Border and Sobel (1987) and Reinganum and Wilde (1985).

deterministic function of the reported state. As suggested by an example in Townsend (1979), however, contracts with stochastic auditing can generally improve upon deterministic ones. Two approaches have been used in the study of mechanisms with stochastic auditing. A first approach, followed by Mookherjee and Png (1989), assumes the agent is risk averse. A second approach, followed by Border and Sobel (1987), assumes the agent is risk neutral and places an arbitrary bound on the transfer from agent to principal. A third approach would be to allow for a risk-neutral agent but to remove any bounds on the transfer (besides those that are required to preserve the agent's limited liability). This third approach has been avoided due to issues of *existence* and *triviality*.

The existence problem was highlighted by Border and Sobel: they construct a simple numerical example in which the principal's expected revenue can be made arbitrarily close to an upper bound, but there exists no feasible contract that allows the principal to obtain the upper bound exactly. Thus the optimal contract does not exist. The triviality problem, postulated by Mookherjee and Png, is that the interesting features of the costly-state-verification model are lost under risk neutrality and stochastic auditing: in this setting, contracts can come arbitrarily close to the first best, where the first best is the solution that could be obtained if there were no informational asymmetry.

In this paper, it is shown that models allowing for stochastic auditing do not suffer from the triviality problem. Proposition 1 states that the total surplus provided by feasible contracts is below a bound itself strictly below the first-best surplus level. The bound is straightforward to compute: examples are provided in which the bound is computed assuming the project's return is a uniform or exponential random variable. In many instances – e.g., if the density function associated with the project's return is strictly positive on its range – the bound provided in Proposition 1 is not tight and thus understates the difference between feasible outcomes and the first best.

To fix ideas, the paper will focus on the design of financial contracts, with a firm borrowing from a lender to finance a project providing a random return, though the results will apply more generally to the other contexts (insurance, tax compliance) as well. Intuitively, the optimal feasible contract is designed to minimize expected auditing costs subject to an individual-rationality constraint for the lender and incentive-compatibility and limited-liability constraints for the firm. One possible scheme would be to let the auditing probability approach zero while letting the punishment for understating the project's return become infinitely large. The problem with such a scheme is that limited liability restricts the severity of punishment. This alone does not prove that feasible contracts cannot approach the first best: it may be possible to implement schemes which let the auditing probability approach zero while letting the reward for truthfully revealing the project's return become infinitely large. Proposition 1 shows that these latter schemes are also infeasible. To derive some intuition for this result, suppose x is the project's realized return, the realization of a random variable distributed on

$[L, H]$. The maximum repayment that the lender can extract from the firm while preventing the firm from understating x depends on the auditing probability specified for return realizations below x. In particular, if the auditing probability is allowed to approach zero even for return realizations near L, then the maximum repayment for any return realization approaches L. To see this, note the firm can always pretend to have earned L; with probability approaching one the firm would not be audited and would have to pay at most L. In general, this repayment will not be sufficient to guarantee the lender's participation in the project. Thus, the auditing probability for low values of the project's return must be bounded away from zero.

2 Model

There are two players in the model, a lender and an investor.[2] The lender and investor will be referred to collectively as the venture. In period 1, the investor can undertake a project requiring sunk investment $K > 0$. The investor has no retained earnings and so must borrow K from the lender. In a prior period (period 0), the investor and lender may sign a financial contract. It is assumed the investor makes a take-it-or-leave-it contract offer to the lender.[3] In period 2 the project produces a random return X, distributed on the interval $[L, H] \subseteq [0, \infty)$ according to cdf F. Let x be a realization of the random variable X. Denote the mean return by $\bar{X} \equiv \int_L^H x\, dF(x)$. For simplicity, assume there is no discounting between the time the investment is made and the return is realized.[4]

Several assumptions will serve to eliminate uninteresting cases. First, assume that the project's expected net present value is positive, i.e., $\bar{X} > K$. This assumption eliminates the case in which the investor would not undertake the project even in the first-best case (the case in which its internal funds are sufficient to cover the investment cost K). Second, assume that $\int_L^K dF(x) > 0$, i.e., the set $\{x | x < K\}$ has positive measure. This assumption eliminates the case in which a financial contract could solve the financing problem trivially (the case in which the investor can repay K with certainty in period 2).

The project's realized return x is private information for the investor. The lender cannot observe x, nor is x verifiable to the court, unless the lender

[2] For a discussion of a model in which there are a large number of agents, see Krasa and Villamil (1992, 1994).

[3] There are two reasons to make this assumption. First, the assumption corresponds to the case in which the financial markets are competitive so that lenders accept any contract yielding them non-negative expected profit. Second, the assumption implies that the individual-rationality constraint applies to the lender, the party with unlimited liability. The resulting contract will be more efficient in terms of venture surplus than a contract in which the individual-rationality constraint applied to a party with limited liability.

[4] For a discussion of optimal mechanisms in a model with multiple periods of investment and/or return, see Chang (1990), Townsend (1982), and Webb (1992).

conducts an audit in period 2. An audit causes the true value of x to be observable to all parties and verifiable to the court. The cost of an audit is given by $b(x) > 0$. The audit cost is assumed to be borne by the lender. Let A be the audit indicator, equal to one if the lender audits and zero if the lender does not audit.

General financial contracts are structured as follows. By standard arguments, attention can be restricted to direct revelation mechanisms without loss of generality (Townsend 1988). After the project realizes a return in period 2, the investor makes an announcement, $\tilde{x} \in [L, H]$, of this return. The lender conducts an audit with probability $a(\tilde{x})$, where $a : [L, H] \to [0, 1]$. To allow for stochastic auditing, $a(\tilde{x})$ is allowed to fall strictly within the unit interval.[5] The mechanism then specifies a transfer between the parties, $T(\tilde{x}, x, A)$. As an accounting convention, a positive value of T implies that the investor makes a transfer to the lender and a negative value implies that the lender makes a transfer to the investor. It is straightforward to show that the optimal form for T is

$$T(\tilde{x}, x, A) = \begin{cases} x & \text{if } A = 1 \text{ and } \tilde{x} \neq x \\ t_0(\tilde{x}) & \text{if } A = 0 \\ t_1(x) & \text{if } A = 1 \text{ and } \tilde{x} = x \ . \end{cases} \tag{1}$$

If an audit is conducted and the investor is shown to have lied about x [as in the first line of (1)], then all its assets are seized by the lender. If an audit is not conducted [as in the second line of (1)], then T can only depend on the investor's announcement, $\tilde{x}$. If an audit is conducted and the firm's announcement is revealed to have been truthful [as in the third line of (1)], then $\tilde{x} = x$; so there is no loss in generality in having T depend on x alone.

A *feasible* contract must satisfy four constraints. First, it must preserve incentive-compatibility for the investor. That is, the investor must weakly prefer to announce $\tilde{x} = x$ than any other value of $\tilde{x}$:

$$\begin{gathered} a(x)t_1(x) + [1 - a(x)]t_0(x) \leq a(\tilde{x})x + [1 - a(\tilde{x})]t_0(\tilde{x}) \\ \forall x, \tilde{x} \in [L, H] \text{ such that } x \neq \tilde{x} \ . \end{gathered} \tag{2}$$

Second, the contract must satisfy a limited-liability constraint for the investor. Since the investor is assumed to have no internal funds besides the project's return, limited-liability requires $T(\tilde{x}, x, A) \leq x$. In view of (1), this constraint can be written

$$t_0(x) \leq x \text{ and } t_1(x) \leq x \quad \forall x \in [L, H] \ . \tag{3}$$

Third, $a(\tilde{x})$ must be a proper probability; i.e.,

$$a(\tilde{x}) \in [0, 1] \quad \forall \tilde{x} \in [L, H] \ . \tag{4}$$

Finally, the lender must earn more from signing the contract than not. Normalizing the lender's opportunity utility to zero, this individual-rationality (or participation) constraint can be written

[5] The venture is assumed to have access to a public randomizing device.

$$\int_L^H \{a(x)t_1(x) + [1-a(x)]t_0(x) - a(x)b(x)\}\, dF(x) \geq K \ . \tag{5}$$

The optimal contract maximizes the investor's expected utility subject to the feasibility constraints. The investor is assumed to be risk neutral, so its objective function can be written

$$\bar{X} - \int_L^H \{a(x)t_1(x) + [1-a(x)]t_0(x)\}\, dF(x) \ . \tag{6}$$

Summarizing the analysis of the section, the optimal contract specifies $a(x)$, $t_0(x)$, and $t_1(x)$ solving the following problem:

$$\text{maximize (6) subject to (2), (3), (4), and (5)} \ . \tag{7}$$

3 Bound on venture surplus

3.1 Analysis

As a preliminary step in solving (7), first note that (5) must bind at an optimum. Consider removing constraint (5) from (7). Then the solution would involve $t_0(x) = t_1(x) = 0$ for all $x \in [L, H]$. But this solution violates (5).

Treating (5) as an equality and substituting into (6) yields a new form for the objective function: $\bar{X} - K - \int_L^H a(x)b(x)\, dF(x)$. Now the first two terms of this expression are constants. So (7) is equivalent to choosing $a(x)$, $t_0(x)$, and $t_1(x)$ to solve

$$\text{minimize} \int_L^H a(x)b(x)\, dF(x) \text{ subject to (2), (3), (4), and (5),} \tag{8}$$

where (5) can be treated as an equality. It is evident from (8) that the objective of the optimal contract is to minimize expected auditing costs.

The analysis proceeds by deriving a lower bound on $a(x)$, in turn implying that the venture's surplus is bounded away from its first-best surplus. To this end, note that condition (5) implies

$$K \leq \int_L^H \{a(x)t_1(x) + [1-a(x)]t_0(x)\}\, dF(x) \ . \tag{9}$$

Now, for all $\tilde{x}, x \in [L, H]$,

$$\begin{aligned} a(x)t_1(x) + [1-a(x)]t_0(x) &\leq a(\tilde{x})x + [1-a(\tilde{x})]t_0(\tilde{x}) \\ &\leq a(\tilde{x})x + [1-a(\tilde{x})]\tilde{x} \\ &\leq a(\tilde{x})x + \tilde{x} \ , \end{aligned} \tag{10}$$

where the first line holds by (2), the second line holds by (3), and the third line holds since $\tilde{x} \geq L \geq 0$. Therefore, by (9) and (10), $K \leq \int_L^H [a(\tilde{x})x + \tilde{x}]\, dF(x)$ for all $\tilde{x} \in [L, H]$. Integrating and dividing through by $\bar{X}$, $a(\tilde{x}) \geq (K - \tilde{x})/\bar{X}$. Combining this inequality with the fact that $a(\tilde{x}) \geq 0$ yields the following lower bound on the auditing probability:

$$a(\tilde{x}) \geq \max\left(0, \frac{K - \tilde{x}}{\bar{X}}\right) \ . \tag{11}$$

Since the auditing probability is bounded away from zero, the return from any financial contract is bounded away from the first best. Formally,

$$\begin{aligned}\int_L^H a(x)b(x)\, dF(x) &\geq \int_L^H \max\left(0, \frac{K - x}{\bar{X}}\right) b(x)\, dF(x) \\ &= \frac{1}{\bar{X}} \int_L^K (K - x) b(x)\, dF(x) \\ &\equiv \Delta \ .\end{aligned}$$

The integrand $(K - x)b(x)$ is strictly positive for all $x \in [L, K)$. Furthermore, $[L, K)$ has positive measure by assumption. Therefore, by Billingsley (1979, Theorem 15.2), $\Delta > 0$. Summarizing the analysis,

Proposition 1 *Suppose a feasible contract is used to fund investment. Expected venture surplus is bounded away from $\bar{X} - K$, the level that can be obtained in the first best. In particular, the difference between $\bar{X} - K$ and expected venture surplus is at least Δ.*

There are a broad range of cases in which the bound on venture surplus given in Proposition 1 is not tight; i.e., the difference between the first-best surplus and the expected venture surplus strictly exceeds Δ. For instance, if X is a continuous random variable with a strictly positive density function on (L, H), then the bound in Proposition 1 is not tight.[6]

3.2 Examples

It is straightforward to compute Δ in applications. For example, suppose X is distributed uniformly on $[0, H]$ and $b(x)$ is a constant, i.e., $b(x) = b$, for all $x \in [0, H]$. Then $\Delta = bK^2 H^{-2}$. Expressed as a percentage of venture surplus in the first best,

$$\frac{\Delta}{\bar{X} - K} = \frac{2bK^2}{H^2(H - 2K)} \ .$$

Both of the previous expressions are increasing in b and K and declining in H.

Suppose, alternatively, that X is an exponential random variable with mean $1/\lambda$. Then $\Delta = b(\lambda K + \mathrm{e}^{-\lambda K} - 1)$. Expressed as a percentage of venture surplus in the first best,

$$\frac{\Delta}{\bar{X} - K} = \frac{\lambda b(\lambda K + \mathrm{e}^{-\lambda K} - 1)}{1 - \lambda K} \ .$$

[6] This statement is proved formally in a working-paper version of the article available from the author on request.

Both of these expressions are increasing in b, K, and λ in the relevant range of parameters.

4 Conclusion

The discussion above has been couched in terms of an investment-finance example. The result from Proposition 1 – that the first best cannot be approached even if contracts are allowed to specify stochastic auditing – is general, however, applying to any case in which it is reasonable to assume the agent's liability is limited. For example, the model fits the case of tax compliance. In this case, random variable X can be reinterpreted as the citizen's income, distributed on $[L, H]$ according to cdf F; and K can be reinterpreted as the government's revenue requirement. (If the government has no revenue requirement, auditing costs can be minimized trivially by setting a tax of zero.) The objective of the optimal tax policy would be the same as in (8): to minimize expected auditing costs.

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