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Loan Commitments and the Debt Overhang Problem

Christopher M. Snyder*

Abstract

The debt overhang problem is shown to arise in the context of an entrepreneurial project that requires a sequence of investments financed by an outside lender. The entrepreneur, not internalizing losses accruing to the lender which financed the initial investments, may inefficiently cancel the project and instead pursue an outside opportunity. It is shown that loan commitments (contracts that allow the entrepreneur to borrow a variable amount at a set interest rate in return for a fixed fee) are the optimal financial contracts in this setting, strictly dominating standard debt. The existence of the fixed fee allows loan commitments to set a relatively low interest rate, improving the entrepreneur's incentives to continue the project. The paper specifies the optimal contract fully, derives robust comparative statics properties (using an extension of Milgrom and Roberts (1994)), and extends the results to more realistic settings (e.g., allowing the market risk-free rate to be stochastic).

I. Introduction

The debt overhang problem, originally posed by Myers (1977), is associated with entrepreneurial projects requiring a sequence of investments. After the initial investments are sunk, financed, for instance, by debt, the interests of the entrepreneur and debt holder begin to diverge. In particular, the entrepreneur does not internalize the losses that accrue to the debt holder if later investments are not made and the project lapses, failing to provide a return with which to repay the debt. The better the entrepreneur's opportunities outside the project, the greater the divergence in the parties' interests. Unless some inducement is given to the entrepreneur to internalize the debt holder's welfare (for example, through renegotiation of the initial debt contract), his investment in later stages of the project may be suboptimal.

In the paper, I argue that loan commitments—contracts specifying a loan maximum, a fixed fee, and an interest rate—are more efficient than standard debt in the presence of the debt overhang problem. The essential beneficial feature of a loan commitment is its fixed fee, a payment from the entrepreneur to the lender that is independent of the amount borrowed. With a fixed fee, a loan

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commitment can provide the lender with the same expected return but at a lower interest rate than a standard debt contract. Since the interest rate determines the entrepreneur's incentives to invest at the margin, the underinvestment associated with the debt overhang problem is ameliorated by the relatively low interest rates associated with the loan commitment.

In the model presented in Section II, the entrepreneur can undertake a project requiring two stages of investment, I_0 and I_1 , before producing a return. Although I_0 is fixed at the outset, I_1 is a random variable, the realization of which is observed only by the entrepreneur and not by outside lenders. The return generated by the project is costlessly verifiable; the entrepreneur's other assets are assumed to be unverifiable.¹ Debt overhang becomes a problem when the realization of I_1 is high, implying that the entrepreneur needs to borrow a great deal to continue the project. With standard debt contracts, the repayment to the lender is proportional to the amount borrowed. Consequently, the repayment may be so high that the entrepreneur decides to abandon the project and pursue his outside opportunity instead. Since the entrepreneur does not internalize the welfare of the initial debt holder (the financier of I_0) in making this decision, he abandons the project too often relative to the social optimum. Loan commitments, by contrast, can partially sever the link between repayment and amount borrowed by introducing a fixed fee. Repayment in high- I_1 states can be reduced, improving the entrepreneur's incentives to continue the project, with the lender compensated by increased repayment in low- I_1 states (states in which the entrepreneur has more than adequate incentives to continue the project).

Not only are loan commitments shown to be strictly more efficient than standard debt, but also loan commitments are shown to be optimal financial contracts in the model. These are striking results in view of the relative simplicity of loan commitments compared to general contractual forms, providing a rationale for the widespread use of loan commitments in practice.² At one extreme, there are cases in which the first best (the expected return realized if the entrepreneur self-finances) can be obtained with loan commitments. At the other extreme, there are cases in which credit rationing is observed, i.e., there exists no financial contract that can be used to finance investment even though the project's net present value is positive in expectation. The conditions under which credit rationing is observed are characterized: intuitively, credit rationing is more likely the higher is the initial sunk investment I_0 relative to the project's expected return.

In cases in which loan commitments are successful in financing investment, this paper provides an analytic solution for the optimal loan commitment. The comparative static properties of this solution are explored using an extension of Milgrom and Roberts (1994). A number of empirically testable results emerge from the analysis: the greater the required initial investment, the lower is the contractual interest rate specified by the loan commitment, the higher is the contractual fixed fee, and the lower is the expected draw down of the loan

¹Under these plausible verifiability assumptions, simple solutions to the debt overhang problem such as issuing equity or renegotiating debt contracts are impractical; see Section V.

²Loan commitments accounted for three quarters of commercial lending in the U.S. in 1989; see Duca and VanHoose (1990).

commitment (where the draw down is the amount borrowed after the initial investment is sunk). The same is true for increases in the market risk-free rate. The contractual interest rate is likely to be positively correlated with the expected draw down. This correlation is not due to upward-sloping loan demand but instead to the variables' being simultaneously determined. The contractual fixed fee should be negatively correlated with the contractual interest rate and with the expected draw down.

The model is tractable enough to admit a number of interesting extensions. In Section VI, the market risk-free rate of return is allowed to be stochastic. The optimal loan commitment ties the contractual interest rate to realizations of the risk-free rate and specifies a fixed fee that tends not to vary with fluctuations in the risk-free rate. Loan commitments used in practice have this form: for example, note issuance facilities (NIFs), used extensively on international capital markets for medium-term financing.

Given the widespread use of loan commitments in practice, it is not surprising that there is a literature providing an economic rationale for the contractual form, in particular as a solution to an informational problem on the capital market.³ Broadly speaking, there are two strands of the literature pointing to two beneficial properties of loan commitments: first, unlike standard debt, they are long-term rather than spot contracts; second, they can specify a lower interest rate than standard debt because of the fixed fee. The first strand of the literature includes Thakor's (1989) model of adverse selection in which long-term contracts such as loan commitments can sort among entrepreneur types at an early stage when incentive compatibility constraints are relatively weak. In Houston and Venkataraman (1994), the lender may obtain inside information about the quality of the project as it progresses, perhaps allowing it to expropriate the entrepreneur's effort investment if refinancing is required. A long-term contract resembling a loan commitment can safeguard the entrepreneur's investment against expropriation.⁴ The present paper is in the second strand of the literature, which also includes Boot, Thakor, and Udell (1987), (1991), Morgan (1993), and Shockley (1995). In Boot, Thakor, and Udell, there is no sequential investment or debt overhang. By introducing a fixed fee paid *ex ante*, loan commitments can reduce the entrepreneur's *ex post* interest payment and increase his effort incentives. In Morgan's costly state verification model, introducing a fixed fee reduces the dependence of the repayment on the state of the world, in turn reducing auditing costs since auditing costs are convex in the repayment level. In Shockley's model, the entrepreneur has sufficient internal resources to finance investment but may seek debt financing to take advantage of the tax deductibility of interest. In contrast to the present paper, the presence of a material adverse change clause (allowing the lender to cancel the contract if its signal of the project's return is unfavorable) may relax the over-borrowing constraint.

³For a more extensive review of the literature on loan commitments, see Bhattacharya and Thakor (1993).

⁴In Petersen and Rajan (1995), by contrast, the fact that a monopolistic creditor can expropriate a borrower's surplus in later periods has a social benefit—allowing it to lend at subsidized rates in early periods.

The most closely related paper is Berkovitch and Greenbaum (1991), which focuses directly on the role of loan commitments in the presence of the debt overhang problem. Several difficulties arise with their analysis that cause some of the intuitive conclusions of the present paper to be missed. The major difficulty is that the authors' formulation of the under-borrowing constraint is overly strong. The effect of the usage fee, a charge for borrowing less than the contractually specified loan maximum, is omitted. Omitting the effect of the usage fee understates the true cost of under-borrowing. The overly strong constraint in Berkovitch and Greenbaum turns out to bind, suggesting that the beneficial properties of loan commitments come from setting high interest rates, directly contrasting the intuition of the second strand of the loan commitment literature discussed in the previous paragraph.⁵ The present paper also advances the literature by deriving explicit expressions for optimal contracts, fully exploring the comparative static properties of these contracts, and considering issues such as credit rationing and stochastic interest rates.

II. Model

The basic set up of the model is due to Berkovitch and Greenbaum (1991). An entrepreneur has the opportunity to undertake a project requiring a sequence of sunk investments before generating a return. In period 0, the project requires investment $I_0 > 0$. In period 1, the project requires another investment, I_1 . The value of I_1 is stochastic, distributed on $[\underline{I}, \bar{I}]$ according to the cumulative distribution function F . Let f be the strictly positive density function associated with F .

The sequential nature of investment will turn out to be crucial in the analysis: if there is only one investment stage, then the entrepreneur can obtain the first best (i.e., his level of surplus given sufficient internal funds to self-finance) with external financing using an appropriately designed financial contract. The debt overhang problem only arises if investment occurs in several stages: then it may be impossible to obtain the first best with external financing; even stronger, the entrepreneur may not be able to obtain any external financing at all.

If both investments are made, the project produces a return in period 2. With probability $q \in (0, 1)$, the return is $R > 0$. With probability $1 - q$, the return is zero. If the entrepreneur does not make the investment in either period 0 or period 1, the project lapses. In this event, the entrepreneur can turn to his best opportunity outside of the project and earn $N > 0$, his opportunity wage.⁶ The entrepreneur is assumed to be risk neutral.

⁵A second difficulty is that the authors omit a term in the entrepreneur's objective function measuring the option value of his being able to cancel the project and pursue his outside opportunity. Omitting this term leads to an overstatement of the costs of debt overhang and leads to an inconsistency between the author's proposed optimum and the maximizer of their proposed objective function. Third, Berkovitch and Greenbaum state that loan commitments can be improved upon by more general contracts, contradicted by the proposition below that loan commitments are optimal. The discrepancy is due to the authors' omission of an essential incentive compatibility constraint (see condition (17) below).

⁶The entrepreneur's opportunity wages in period 0 and in period 2 are normalized to be zero without loss of generality.

Suppose that there is no discounting between periods 0 and 1, i.e., normalize the risk-free market interest rate in period 0 to be zero. Between periods 1 and 2, there is discounting. Denote the risk-free market interest rate in period 1 by r . To simplify notation, let $\delta \equiv 1/(1+r)$ be the associated discount factor.

A. First-Best Benchmark

Consider the case, referred to as the first-best benchmark, in which the entrepreneur has sufficient internal funds to finance the project himself. In period 0, the entrepreneur sinks I_0 . In period 1, his optimal investment decision depends on the realization of I_1 . He invests if and only if the expected net present value of investing, $\delta qR - I_1$, exceeds his opportunity wage N . Thus, there exists a critical level of investment, $I^* \equiv \delta qR - N$, such that he invests if $I_1 \leq I^*$ and not if $I_1 > I^*$. Net of his opportunity wage, the entrepreneur's expected profit from undertaking the project in the first-best benchmark is

$$(1) \quad V \equiv \int_{I^*}^{\bar{I}} (\delta qR - I_1) dF(I_1) + \int_{I^*}^{\bar{I}} N dF(I_1) - I_0 - N.$$

Substituting $\delta qR = I^* + N$ into (1) and simplifying yields

$$(2) \quad V = \int_{I^*}^{\bar{I}} (I^* - I_1) dF(I_1) - I_0.$$

The condition under which the entrepreneur undertakes the project in period 0 in the first-best benchmark can be stated succinctly in terms of V : the entrepreneur sinks I_0 if and only if $V \geq 0$.

The second term on the right-hand side of (1), reflecting an option value generated by the sequential nature of investment, requires some discussion.⁷ Denote this term by $\sigma(I^*)$, where, in general, $\sigma(\bar{I}) \equiv \int_{\bar{I}}^{\bar{I}} N dF(I_1)$. In period 1, the entrepreneur has the option to cancel the project if I_1 is too high. By canceling the project, he avoids additional investment costs (since only I_0 is sunk, not I_1) and earns opportunity wage N . If the entrepreneur had to commit to sinking I_1 before learning its value, this strictly positive option value would be lost. Though the ability of the entrepreneur to cancel the project provides a positive value in the first-best benchmark, this ability has an associated cost in the case in which the entrepreneur needs external financing: the entrepreneur makes the decision to cancel the project without regard to the welfare of the lender, generating the debt overhang problem.

B. External Financing Required

The bulk of the analysis will treat the case in which the entrepreneur's internal funds are limited, so he has to resort to an outside lender to fund investment. For concreteness, the lender is called the *bank*. The entrepreneur and

⁷Berkovitch and Greenbaum (1991) omit this term from the objective function. Without this term, the optimal critical value of I_1 would be δqR rather than $\delta qR - N$, but then it would be inconsistent to define $I^* = \delta qR - N$.

the bank will sometimes be referred to collectively as the *venture*. The bank is assumed to be risk neutral. It operates in a competitive lending sector, modeled by assuming that it accepts any contract from which the present value of its earnings are non-negative in expectation. At any point in the rest of the game, other lenders in the competitive lending sector stand ready to accept any contract offered by the entrepreneur giving them non-negative expected profit.

The timing of the game between the entrepreneur and bank is presented schematically in Figure 1. The entrepreneur's level of internal funds is normalized to zero, implying that the project cannot be undertaken unless he signs a financial contract with the lender to fund investment. In period 0, the entrepreneur makes an offer of a financial contract to the bank. If the contract is not accepted, the project lapses and the entrepreneur pursues his outside opportunity instead. If the contract is accepted, the entrepreneur borrows I_0 from the bank and invests it in the project in period 0. In period 1, another round of investment is required. The required level of investment, given by the value of random variable I_1 , is observed by the entrepreneur at the start of period 1 but not by the bank. The entrepreneur can choose whether or not to make the investment at the end of period 1. If the investment is not made, the entrepreneur earns his opportunity wage and the game ends. The opportunity wage is not verifiable and so cannot be extracted from the entrepreneur by the bank. If the investment is made, the project continues and produces a return in period 2 (R with probability q and 0 otherwise). The bank can costlessly verify up to R of the entrepreneur's period 2 assets (the assets could include funds borrowed from the bank in excess of that needed for the project in addition to the project's return); entrepreneur assets in excess of R cannot be verified and so cannot be extracted from the entrepreneur by the bank.⁸

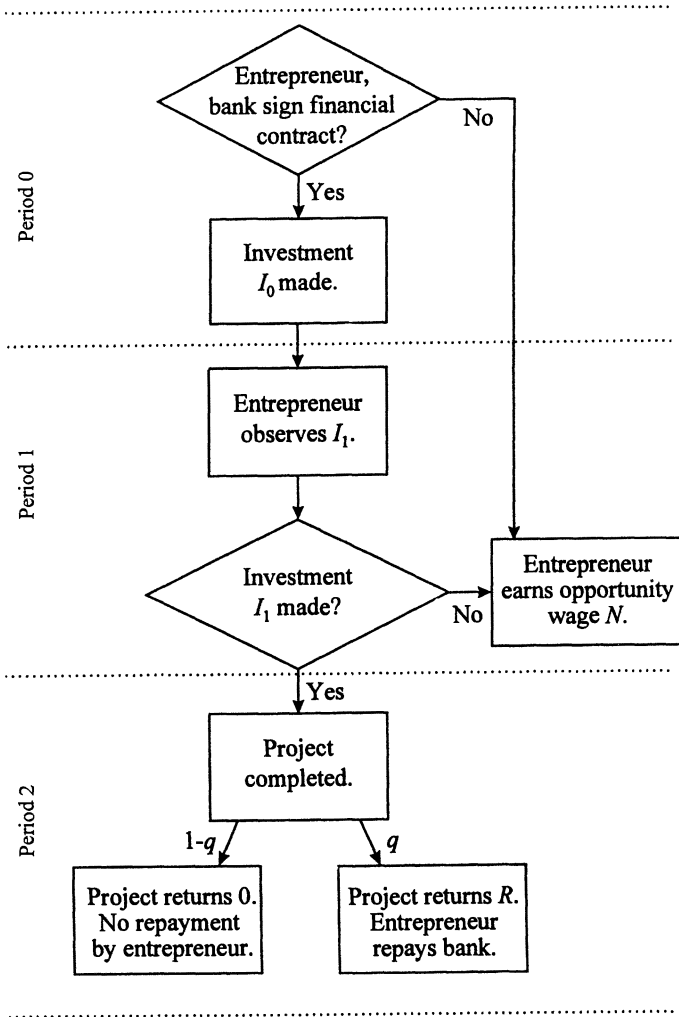
C. Loan Commitment Contracts

Attention is restricted to financial contracts in the class of *loan commitments*. The simple structure of loan commitments and the fact that they are used extensively in practice to finance investment argue for the study of loan commitments in their own right. As shown in Section V, the restriction of financial contracts to loan commitments is made without loss of generality since loan commitments are the optimal feasible contracts.

In the context of the model, the structure of a loan commitment is determined by three parameters, L , A , and i . The lender can borrow the funds necessary for investment in periods 0 and 1 up to the maximum total level specified by the contract, L . If the project is completed and earns a positive return, the bank is repaid a fixed fee A for administering the loan commitment, plus the total principal borrowed, plus interest on the principal, where the contractual interest rate is given by i . Since the investment requirement is private information for the

⁸The ability of the bank to verify the entrepreneur's period 2 assets can be thought of as a limiting case of convex auditing cost function. Townsend (1979) and Gale and Hellwig (1985) discuss the formalization of auditing (or bankruptcy) costs in a costly state verification model. Although Weiss (1990) provides empirical evidence that direct bankruptcy costs (e.g., fees for lawyers and investment bankers) are 3% of asset value on average, the indirect costs of bankruptcy (e.g., transfer of assets to less able managers) can be much larger (see Cutler and Summers (1988)) and are plausibly non-linear.

FIGURE 1
Timing of Game



entrepreneur, the amount he borrows in period 1 may depend on a message he sends to the bank. Attention is restricted to contracts that induce the entrepreneur to borrow exactly the amount needed to fund investment: I_0 in period 0 and I_1 in period 1. I show in Section V that this further restriction does not impair the performance of the contract.

Given a loan commitment with parameters L , A , and i , the entrepreneur's equilibrium decision to continue or cancel the project based on his observation of I_1 can be characterized. His expected surplus if he continues the project is given by $\delta q[R - A - (1 + i)(I_0 + I_1)]$. If he cancels, he receives N . Therefore,

defining the critical level of investment associated with the loan commitment as follows,

$$(3) \quad \widehat{I} = \left(\frac{1}{1+i} \right) \left(R - \frac{N}{\delta q} - A \right) - I_0,$$

the entrepreneur continues the project if $I_1 \leq \widehat{I}$ and cancels it if $I_1 > \widehat{I}$. Net of his opportunity wage, the expected present value of entrepreneur profit in equilibrium can be written,

$$(4) \quad \pi_e = \delta q \int_I^{\widehat{I}} [R - A - (1+i)(I_0 + I_1)] dF(I_1) + \int_{\widehat{I}}^{\bar{I}} N dF(I_1) - N.$$

The first term is the entrepreneur's profit if the project is completed and successfully returns R . The second term is the option value ($\sigma(\widehat{I})$ discussed above) of pursuing his outside opportunity if I_1 is so high that he cancels the project. The expected present value of the bank's net profit in equilibrium with the loan commitment is

$$(5) \quad \pi_b = \delta q \int_I^{\widehat{I}} [(1+i)(I_0 + I_1) + A] dF(I_1) - I_0 - \int_I^{\widehat{I}} I_1 dF(I_1).$$

The optimal loan commitment contract maximizes π_e subject to a participation constraint for the lender and incentive compatibility constraints for the entrepreneur. The competitive bank must be guaranteed a non-negative expected present value from the contract, implying that the participation constraint is

$$(6) \quad \pi_b \geq 0.$$

Incentive compatibility conditions are needed in the presence of the modeled informational asymmetry to ensure that the entrepreneur borrows exactly the amount needed to fund investment.

The incentive compatibility conditions come in two forms, over-borrowing and under-borrowing constraints. First, consider the over-borrowing constraints. Recalling the definition of \widehat{I} , if the true value of I_1 exceeded \widehat{I} , the entrepreneur would cancel the project. The total amount borrowed would only exceed $I_0 + \widehat{I}$ if the entrepreneur overstated his true investment requirements. Without loss of generality, then, the following condition can be imposed on incentive compatible loan commitments,

$$(7) \quad L \leq I_0 + \widehat{I}.$$

Condition (7) is not sufficient to prevent over-borrowing. If the interest rate is set too low, the entrepreneur may be inclined to overstate his true investment needs and invest the difference at the risk-free rate. Suppose that the entrepreneur borrows one dollar more than the required level of period 1 investment I_1 . With probability q , the project is successful in period 2, in which case the entrepreneur would earn an extra $1+r$ (one dollar invested at the risk-free rate) but would have

to repay the bank an additional $1+i$.⁹ Comparing marginal benefit and marginal cost and recalling the definition $\delta \equiv 1/(1+r)$, over-borrowing is prevented if and only if

$$(8) \quad 1+i \geq \frac{1}{\delta},$$

or, equivalently, $i \geq r$.

An under-borrowing constraint is required to prevent the entrepreneur from borrowing less than I_1 in period 1 from the bank with which he signed the loan commitment and borrowing the residual from some other lender. Suppose the entrepreneur replaces one dollar of borrowing from the loan commitment with one dollar borrowed from an outside lender. Let ρ be the nominal interest rate paid to outside lenders. Since outside lenders are assumed to be risk neutral and competitive, they would bid down ρ until the risk-adjusted return on the loan equaled the return from one dollar invested in the risk-free asset,

$$(9) \quad q(1+\rho) = 1+r.$$

In the event that the completed project is successful, borrowing one dollar less from the bank would save the entrepreneur $1+i$; borrowing one dollar from the outside lender would cost $1+\rho$. Substituting from (9) gives a condition guaranteeing the cost exceeds the savings, which is necessary and sufficient to prevent under-borrowing,¹⁰

$$(10) \quad 1+i \leq \frac{1}{\delta q}.$$

Contracts that satisfy constraints (6), (7), (8), and (10) will be called *feasible* loan commitments.

III. Optimal Loan Commitments

A loan commitment differs from a standard debt contract in two main ways. First, the loan commitment allows the level of financing to vary with I_1 , the investment requirement of the entrepreneur. This flexibility is useful if I_1 is not known at the time of contracting, for then the standard debt contract would need to set the size of the loan to be the maximum of the range of possible investment requirements (\bar{I} in the notation).

Second, the loan commitment can specify relatively low interest rates because of the existence of the fixed fee. This feature is central to the ability of loan

⁹Letting the amount of over-borrowing be represented more generally by $dI > 0$, the amount repaid to the bank would only increase if $A + (1+i)(I_0 + I_1 + dI) \leq R$. If $A + (1+i)(I_0 + I_1 + dI) > R$, the amount to be extracted from the entrepreneur would exceed the limit that the bank can verify, in which case, the amount repaid to the bank would not increase with over-borrowing but would remain a constant (R). Note, however, that condition (7) rules out the latter inequality.

¹⁰The analogous under-borrowing constraint in Berkovitch and Greenbaum (1991) is inconsistent with (10). If an omitted term (involving what is termed in their model the usage fee, α) is correctly accounted for, the resulting condition can be seen to be consistent with (10). Ignoring the omitted term produces an overly strong constraint that binds at the optimum for some parameters; it is shown below that the correct constraint never binds.

commitments to ameliorate the debt overhang problem. The debt overhang problem arises from the sequential nature of investment. In the period 1 investment stage, the entrepreneur does not regard the period 0 investment as sunk since he can cancel the project, take his outside opportunity instead, and avoid repaying the bank for the period 0 investment. The existence of the period 0 investment—sunk by the bank but not regarded as sunk by the entrepreneur—implies that the entrepreneur has suboptimal incentives to invest in period 1 at the margin. Loan commitments can improve the entrepreneur's incentives by lowering the interest rate—the effective marginal cost of borrowing—and compensating the bank for the reduced interest receipts with a positive fixed fee.

Collecting the feasibility constraints outlined in the previous section, the program generating the optimal loan commitment can be expressed as

$$(11) \quad \max_{L,A,i} \pi_e \quad \text{subject to} \quad (3), (6), (7), (8), \text{ and } (10).$$

A series of lemmas will help simplify (11). Proofs of the lemmas and all subsequent propositions are provided in the Appendix.

Lemma 1. At the optimum, the bank's participation constraint (6) binds; i.e., $\pi_b = 0$.

The proof of Lemma 1 can be seen intuitively. If $\pi_b > 0$, the entrepreneur can always increase π_e by lowering the fixed fee A .

Setting the expression for π_b in (5) equal to 0, substituting into the expression for π_e in (4), substituting $I^* = \delta qR - N$, and rearranging, the objective function can be written

$$(12) \quad \pi_e = \int_I^{\hat{I}} (I^* - I_1) dF(I_1) - I_0.$$

The particular form of the objective function in (12) makes it clear that the debt overhang problem is a problem of underinvestment in period 1. It is immediate that π_e is maximized for $\hat{I} = I^*$. If $\hat{I} = I^*$ then, as can be seen from a comparison of (12) with (2), the first best is attained with a loan commitment. The entrepreneur's surplus using a loan commitment π_e falls short of the first-best surplus level V to the extent that \hat{I} , the critical level of period 1 investment, falls short of I^* , the critical level of investment in the first best.

As discussed above, loan commitments enhance the incentives to invest in period 1 by subsidizing a reduction in the marginal cost of borrowing, compensated by charging a fixed fee. Intuitively, then, loan commitments should specify the lowest interest rate possible (subject to relevant constraints); so an upper bound on interest rates, such as that specified by (10), should not constrain the solution. This intuition is borne out by Lemma 2.

Lemma 2. The under-borrowing constraint (10) does not bind at an optimum.

The remaining constraints are those preventing over-borrowing: (7) and (8). Increasing the interest rate is a deterrent to over-borrowing only if the associated increase in the repayment required from the entrepreneur can be successfully

extracted from him. Since the bank can verify no more than R of his assets and, therefore, can extract no more than R from him in period 2, the entrepreneur could pursue a strategy of borrowing an unbounded amount (\bar{I} , if \bar{I} is finite), paying the bank the maximum possible that can be verified, and consuming the remainder. By placing a limit L on the total amount borrowed, (7) prevents this strategy; indeed, preventing this strategy is the *only* function of L in the contract. The entrepreneur's surplus is clearly maximized when (7) holds as an equality. A lower value of L would constrain the period 1 level of investment to be strictly below \hat{I} ; since the expression for π_e in (12) is increasing in its upper limit of integration in the relevant range, this constraint would reduce π_e . Summarizing this discussion:

Lemma 3. An optimum for the borrowing limit on the loan commitment is $L^* = I_0 + \hat{I}$.

The form of the optimal loan commitment depends crucially on the remaining constraint, (8). As discussed above, loan commitments address the debt overhang problem by reducing the contractual interest rate to enhance the entrepreneur's period 1 incentives to invest. For some parameter constellations, he can be given sufficient incentives for the first-best level of investment with an interest rate above r . For the remaining parameter constellations, even if i is set at r (the lowest possible level subject to (8)), he has suboptimal incentives to invest in period 1. For these remaining parameter constellations, it is impossible to obtain the first best; the second best involves setting i as low as possible subject to (8), i.e., $i = r$. The following three propositions characterize the conditions under which the first best can be obtained and calculates the optimal interest rate i^* and fixed fee A^* .

Proposition 1. The first best can be obtained using loan commitment contracts if and only if

$$(13) \quad q \leq \frac{V}{I_0 + V}.$$

Proposition 2. Suppose (13) holds. The optimal loan commitment specifies $L^* = I_0 + I^*$,

$$1 + i^* = \frac{1}{\delta q} \left(\frac{V}{I_0 + V} \right),$$

$$\text{and } A^* = \frac{I_0}{\delta q} \left(\frac{I^* - V}{I_0 + V} \right).$$

Proposition 3. Suppose (13) does not hold. If a feasible loan commitment exists, the optimal loan commitment specifies $L^* = I_0 + \hat{I}$, $1 + i^* = 1/\delta$, and

$$A^* = \frac{1}{\delta q F(\hat{I})} \left\{ [1 - qF(\hat{I})] I_0 + (1 - q) \int_{\hat{I}}^{\bar{I}} I_1 dF(I_1) \right\},$$

for $\hat{I} = I^*/q - \delta A^* - I_0$.

The existence question raised in Proposition 3 will be deferred to the end of the section.

A. Comparative Statics

By virtue of having explicit solutions for the optimal loan commitment provided by Propositions 2 and 3, it is possible to analyze the effect of changes in the structural variables on equilibrium in the model. The generality (e.g., independence of second-order conditions) of the subsequent comparative statics results derives from the application of Theorem 1, contained in the Appendix.¹¹ To capture changes in the distribution of F on equilibrium, one additional structural variable, θ , will be introduced. Suppose that the distribution function can be written $F(I_1, \theta)$, with $\theta \in \Theta \subseteq \mathbf{R}$. Suppose further that for all $\theta \in \Theta$, $\partial F(I_1, \theta)/\partial \theta \geq 0$, and $\partial F(\bar{I}, \theta)/\partial \theta = \partial F(\underline{I}, \theta)/\partial \theta = 0$. Hence, increases in θ improve the distribution of I_1 in the sense of first-order stochastic dominance: i.e., low values of I_1 are more likely the higher is θ . The list of structural variables for which comparative statics results will be considered includes I_0 , N , δ , q , R , and θ .

Table 1 summarizes the comparative statics results. Roughly speaking, the structural variables can be divided into two groups. For the first group, an increase in a structural variable exacerbates the debt overhang problem in the sense that the entrepreneur is more inclined to cancel the project in later stages and the entrepreneur's net surplus falls. Condition (13) is strengthened, implying that the first best is more difficult to obtain. To counteract the decline in the entrepreneur's investment incentives, the link between the amount he borrows and the amount he repays to the bank must be severed: this is done in the loan commitment by lowering the contractual interest rate and increasing the contractual fixed fee. For the second group, an increase in a structural variable ameliorates the debt overhang problem, improving the entrepreneur's incentives to invest in later stages of the project, increasing his net surplus, and allowing him to obtain the first best in a larger number of cases. The loan commitment used to finance the project can specify a higher interest rate and a lower fixed fee and still maintain appropriate investment incentives.

Consider first the structural variable I_0 , measuring the investment that must be sunk before the total investment requirement $I_0 + I_1$ is known. An examination of condition (13) shows that the existence of this preliminary round of investment is central to the debt overhang problem. If $I_0 = 0$, then condition (13) reduces to $q \leq 1$, which is trivially true. Thus, if $I_0 = 0$, the entrepreneur can always obtain the first best and there is no debt overhang problem. Increases in I_0 exacerbate the debt overhang problem.

An increase in N , the entrepreneur's surplus from pursuing his outside opportunity, also exacerbates the debt overhang problem. Faced with the decision of continuing the project or canceling it and pursuing his outside opportunity,

¹¹Theorem 1 extends work on robust comparative statics of Milgrom and Roberts (1994) to the case in which the function in question may be negative at both endpoints, implying that the existence of a zero of the function is not guaranteed.

TABLE 1
Summary of Basic Comparative Statics Results

Parameter Increased	First-Best Attainment	Critical Investment Level \hat{I}	Entrepreneur Net Profit π_e	Relative Contractual Interest Rate $\delta(1+i^*)$	Contractual Fixed Fee A^*
I_0	(13) stronger	(-)	(-)	(-)	(+)
N	(13) stronger	(-)	(-)	(-)	(+)
R	(13) weaker	(+)	(+)	(+)	(-)
θ	(13) weaker	(+)	(+)	(+)	(-)
δ	(13) weaker	(+)	(+)	(+)	(-)
q	ambiguous	(+)	(+)	(+) or (-)	(-)

In the column for first-best attainment, strengthening (weakening) (13) means the first best is more difficult (easier) to obtain. In the last four columns, a positive (negative) sign reflects a weak increase (decrease).

the entrepreneur is more inclined to cancel it the more valuable is the outside opportunity.

The fact that an increase in the entrepreneur's outside opportunity exacerbates the debt overhang problem implies that π_e , his net surplus, is declining in N . This is not surprising since net surplus involves the subtraction of N . An interesting question regards whether the entrepreneur's gross surplus, $\pi_e + N$, is declining in N as well. There are two offsetting effects. The positive effect of an increase in N is to increase the entrepreneur's earnings if the project is canceled. The negative effect is to reduce the entrepreneur's incentive to continue the project. It can be shown that the latter effect can outweigh the former so that $\partial(\pi_e + N)/\partial N < 0$ for some parameters.¹² This result is interesting because it implies the entrepreneur may benefit ex ante from taking measures that reduce the value of his outside option. Such measures may include destruction of human or physical capital. Less extreme measures may involve excessive investment (from an ex post view) in sunk assets specific to the project, i.e., assets more valuable in connection with the project than in outside opportunities.

Increases in δ (the market risk-free discount factor) and R (the project's return conditional on success) ameliorate the debt overhang problem. The higher are these variables, the greater is the expected present value of the project's return (δqR) relative to the cost of investment, improving the entrepreneur's investment incentives.¹³ An increase in θ , which is equivalent to a decrease in the expected investment required after the initial stage, also ameliorates the debt overhang problem. If only a small investment is required after the initial stage,

¹²To see this, note

$$\frac{\partial(\pi_e + N)}{\partial N} = 1 - F(\hat{I}) - \frac{(I^* - \hat{I})f(\hat{I})F(\hat{I})}{\delta[qF(\hat{I}) - f(\hat{I})(I^* - \hat{I})]}$$

Assuming the functional forms and parameter values in Section IV along with $q=0.95$, this derivative is negative.

¹³The effect of δ on the contractual interest rate is unambiguously positive with respect to the relative interest rate $\delta(1+i^*)$ rather than i^* itself.

the entrepreneur has little reason not to continue the project, and so the debt overhang problem vanishes.

The structural variable q , which can be interpreted as the conditional probability of project success (conditional on having made the required investments in the project), does not fall neatly into one of the two categories. An increase in this conditional probability increases the project's expected return and, therefore, increases the entrepreneur's investment incentives and surplus. However, the first-best benchmark also increases with q , and at an increasing rate;¹⁴ so the first best may be impossible to obtain for values of q near one. Indeed, in the example in Section IV, the first best is not attainable for either very low or very high values of q . As demonstrated by the example in Section IV, the effect of q on the interest rate specified by the loan commitment is ambiguous; however, it can be shown that the fixed fee falls as q increases.

The results in Table 1 suggest that the theory can be tested by regressing endogenous variables \widehat{I} , π_e , $\delta(1+i^*)$, and A^* on structural variables I_0 , N , R , θ , δ , and q using data from a cross section of loan commitments. In principal, the initial loan amount I_0 , risk-free discount factor δ , return R , and contractual interest rate i^* and fixed fee A^* are readily observable. Though the cutoff level of investment, \widehat{I} , is not directly observable, it is proportional to the expected draw down of the loan commitment, where *draw down* is the amount borrowed after the initial investment I_0 is sunk. Formally, the expected draw down is $\int_I^J I_1 dF(I_1)$. Other variables may be difficult to observe, especially the value of the entrepreneur's outside opportunity, N .¹⁵

In the absence of good proxies for all the structural variables, the proposed regressions may suffer from omitted variable bias. However, the model's most interesting empirical implications regard the reduced form relationships among the endogenous variables, implications that could be tested without information on the structural variables and, thus, would be immune to omitted variable bias. Comparing the third and fourth columns of Table 1 suggests that the contractual interest rate may be negatively correlated with the contractual fixed fee.¹⁶ Comparing the first and third columns of Table 1 suggests that the contractual interest rate may be positively correlated with the expected draw down; that is, the entrepreneur should borrow more in later stages of the project the higher is the interest rate.¹⁷ This positive correlation is not due to the entrepreneur's having a positively sloped loan demand curve. Rather, the two variables respond similarly to movements in underlying structural variables. For example, the group of structural variables that exacerbate the debt overhang problem tend

¹⁴Differentiating the expression for V in (12) yields $\partial^2 V / \partial q^2 = (\delta R)^2 f(I^*) > 0$.

¹⁵One possible proxy for N is the specificity of the entrepreneur's human capital investment in the project (the two should be inversely related). See Anderson and Schmittlein (1984) for one attempt to measure human capital specificity empirically.

¹⁶The theory is consistent with, but does not necessitate, a negative correlation between the contractual interest rate and fixed fee: an increase in q may decrease both $\delta(1+i^*)$ and A^* .

¹⁷The theory is consistent with, but does not necessitate, a positive correlation between the contractual interest rate and expected draw down $\int_I^J I_1 dF(I_1, \theta)$: an increase in q increases \widehat{I} but may decrease $\delta(1+i^*)$; further, an increase in θ increases $\delta(1+i^*)$ but may decrease $\int_I^J I_1 dF(I_1, \theta)$ because of the effect of θ on the distribution F .

to reduce the entrepreneur’s investment incentives and, thus, his expected draw down of the loan commitment. To counter his reduced investment incentives, the contract specifies a lower interest rate and a higher fixed fee, though these contractual measures cannot fully offset the reduction in investment incentives, so the expected draw down still falls. The group of structural variables that ameliorate the debt overhang problem tend to cause the expected draw down and the contractual interest rate to rise and the fixed fee to fall. For either group of structural variables, the draw down and contractual interest rate tend to move in the same direction and in the opposite direction from the fixed fee.

B. Comparison to Standard Debt

This subsection highlights the beneficial properties of loan commitments by comparing them to an alternative form of financing: a sequence of standard debt contracts. A sequence of standard debt, abbreviated SSD, has the following structure: in period 0, the bank lends the entrepreneur I_0 at interest rate i_0 . In period 1, the bank lends the entrepreneur I_1 at interest rate i_1 . In period 2, if the project is successful, the bank is repaid $(1 + i_0)I_0$ from the first standard debt contract and $(1 + i_1)I_1$ from the second. Define π_b^0 and π_b^1 to be the bank’s expected net profit from the period 0 standard debt contract and from the period 1 standard debt contract, respectively. The bank’s individual rationality constraint can then be written $\pi_b^0 \geq 0$ and $\pi_b^1 \geq 0$.¹⁸

It is a straightforward exercise to characterize fully the optimal SSD.

Proposition 4. A feasible SSD exists if and only if the equation,

$$(14) \quad I_0 = (I^* - I^s)F(I^s),$$

has a solution for I^s . The greatest value of I^s solving (14) is the critical investment level associated with the optimal SSD: the entrepreneur continues the project if and only if $I_1 \leq I^s$. The optimal interest rates i_0^s and i_1^s are given by $1 + i_0^s = 1/[\delta q F(I^s)]$ and $1 + i_1^s = 1/(\delta q)$. The entrepreneur’s profit (net of the opportunity wage N) is

$$(15) \quad \pi_e^s = \int_{I_1}^{I^s} (I^* - I_1) dF(I_1) - I_0.$$

As with loan commitments, with SSDs a variable of interest is the critical level of investment (denoted I^s in the case of SSDs). The objective of the optimal SSD is to give the entrepreneur appropriate incentives to raise I^s as close as possible to the first-best level, I^* . As can be seen from (14), $I^s < I^*$; so the first best cannot be obtained with SSDs. The fact that the bank needs to

¹⁸Note first that a necessary condition for the bank to participate is $\pi_b^0 + \pi_b^1 \geq 0$. Suppose, in addition, that $\pi_b^0 > 0$ and $\pi_b^1 < 0$. Then the bank would gain from refusing to sign the period 1 standard debt contract, forcing the entrepreneur to resort to another lender. Suppose instead that $\pi_b^0 < 0$ and $\pi_b^1 > 0$. Then, the entrepreneur would gain from signing a period 1 standard debt contract with another lender. Therefore, the individual rationality constraint must specify $\pi_b^0 \geq 0$ and $\pi_b^1 \geq 0$.

be repaid for funding period 0 investment reduces the entrepreneur’s incentives to continue the project in period 1. Comparing expressions (12) and (15), it is immediate that one form of financing is more efficient than the other if and only if it specifies a higher critical level of investment. In fact, it is intuitively clear from an examination of the interest rates associated with the financial contracts that $\hat{I} > I^*$; and so loan commitments are more efficient than SSDs. Recall the previous argument that the interest rate determines the entrepreneur’s marginal incentive to invest in period 1. But the period 1 interest rate with the optimal SSD is $i_1^* = 1/(\delta q)$ (see Proposition 4) and with the optimal loan commitment is $i^* < 1/(\delta q)$ (see Propositions 2 and 3). Consequently, the marginal investment incentives are greater with the optimal loan commitment than with SSDs. Formally,

Proposition 5. Suppose there exists a feasible SSD. Then there exists a loan commitment that is strictly more efficient (i.e., provides the entrepreneur with strictly higher surplus) than the optimal SSD.

C. Credit Rationing

As suggested by Proposition 3, for some values of the parameters, there may exist no feasible loan commitment contract, i.e., the debt overhang problem leads to credit rationing in equilibrium. The fact that constraint (8) bounds feasible interest rates from below gives rise to credit rationing. High interest rates reduce the probability that the entrepreneur continues the project, thereby reducing the expected return from sinking the initial investment in period 0. If constraint (8) binds too tightly, the probability of continuing the project may be so low that the initial investment cost I_0 cannot be recouped.¹⁹

The following proposition characterizes the conditions under which feasible loan commitments exist.

Proposition 6. Define

$$(16) \quad H(x) \equiv (1 - q) \int_I^x (x - I_1) dF(I_1) + (I^* - x)F(x) - I_0.$$

There exists a feasible loan commitment contract if and only if $\{x \in [I, I^*] | H(x) \geq 0\} \neq \emptyset$. The set of parameters such that there exists no feasible loan commitment but $V > 0$ is non-empty. Holding parameters besides R constant, there exists $\tilde{R} \in \mathbf{R}^+$ such that feasible loan commitments exist if and only if $R \geq \tilde{R}$.

¹⁹The role played by interest rates in the present model is similar to the role played in Stiglitz and Weiss (1981). In both models, increasing the interest rate directly benefits the bank by increasing the repayment to the bank if the project is successful. In both, increasing the interest rate has an indirect cost to the bank, a cost that arises due to the existence of moral hazard on the part of the entrepreneur. In the present model, increasing the interest rate raises the probability that the entrepreneur cancels the project and takes his outside opportunity, in turn, reducing the probability that the bank is repaid. In Stiglitz and Weiss, increasing the interest rate induces the entrepreneur to undertake riskier projects, also leading to a reduction in the probability the bank is repaid. To alleviate the moral hazard problem in their model, the optimal contract involves credit rationing. In the present model, credit rationing in period 1 would merely reinforce the entrepreneur’s tendency to underinvest and so would be harmful. The bank may ration credit in period 0 by simply refusing to sign the loan commitment.

Holding parameters besides I_0 fixed, there exists $\tilde{I}_0 \in \mathbf{R}^+$ such that feasible loan commitments exist if and only if $I_0 \leq \tilde{I}_0$.

Although the first result (the necessary and sufficient condition for existence) is fairly abstract, it is quite useful: it can be used to prove a number of corollaries including the other three results in Proposition 6. To build more intuition for when the existence condition holds, Section IV provides sets of parameters that satisfy the condition and sets that violate the condition in an example with explicit functional forms.

Turning to the second result of the proposition, there are non-trivial cases in which no feasible loan commitment exists even though the project's expected net present value is positive. The entrepreneur would undertake such projects if he had sufficient internal funds, but cannot obtain external funds to finance them. The remaining results state that feasible contracts exist if and only if R is sufficiently high or I_0 is sufficiently low.

IV. Numerical Example

Consider a simple example that illustrates several interesting features of the optimal loan commitment. Let I_1 be distributed uniformly on $[0, 1]$; let $r = 0$, $R = 7/8$, $N = 1/8$ and $I_0 = 1/36$; and let q vary.²⁰

In this example, if the project is self-financed by the entrepreneur, it has positive net present value for $q > 0.41$. Loan commitments are not feasible unless $q > 0.47$. Thus, there exists an interval, $q \in (0.41, 0.47)$, in which the debt overhang problem leads to underinvestment in period 0. Figure 2 graphs several variables on the interval $q \in [0.47, 1.00]$. Each panel is divided into three regions. In the middle region of each panel, q is such that (13) is satisfied, implying the first best can be obtained using loan commitments.

Panel A graphs the optimal fixed fee and interest rate vs. q . The fixed fee declines for all q ; the interest rate at first increases, then decreases as project success becomes more certain. Interestingly, constraint (8) binds in two disjoint regions, for both low and high q .²¹ The picture for the critical investment level (Panel B) confirms that $\hat{I} < I^*$ when (8) binds but that $\hat{I} = I^*$ in the region in which (8) is slack (the region in which the first best is obtained in equilibrium). Notice that \hat{I} is increasing over the whole interval, so the entrepreneur prefers higher values of q even though this prevents him from obtaining the first best. This point is verified in Panel C: entrepreneur welfare is everywhere increasing.

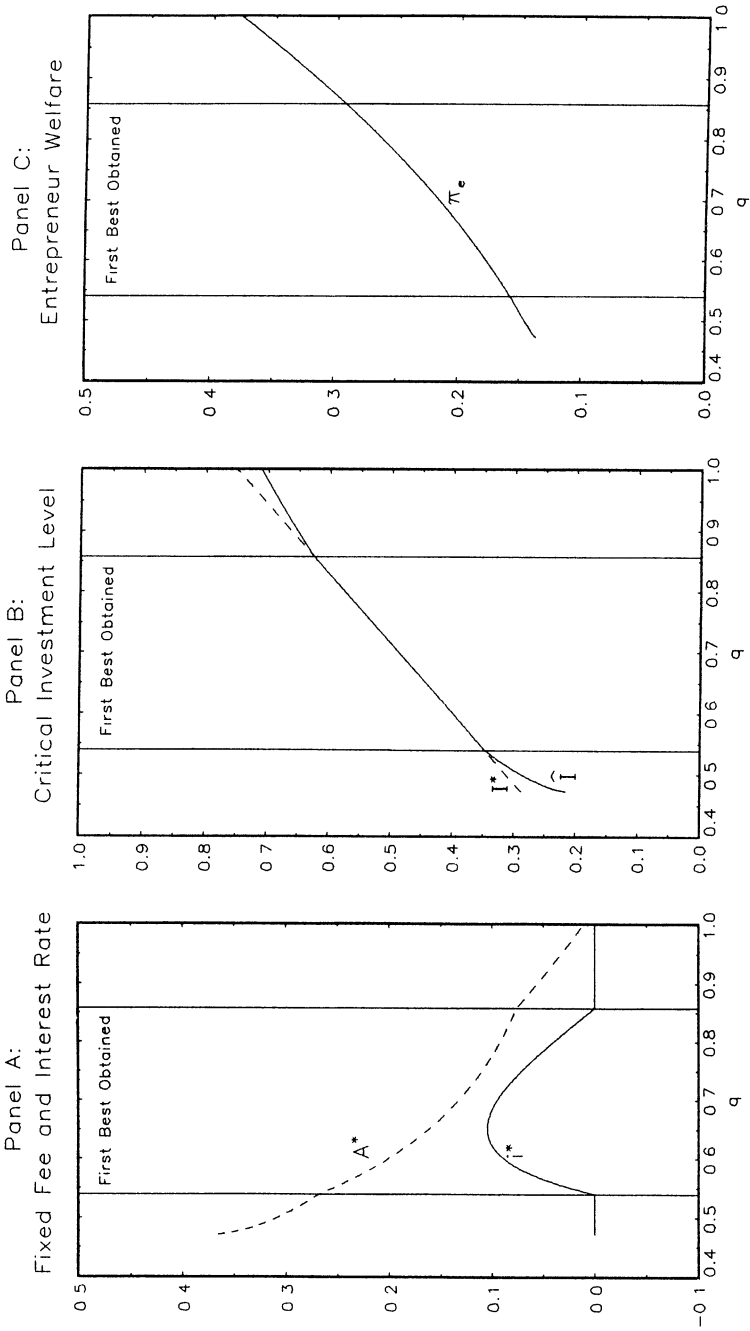
V. General Financial Contracts

This section justifies the restriction of attention to loan commitments by showing that loan commitments are in fact optimal financial contracts. By the Revelation Principle (Myerson (1983)), attention can be restricted to direct revelation mechanisms in which the entrepreneur is induced to make a truthful

²⁰All calculations are presented to an accuracy of two decimal places.

²¹Thus, I have a counterexample to Berkovitch and Greenbaum's (1991) claim that the first best can be obtained if q is close enough to one (see the discussion associated with their condition (12)).

FIGURE 2
Optimal Loan Commitment in a Numerical Example



report \tilde{I}_1 of his investment requirement, I_1 . In the context of the model, a general direct revelation mechanism can be specified by the functions T_0 , $T_1(\tilde{I}_1)$, $S_2(\tilde{I}_1)$, and $D_2(\tilde{I}_1, I_1)$. T_0 is the transfer from the bank to the entrepreneur in period 0. $T_1(\tilde{I}_1)$ is the transfer from the bank to the entrepreneur in period 1. $S_2(\tilde{I}_1)$ is the repayment from the entrepreneur to the bank in period 2 conditional on the project's success (i.e., the project's returning R) and on \tilde{I}_1 . $D_2(\tilde{I}_1, I_1)$ is the repayment from the entrepreneur to the bank in period 2 conditional on the project's failure (i.e., the project's returning 0). If the project is not successful, the bank can deduce the true value of I_1 by verifying the entrepreneur's period 2 assets.²² Thus, $D_2(\tilde{I}_1, I_1)$ can effectively be conditioned on the true value I_1 as well as the entrepreneur's announcement \tilde{I}_1 . If the project is successful, the bank is not able to verify the entrepreneur's residual assets over R ; so $S_2(\tilde{I}_1)$ cannot be conditioned on I_1 . The contract, being a direct revelation mechanism, will ensure truthful announcement, $\tilde{I}_1 = I_1$, in equilibrium.

First, note that there is no loss of generality in restricting the period 0 and period 1 transfers to equal the loan requirements in each period: $T_0 = I_0$ and $T_1(\tilde{I}_1) = \tilde{I}_1$. If $T_0 < I_0$ or $T_1(\tilde{I}_1) < \tilde{I}_1$, the project will not be completed. Rather than having $T_0 > I_0$ or $T_1(\tilde{I}_1) > \tilde{I}_1$, an equivalent outcome can be obtained by lowering the transfers in periods 0 and 1 so that $T_0 = I_0$ and $T_1(\tilde{I}_1) = \tilde{I}_1$ and concomitantly lowering the period 2 repayment $S_2(\tilde{I}_1)$ to make all parties indifferent between the contracts. Second, note that there is no loss of generality in having the bank extract all the entrepreneur's assets if the project fails, i.e., $D_2(\tilde{I}_1, I_1) = \max(0, \tilde{I}_1 - I_1)/\delta$. This specification punishes the entrepreneur maximally for overstating his true loan demand off the equilibrium path; along the equilibrium path, this specification relaxes the bank's individual rationality constraint to the greatest extent possible.

Define $C \subseteq [L, \bar{I}]$ to be the set of investment levels such that the entrepreneur continues the project if and only if $I_1 \in C$. Truth telling requires that $C = [L, \hat{I}]$ for some critical investment level, $\hat{I} = \sup C$. If, to the contrary, there exists $I'_1 \in (L, \hat{I})$ such that $I'_1 \notin C$, the entrepreneur can gain by reporting that its period 1 investment requirement is I'_1 for some $I''_1 \in C \cap (I'_1, I^*)$. Truth telling also requires that the entrepreneur cannot gain by overstating I_1 and investing the residual $\tilde{I}_1 - I_1$ at the risk-free rate,

$$\delta q[R - S_2(I_1)] \geq \delta q \left[R - S_2(\tilde{I}_1) + \frac{1}{\delta}(\tilde{I}_1 - I_1) \right],$$

for all $I_1 \in [L, \hat{I}]$ and $\tilde{I}_1 \in [I_1, \hat{I}]$. Rearranging and taking the limit as \tilde{I}_1 approaches I_1 from above implies

$$(17) \quad \check{S}_2(I_1) \geq \frac{1}{\delta} \quad \forall I_1 \in [L, \hat{I}],$$

where $\check{S}_2(I_1) \equiv \lim_{h \rightarrow 0^+} [S_2(I_1) - S_2(I_1 - h)]/h$. As a consequence of (17), $S_2(I_1)$ must be differentiable almost everywhere and increasing. The countable

²²Letting Y_2 be the verified assets of the entrepreneur in period 2, the true value of I_1 can be deduced by using the formula $I_1 = T_1(I_1) - \delta Y_2$.

discontinuities of $S_2(I_1)$ must be upward jumps, i.e., $\check{S}_2(I_1) = \infty$ at points of discontinuity.

To build some intuition for general financial contracts, note that they can equivalently be structured as loan commitments with variable interest rates $i(I_1)$. The variable interest rate can be computed by solving the identity $S_2(I_1) \equiv [1 + i(I_1)](I_0 + I_1)$ for $i(I_1)$. The truth telling constraint (17) can then be written $\check{i}(I_1)(I_0 + I_1) \geq r - i(I_1)$, where $\check{i}(I_1)$ is the right-hand limit analogous to $\check{S}_2(I_1)$. Simply bounding $i(I_1)$ is not sufficient to ensure truth telling; the rate of change of $i(I_1)$ must also be restricted.²³

Proposition 7 states that fixed interest rate loan commitments are optimal in the class of feasible contracts. It might seem intuitive that the additional flexibility of allowing $S_2(I_1)$ to be non-linear (or, equivalently, allowing $i(I_1)$ to be non-constant) should improve the efficiency of financial contracts. Recall, however, that the goal of financial contracts is to increase marginal investment incentives (the entrepreneur's incentives to continue the project for values of I_1 near I^*), thus ameliorating the underinvestment problem. In terms of general contractual forms, this goal is accomplished by having low values of $S_2(I_1)$ for I_1 near I^* . Low values of $S_2(I_1)$ can be obtained by having the function grow slowly with I_1 . Given constraint (17), the slowest $S_2(I_1)$ can grow is at rate $1/\delta$, implying that $S_2(I_1)$ is affine and that the overall contract is identical to a fixed interest rate loan commitment at an optimum.

Proposition 7. A fixed interest rate loan commitment is an optimal feasible contract.

In view of Proposition 7, the previous results derived above (Lemma 1 through Proposition 3) are far more general than originally stated, characterizing equilibrium with no restrictions on financial contracts other than feasibility.

The discussion in this section clearly identifies the beneficial properties of loan commitments. It may be less clear why other potential solutions to the debt overhang problem—contractual covenants requiring sequential investment in certain states, renegotiation of the initial financial contract, equity financing (see Myers (1977) for a discussion)—are inferior to loan commitments in the model. The crucial assumption is that I_1 is unobservable to all parties but the entrepreneur. This assumption implies that it is impossible to write a debt covenant requiring investment in certain I_1 states. This assumption implies that renegotiation would be harmful rather than beneficial, owing to the familiar result that incentive compatibility is more difficult to maintain in a regime with renegotiation than with commitment (see, e.g., Laffont and Tirole (1993), chapter 10). This assumption implies that equity financing of I_0 would be of limited value since the equity holders would not have the necessary information to vote for project continuation/cancelation in later stages of the project.²⁴

²³Berkovitch and Greenbaum's (1991) result that variable interest rate loan commitments improve on fixed rate ones (see their Proposition 5) is due to the fact that they neglect the constraint on $\check{i}(I_1)$.

²⁴Given that such a vote is impossible, it can be shown that equity financing of I_0 is identical to standard debt financing.

VI. Stochastic Risk-Free Rate

The model developed in Section II is tractable enough to admit various extensions. In the extension analyzed in this section, the risk-free interest rate r is allowed to be stochastic.²⁵ Specifically, let $\delta \equiv 1/(1+r)$ be distributed on $[\underline{\delta}, \bar{\delta}]$ according to distribution function $\Phi(\delta)$. In period 1, δ is realized. It is assumed that this realization is verifiable, so that contracts can be conditioned on δ . Loan commitments will be allowed to specify a borrowing limit $L(\delta)$, a fixed fee $A(\delta)$, and an interest rate $i(\delta)$, all functions of δ .²⁶

Defining the expectations operator $E_\delta x(\delta) \equiv \int_{\underline{\delta}}^{\bar{\delta}} x(\delta) d\Phi(\delta)$, the entrepreneur chooses $L(\delta)$, $A(\delta)$, and $i(\delta)$ to maximize $E_\delta \pi_e(\delta)$.²⁷ Following the logic of Section III, the crucial constraints are the individual rationality constraint,

$$(18) \quad E_\delta \pi_b(\delta) \geq 0,$$

and the over-borrowing constraint,

$$(19) \quad 1 + i(\delta) \geq \frac{1}{\delta} \quad \forall \delta \in [\underline{\delta}, \bar{\delta}].$$

It is straightforward to extend Proposition 1 to the stochastic δ case: the condition under which the first best $E_\delta V(\delta)$ can be obtained using loan commitments becomes

$$(20) \quad q \leq \frac{E_\delta V(\delta)}{E_\delta V(\delta) - I_0},$$

a natural generalization of condition (13).

The interesting question regards the structure of optimal loan commitments if (20) does not hold so that the first best cannot be obtained. One possible solution would be to specify the contract given in Propositions 2 or 3 for each δ , essentially treating each δ state as independent. For concreteness, this solution will be referred to as a *sequence of δ contracts*. A sequence of δ contracts can be improved upon by recognizing that the δ states are linked by constraint (18): the bank's profits must be non-negative, on average, across δ states rather than non-negative in each state. Intuitively, the bank can be subsidized with positive profit in the high δ states to compensate it for losses in the low δ states. This cross-subsidization can be accomplished by increasing the fixed fee in high δ states and decreasing the fixed fee in low δ states (where the changes in the fixed fee are measured relative to its level in a sequence of δ contracts). In states in which δ is so high that $\hat{I}(\delta)$ is near $I^*(\delta)$, increasing $A(\delta)$ only causes a second-order decrease in $\pi_e(\delta)$ but causes a first-order increase in $\pi_b(\delta)$. Consequently, slack can be introduced in constraint (18) with little loss to the entrepreneur.²⁸

²⁵I am grateful to a referee for suggesting this extension.

²⁶An alternative specification would constrain A to be independent of δ . Owing to the cross-subsidization result discussed below, $A(\delta)$ will tend not to vary with δ under either specification.

²⁷In this section, the terms π_e , π_b , V , I^* , and \hat{I} will be written as functions to stress their dependence on the stochastic state variable δ .

²⁸For an interpretation of optimal contracts as trading slack in the individual rationality constraint across states, see Maskin and Tirole (1990).

Proposition 8. Suppose δ is a random variable as modeled in the present section. Then the first best can be obtained using a loan commitment if and only if (20) holds. If (20) does not hold, the following statements are true for all $\delta \in [\underline{\delta}, \bar{\delta}]$. First, the optimal contract specifies $1 + i(\delta) = 1/\delta$. Second, $\widehat{l}(\delta) < l^*(\delta)$. Third, letting λ be the multiplier on constraint (18), $A(\delta)$ is increasing and $\widehat{l}(\delta)$ is decreasing in λ .

According to Proposition 8, if the first best cannot be obtained, the optimal contract sets the interest rate to equal the risk-free rate in each δ state. Furthermore, the more stringent is the bank's individual rationality constraint (due, for example, to an increase in the probability of low δ states), the more cross-subsidization between states. This is evidenced by the fact that for high δ states, the fixed fee specified by the optimal loan commitment is already greater than that specified by a sequence of δ contracts; the higher is λ , the greater the difference in the fixed fees since the fixed fee in a sequence of δ contracts is independent of λ . Table 1 implies that the fixed fee in a sequence of δ contracts is declining in δ . Since this negative correlation is counteracted by the cross-subsidization with the optimal loan commitment, the fixed fee will tend to vary less with δ in the optimal contract than in a sequence of δ contracts. For values of λ high enough, $A(\delta)$ will tend to be independent of δ in the optimal contract.

Many financial contracts used in practice have a similar structure, with a fixed fee independent of the size of the loan and an interest rate tied to the market rate. Note issuance facilities (NIFs) are used extensively on international capital markets to provide medium-term financing.²⁹ NIFs allow the borrower to request loans of variable sizes over time at an interest rate that closely approximates the market interest rate for a risk-free loan (ranging from the LIBOR to 50 basis points below the LIBID). The lender is paid a fee, which is often independent of the size of the amount borrowed, for underwriting and managing the NIF (Cross et al. 1986).

VII. Conclusion

The model developed in this paper is a realistic setting in which commonplace contracts—loan commitments—are optimal. The virtue of loan commitments is that they specify relatively low interest rates, improving the entrepreneur's marginal investment incentives, and compensate the lender with a fixed fee. The model is tractable enough to allow various extensions. In the extension to stochastic risk-free rates, the optimal contract closely resembles a note issuance facility (NIF). Other extensions are also possible: for example, the model could be extended to allow for multiple rounds of sequential investment (rather than just two as in the present model). Intuitively, increasing the number of investment stages, assuming these stages are stochastic, should exacerbate the debt overhang problem: a greater investment needs to be sunk before the total investment required for the project is realized. An offsetting effect is that the value of the entrepreneur's outside option may decline with time as he progresses

²⁹Approximately \$75 billion in NIFs were outstanding in Europe in 1985 (Cross et al. (1986)).

with the investment stages, increasing his incentive to continue the project. The net direction of the two effects is ambiguous.

The paper provides an important application of a new result on robust comparative statics (robust in the sense that they hold without reference to second-order conditions or other conditions usually assumed to guarantee uniqueness of a solution). Theorem 1 extends the work of Milgrom and Roberts (1994) to cases in which existence of a solution is not guaranteed such as is the case in the present model when credit rationing occurs. Since existence of a solution is often an issue in corporate finance models (and principal-agent models with informational asymmetries more generally), Theorem 1 can be applied in a wide range of cases.

Appendix

Theorem 1 is used in the proofs of several of the propositions.

Theorem 1. Let $g(x, t)$ be a function from $[\underline{x}, \bar{x}] \times T$ to \mathbf{R} , with $[\underline{x}, \bar{x}] \subset \mathbf{R}$ and $T \subseteq \mathbf{R}$. Define $x_H(t) \equiv \sup\{x \in [\underline{x}, \bar{x}] | g(x, t) \geq 0\}$. Let $t', t'' \in T$ be such that $t' < t''$. Then the following hold:

1. Suppose $\{x \in [\underline{x}, \bar{x}] | g(x, t') \geq 0\} \neq \emptyset$ and $g(x, t)$ is non-decreasing in t . Then $x_H(t') \leq x_H(t'')$.

2. Suppose $\{x \in [\underline{x}, \bar{x}] | g(x, t'') \geq 0\} \neq \emptyset$ and $g(x, t)$ is non-increasing in t . Then $x_H(t') \geq x_H(t'')$.

If in addition, $g(x, t)$ is continuous in x , $g(\bar{x}, t) < 0 \forall t \in T$, and $g(x, t)$ is strictly monotone in t (either strictly decreasing in the case of statement 1 or strictly increasing in the case of statement 2), the preceding inequalities regarding x_H are strict.

Proof. I will prove statement 1 of the theorem; statement 2 is proved analogously. Take any $y \in \{x \in [\underline{x}, \bar{x}] | g(x, t') \geq 0\}$. (Note that y exists since $\{x \in [\underline{x}, \bar{x}] | g(x, t') \geq 0\} \neq \emptyset$ by assumption.) Then $0 \leq g(y, t') \leq g(y, t'')$, where the second inequality follows from the assumption that $g(x, t)$ is non-decreasing in t . Thus, $\{x \in [\underline{x}, \bar{x}] | g(x, t') \geq 0\} \subseteq \{x \in [\underline{x}, \bar{x}] | g(x, t'') \geq 0\}$. Therefore, $x_H(t') \leq x_H(t'')$.

Next, it is proved that $x_H(t'') > x_H(t')$ assuming that $g(x, t)$ is continuous in x , $g(\bar{x}, t) < 0 \forall t \in T$, and $g(x, t)$ is strictly increasing in t . Since $g(x, t)$ is continuous in x and $g(\bar{x}, t') < 0$, $g(x_H(t'), t') = 0$. Now $g(x, t)$ is strictly increasing in t , so $g(x_H(t'), t'') > g(x_H(t'), t')$, implying $g(x_H(t'), t'') > 0$. By continuity, $\exists \epsilon > 0$, with $x_H(t') + \epsilon < \bar{x}$, such that $g(x_H(t') + \epsilon, t'') > 0$. Therefore, $x_H(t'') \geq x_H(t') + \epsilon > x_H(t')$.

Proof of Lemma 1. First, I show that $\partial \pi_e / \partial A < 0$. Differentiating expression (12) yields

$$\frac{\partial \pi_e}{\partial A} = (I^* - \widehat{I})f(\widehat{I}) \left(\frac{\partial \widehat{I}}{\partial A} \right).$$

In view of the expression for \widehat{I} in (3), $\partial \widehat{I} / \partial A = -A / (1+i) < 0$. Thus, $\partial \pi_e / \partial A < 0$.

Take any loan commitment with parameters L' , A' , and i' such that the associated surplus for the bank, π'_b , is strictly positive. Consider a new loan commitment with parameters $L'' = L'$, $i'' = i'$, and $A'' = A' + dA$, where $dA > 0$ is chosen small enough that the associated surplus for the bank, π''_b , is still strictly positive. The new contract satisfies all the feasibility constraints. Since $\partial\pi_e/\partial A < 0$, the entrepreneur's surplus is higher with the new contract than the original one, proving that any contract with $\pi_b > 0$ cannot be optimal.

Proof of Lemma 2. If constraint (8) binds, then the optimal interest rate i^* satisfies $1 + i^* = 1/\delta \leq 1/(\delta q)$, and the lemma is proved. Suppose then that (8) does not bind. I will show that the solution constrained only by (6) (the bank's participation constraint) satisfies (10).

In view of (12), π_e is maximized for $\widehat{I} = I^*$. The solution constrained only by (6) can be obtained by solving the system of equations $\widehat{I} = I^*$ and $\pi_b = 0$ simultaneously for A^* and i^* and setting $L^* = I_0 + I^*$. After some algebra,

$$(21) \quad 1 + i^* = \frac{1}{\delta q} \left[1 - \frac{I_0}{\int_{I^*}^{I^*} (I^* - I_1) dF(I_1)} \right].$$

Thus, $1 + i^* \leq 1/(\delta q)$.

Proof of Proposition 1. (\implies) Suppose the first best can be obtained using loan commitments. Then i^* and A^* must satisfy $\widehat{I} = I^*$. By Lemma 1, i^* and A^* must also satisfy $\pi_b = 0$. Solving $\widehat{I} = I^*$ and $\pi_b = 0$ simultaneously yields

$$(22) \quad 1 + i^* = \frac{1}{\delta q} \left(\frac{V}{V + I_0} \right),$$

$$(23) \quad \text{and } A^* = \frac{I_0}{\delta q} \left(\frac{I^* - V}{I_0 + V} \right).$$

Since i^* must satisfy both (22) and (8), (13) must hold.

(\impliedby) Consider the solution for i^* and A^* given by (22) and (23). By construction, this solution satisfies constraint (6). Constraints (7) and (10) can be ignored by Lemmas 2 and 3. Since $\widehat{I} = I^*$ with this solution, the first best is attained provided the remaining constraint, (8), is satisfied. Supposing (13) holds, this solution for i^* and A^* does satisfy (8).

Proof of Proposition 2. If (13) holds, the proof of Proposition 1 shows that the optimal values of i^* and A^* are given by (22) and (23). Lemma 3 provides a solution for the optimal value of L^* .

Proof of Proposition 3. Suppose (13) does not hold. Then (8) must bind at the optimum. (If (8) did not bind at the optimum, the solution for i^* and A^* in (22) and (23) would be feasible and the first best could be obtained. By Proposition 1, (13) would hold, a contradiction.) Furthermore, by Lemma 1, $\pi_b = 0$ at the optimum. Therefore, the optimal loan commitment, if it exists, must set i^* and A^* to satisfy $\pi_b = 0$ and $1 + i^* = 1/\delta$. Solving $\pi_b = 0$ for A and substituting $1 + i^* = 1/\delta$ yields the expression for A^* given in the statement of the proposition. Lemma 3 provides a solution for the optimal value of L^* .

Verification of Table 1. I examine the effect on equilibrium of an increase in I_0 , holding all other parameters constant. The proof for the other parameters is similar. First, I show that increasing I_0 strengthens (13). Let Q represent the right-hand side of (13), i.e., $Q \equiv V/(I_0 + V)$. Then $\partial Q/\partial I_0 = -1/(I_0 + V) < 0$, implying that (13) is strengthened.

Second, it is shown that, if (13) holds, then \widehat{I} , π_e , and i^* (weakly) decline and A^* increases in response to an increase in I_0 . If (13) holds, $\widehat{I} = I^*$, and I^* is independent of I_0 . Further, $\pi_e = V$, and $\partial V/\partial I_0 = -1 < 0$. Differentiating the expression in Proposition 2 shows that $\partial i^*/\partial I_0 \propto dQ/dI_0$ (where the symbol \propto is the relational operator “has the same sign as”). But, as shown in the previous paragraph, $dQ/dI_0 < 0$. Differentiating the expression in Proposition 2, $\partial A^*/\partial I_0 \propto I^* - V + I_0 > 0$.

Third, it is shown that I^* , π_e , and i^* (weakly) decline and A^* (weakly) increases in response to an increase in I_0 if (13) does not hold (provided that a feasible loan commitment exists). I first employ Theorem 1 to show $\partial \widehat{I}/\partial I_0 < 0$. Upon substitution for I^* and A^* in the expression for \widehat{I} and rearranging, Proposition 3 implies $\widehat{I} = \sup\{x \in [L, \bar{I}] | g(x, I_0) \geq 0\}$, where I define

$$g(x, I_0) \equiv (1 - q) \int_L^x (x - I_1) dF(I_1) + (I^* - x)F(x) - I_0.$$

Note $g(I^*, I_0) = (1 - q)V - I_0$. Hence, $g(I^*, I_0) < 0$ if (13) does not hold. Additionally, $\partial g(x, I_0)/\partial x = -qF(x) + (I^* - x)f(x)$, a negative expression for $x \geq I^*$. Thus, $g(\bar{I}, I_0) < 0$. Combined with the facts that $g(x, I_0)$ is continuous in x and that $\partial g(x, I_0)/\partial I_0 < 0$, Theorem 1 implies $\partial \widehat{I}/\partial I_0 < 0$. In view of equation (12), $\partial \pi_e/\partial I_0 = (I^* - \widehat{I})f(\widehat{I})\partial \widehat{I}/\partial I_0 - 1 < 0$. By Proposition 3, I^* is independent of I_0 if (13) does not hold. Finally, differentiating the equation for A^* from Proposition 3,

$$\frac{\partial A^*}{\partial I_0} = \frac{1 - qF(\widehat{I})}{\delta qF(\widehat{I})} + \frac{\partial \widehat{I}}{\partial I_0} \left[\frac{1}{\delta qF(\widehat{I})^2} \right] \left[(1 - q) \int_L^{\widehat{I}} (\widehat{I} - I_1) dF(I_1) - I_0 \right].$$

Assuming (13) does not hold,

$$(1 - q) \int_L^{\widehat{I}} (\widehat{I} - I_1) dF(I_1) - I_0 < (1 - q)V - I_0 < 0.$$

Thus $\partial A^*/\partial I_0 > 0$.

Proof of Proposition 4. If the project is financed with a SSD, the entrepreneur continues the project if and only if the surplus from continuing, $\delta q[R - (1 + i_0)I_0 - (1 + i_1)I_1]$, exceeds the surplus from canceling, N . Thus, there exists a critical level of period 1 investment,

$$(24) \quad I^s = \left(\frac{1}{1 + i_1} \right) \left[R - \frac{N}{\delta q} - (1 + i_0)I_0 \right],$$

such that the entrepreneur continues the project if and only if $I_1 \leq I^s$.

First, consider the solution for i_1^s . Noting that $\pi_b^1 = \delta q(1 + i_1)I_1 - I_1$, it is obvious that the constraint $\pi_b^1 \geq 0$ binds at the optimum, implying $i_1^s = 1/(\delta q)$. (This solution automatically satisfies the constraints on over- and under-borrowing.) Next, consider the solution for i_0^s . Noting that $\pi_b^0 = \delta qF(I^s)(1 + i_0)I_0 - I_0$, it is obvious that the constraint $\pi_b^0 \geq 0$ binds at the optimum, implying $i_0^s = 1/[\delta qF(I^s)]$. Substituting these solutions into (24) and substituting $I^* = \delta qR - N$,

$$(25) \quad I^s = I^* - \frac{I_0}{F(I^s)}.$$

There exists a SSD that can fund the project if and only if a solution to (25) exists in $[L, \bar{I}]$. Rearranging (25) gives (14).

To compute π_e^s , note

$$\begin{aligned} \pi_e^s &= \delta q \int_L^{I^s} [R - (1 + i_0^s)I_0 - (1 + i_1^s)I_1] dF(I_1) + \int_{I^s}^{\bar{I}} N dF(I_1) - N \\ &= \int_L^{I^s} (I^* - I_1) dF(I_1) - I_0, \end{aligned}$$

where the second line holds by substituting for i_0^s and i_1^s and substituting $I^* = \delta qR - N$.

Proof of Proposition 5. It is left to show $\hat{I} > I^s$. Suppose first that (13) holds. Then, $\hat{I} = I^*$. By (14), $I^s < I^*$, and the proposition is proved.

Suppose instead that (13) does not hold. Following the verification of Table 1 above, I can write $\hat{I} = \sup\{x \in [L, \bar{I}] | h^\ell(x) \geq 0\}$ for

$$h^\ell(x) \equiv (1 - q) \int_L^x (x - I_1) dF(I_1) + (I^* - x)F(x) - I_0.$$

I can also write $I^s = \sup\{x \in [L, \bar{I}] | h^s(x) \geq 0\}$ for $h^s(x) \equiv (I^* - x)F(x) - I_0$. Nesting the two together, the critical investment level is $\sup\{x \in [L, \bar{I}] | h(x, \alpha) \geq 0\}$ where

$$h(x, \alpha) \equiv \alpha \int_L^x (x - I_1) dF(I_1) + (I^* - x)F(x) - I_0,$$

and where $\alpha = 0$ in the case of SSD and $\alpha = 1 - q$ in the case of loan commitments. Now $h(I^*, \alpha) = \alpha V - I_0 \leq (1 - q)V - I_0 < 0$ if (13) does not hold and $\alpha \leq 1 - q$. Additionally, $\partial h(x, \alpha) / \partial x = (\alpha - 1)F(x) + (I^* - x)f(x) < 0$ for $x \geq I^*$ and $\alpha \leq 1 - q$. Hence, $h(\bar{I}, \alpha) < 0$. In combination with the facts that $h(x, \alpha)$ is continuous in x and that $\partial h(x, \alpha) / \partial \alpha = \int_L^x (x - I_1) dF(I_1) > 0$ for $x > L$, Theorem 1 implies the critical investment level is strictly increasing in α , in turn implying $\hat{I} > I^s$.

Proof of Proposition 6. Suppose (13) does not hold. In the verification of Table 1 above, it is shown that for the critical investment level associated with the optimal loan commitment is given by $\hat{I} = \sup\{x \in [L, \bar{I}] | h(x) \geq 0\}$, where

$H(x)$ is defined in (16). Hence, a feasible loan commitment exists if and only if $\{x \in [L, \bar{I}] | H(x) \geq 0\} \neq \emptyset$. Suppose (13) holds. Then a feasible loan commitment exists by Proposition 2. Additionally, $H(I^*) = (1 - q)V - I_0 \geq 0$, implying $\{x \in [L, \bar{I}] | H(x) \geq 0\} \neq \emptyset$. This proves the first statement of Proposition 2.

To prove the second statement of the proposition, consider the value of period 0 investment, labeled I'_0 , such that $V = 0$, i.e., $I'_0 = \int_{\underline{I}}^{I^*} (I^* - I_1) dF(I_1)$. Substituting I'_0 into $H(x)$ and rearranging,

$$H(x)|_{I_0=I'_0} = -q \int_{\underline{I}}^x (x - I_1) dF(I_1) - \int_x^{I^*} (I^* - I_1) dF(I_1),$$

a negative expression $\forall x \in [L, I^*]$. Thus, $H(x) < 0 \forall x \in [L, I^*]$. By continuity, $\exists \epsilon > 0$ such that, $\forall I_0 \in (I'_0, I'_0 + \epsilon)$, $V > 0$, but $H(x) < 0 \forall x \in [L, I^*]$.

The third statement of the proposition follows since $\forall x \in (L, I^*)$, $\lim_{R \rightarrow 0} H(x) < 0$, $\lim_{R \rightarrow \infty} H(x) = \infty$, and $\partial H(x) / \partial R > 0$. The fourth statement follows since $\forall x \in (L, I^*)$, $\lim_{I_0 \rightarrow 0} H(x) > 0$, $\lim_{I_0 \rightarrow \infty} H(x) < 0$, and $\partial H(x) / \partial I_0 < 0$.

Proof of Proposition 7. Consider an arbitrary feasible contract, the terms of which will be superscripted by o . This contract can be replaced with a new contract (the terms of which are superscripted by n) that is feasible, does not reduce π_e , but has affine $S_2^n(I_1) \forall I_1 \in [L, \hat{I}^o]$, where \hat{I}^o denotes the critical level of investment associated with the original contract. In particular, define

$$S_2^n(I_1) = \begin{cases} S_2^o(I) + m^o (I_1 - I) + \Delta & \forall I_1 \in [L, \hat{I}^o] \\ \infty & \forall I_1 \in (\hat{I}^o, \bar{I}] \end{cases},$$

where $m^o = \min_{I_1 \in [L, \hat{I}^o]} \check{S}_2^o(I_1)$ and where Δ is given implicitly by

$$\int_{\underline{I}}^{\hat{I}^o} S_2^n(I_1) dF(I_1) = \int_{\underline{I}}^{\hat{I}^o} [S_2^o(I) + m^o (I_1 - I) + \Delta] dF(I_1).$$

Intuitively, the new contract is constructed by having $S_2^n(I_1)$ grow at the slowest rate grown by $S_2^o(I_1)$ over the whole interval $[L, \hat{I}^o]$. The new contract scales up the repayment level by a constant to guarantee the bank's individual rationality constraint remains satisfied.

Indeed, it can be verified from the definition of Δ that individual rationality is satisfied by the new contract. Truth telling is satisfied since $\check{S}_2^n(I_1) = m^o \geq 1/\delta$, where the last inequality follows since the original contract satisfied (17). It is left to check that marginal investment incentives are the same with the new contract as with the original (i.e., $\hat{I}^n = \hat{I}^o$) so that the two contracts would produce the same value for π_e . A sufficient condition for $\hat{I}^n = \hat{I}^o$ is $S_2^n(\hat{I}^o) = S_2^o(\hat{I}^o)$. By the Fundamental Theorem of Calculus,

$$\begin{aligned} S_2^n(\hat{I}^o) - S_2^o(\hat{I}^o) &= S_2^o(\underline{I}) + \int_{\underline{I}}^{\hat{I}^o} \check{S}_2^o(x) dx - S_2^o(\underline{I}) - m^o (\hat{I}^o - \underline{I}) - \Delta \\ &= \int_{\underline{I}}^{\hat{I}^o} [\check{S}_2^o(x) - m^o] dx - \Delta. \end{aligned}$$

Applying the Fundamental Theorem of Calculus to the definition of Δ ,

$$\int_{\underline{I}}^{\widehat{I}^o} \Delta dF(I_1) = \int_{\underline{I}}^{\widehat{I}^o} \int_{\underline{I}}^{I_1} [\check{S}_2^o(x) - m^o] dx dF(I_1).$$

Thus,

$$\begin{aligned} \Delta &= \frac{1}{F(\widehat{I}^o)} \int_{\underline{I}}^{\widehat{I}^o} \int_{\underline{I}}^{I_1} [\check{S}_2^o(x) - m^o] dx dF(I_1) \\ &\leq \frac{1}{F(\widehat{I}^o)} \int_{\underline{I}}^{\widehat{I}^o} \int_{\underline{I}}^{\widehat{I}^o} [\check{S}_2^o(x) - m^o] dx dF(I_1) \\ &= \int_{\underline{I}}^{\widehat{I}^o} [\check{S}_2^o(x) - m^o] dx, \end{aligned}$$

where the second line holds since $\check{S}_2^o(I_1) \geq m^o \forall I_1 \in [\underline{I}, \widehat{I}^o]$. Therefore, $S_2^o(\widehat{I}^o) \geq S_2^n(\widehat{I}^o)$.

Proof of Proposition 8. The first best can be obtained if and only if, fixing $\widehat{I}(\delta) = I^*(\delta)$, the maximum value of $E_\delta \pi_b(\delta)$ is non-negative. $E_\delta \pi_b(\delta)$ is maximized subject to the under-borrowing constraint by setting $1 + i(\delta) = 1/\delta \forall \delta$. Now $\widehat{I}(\delta) = I^*(\delta)$ and $1 + i(\delta) = 1/\delta$ implies $\delta q A(\delta) = (1 - q)I^*(\delta) - qI_0$. Substituting these values into $\pi_b(\delta)$ and simplifying implies that the maximum value of $\pi_b(\delta)$ is $(1 - q) \int_{\underline{I}}^{I^*(\delta)} [I^*(\delta) - I_1] dF(I_1) - I_0 = (1 - q)V(\delta) - I_0$. Hence, the first best can be obtained if and only if $(1 - q)E_\delta V(\delta) - I_0 \geq 0$, a condition equivalent to (20).

Suppose (20) does not hold. Consider any loan commitment with $E_\delta \pi_b(\delta) > 0$. This contract can be improved by reducing $A(\delta)$ by dA in all δ states. Thus, constraint (18) can be taken to bind without loss of generality. Substituting $E_\delta \pi_b(\delta) = 0$ into the entrepreneur's objective function implies

$$(26) \quad E_\delta \pi_e(\delta) = E_\delta \int_{\underline{I}}^{\widehat{I}(\delta)} [I^*(\delta) - I_1] dF(I_1) - I_0.$$

Suppose $\exists \delta \in [\underline{\delta}, \bar{\delta}]$ such that $1 + i(\delta) > 1/\delta$. Then the contract can be improved by decreasing $i(\delta)$ by di and increasing $A(\delta)$ by $dA = [\widehat{I}(\delta) + I_0] di$, where di and dA have been calculated to leave $\widehat{I}(\delta)$ unchanged. However, constraint (18) is relaxed,

$$\begin{aligned} d\pi_b(\delta) &= \delta q \int_{\underline{I}}^{\widehat{I}(\delta)} [dA - (I_0 + I_1) di] dF(I_1) \\ &= \left\{ \delta q \int_{\underline{I}}^{\widehat{I}(\delta)} [\widehat{I}(\delta) - I_1] dF(I_1) \right\} di, \end{aligned}$$

implying $d\pi_b(\delta) > 0$. Thus, at an optimum, $1 + i(\delta) = 1/\delta \forall \delta \in [\underline{\delta}, \bar{\delta}]$.

Substituting $1 + i(\delta) = 1/\delta$ into (18) and solving for $A(\delta)$ in terms of $\widehat{I}(\delta)$ —i.e., $A(\delta) = [I^*(\delta)/q - \widehat{I}(\delta) - I_0]/\delta$ —the program determining the optimal loan commitment reduces to choosing $\widehat{I}(\delta)$ to maximize $E_\delta \pi_c(\delta)$, subject to $E_\delta \pi_b(\delta) = 0$. This can be solved by choosing $\widehat{I}(\delta)$ for each $\delta \in [\underline{\delta}, \bar{\delta}]$ to maximize the following Lagrangian,

$$\mathcal{L} = \int_I^{\widehat{I}(\delta)} \left\{ I^*(\delta) - I_1 + \lambda \left[I^*(\delta) - q\widehat{I}(\delta) - (1 - q)I_1 \right] \right\} dF(I_1).$$

Upon rearranging, the first-order condition becomes

$$\widehat{I}(\delta) = I^*(\delta) - \left(\frac{\lambda q}{1 + \lambda} \right) \left[\frac{F(\widehat{I}(\delta))}{f(\widehat{I}(\delta))} \right].$$

Clearly, $I^*(\delta) > \widehat{I}(\delta)$. To show that $A(\delta)$ is increasing in λ , note $\widehat{I}(\delta) = \sup\{x \in [\underline{I}, \bar{I}] | K(x, \lambda) \geq 0\}$, where

$$K(x, \lambda) \equiv I^*(\delta) - x - \left(\frac{\lambda q}{1 + \lambda} \right) \left[\frac{F(x)}{f(x)} \right].$$

Now $K(x, \lambda) < 0 \forall x \in [I^*(\delta), \bar{I}]$. Further, $\partial K(x, \lambda)/\partial \lambda < 0$. Therefore, Theorem 1 implies $\widehat{I}(\delta)$ is declining in λ . Since $1 + i(\delta)$ is fixed at $1/\delta$, $A(\delta)$ is increasing in λ .

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⁷ **The Loan Commitment as an Optimal Financing Contract**

Elazar Berkovitch; Stuart I. Greenbaum

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