Optimal Vaccine Subsidies for Epidemic Diseases

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Abstract: We analyze optimal vaccine subsidies in a model integrating disease epidemiology into a market with rational economic agents. The focus is on an intensive vaccine campaign to quell an epidemic in the short run. Across a range of market structures, positive vaccine externalities and optimal subsidies peak for diseases that spread quickly, but not so quickly that everyone is driven to be vaccinated. We assess the practical relevance of this peak—as well as the existence of increasing social returns to vaccination and optimality of universal vaccination—in calibrations to the Covid-19 pandemic.

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1. Introduction

Technologies such as vaccines, condoms, and mosquito nets that protect individuals against infectious diseases can generate positive externalities by reducing transmission to others. While textbook economic models provide a justification for public subsidies of such preventive technologies, the appropriate magnitude of these subsidies remains understudied. Without a deeper understanding, it is difficult for economists to provide guidance—even at a conceptual level—on the optimal level of government subsidies for infectious-disease control and how such subsidies should vary across diseases. Furthermore, much existing work by economists has been oriented toward policies combating endemic diseases playing out over generations such as vaccination campaigns to eradicate polio or circumcision campaigns to reduce the spread of HIV. The Covid-19 pandemic, however, has underscored the urgency of quelling global outbreaks of novel diseases.

To address these questions, we construct a tractable model that integrates epidemiological and economic considerations. For concreteness, we focus on the market for a vaccine, but the analysis applies to other aforementioned preventive technologies. Consumers and producers base their economic decisions on rational expectations of disease dynamics based on a susceptible-infected-recovered (SIR) model, standard in the epidemiology literature. We analyze a vaccine campaign introduced at a single point in time into a population without turnover, modeling choices intended to capture relevant features of epidemics like Covid-19 that may rise and fall within a generation, calling for a concentrated policy response.

A key finding—robustly holding across market structures ranging from perfect competition to Cournot competition to monopoly—is that the positive externality exerted by the marginal consumer's vaccination on others peaks for intermediate rather than the highest values of $\mathcal{R}_0$, the disease's *basic reproductive number*, a widely used measure of infectiousness. For low values of $\mathcal{R}_0$, the marginal externality is low because there is little disease transmission between people. For high

values of $\mathcal{R}_0$, vaccinating a given consumer does not provide much protection to others since they are almost certain to contract the disease from someone else anyway. To be sure, a consumer's vaccination provides a substantial social benefit when $\mathcal{R}_0$ is extremely high, but most of that benefit is internalized by the consumer. The optimal subsidy, which corrects for the marginal externality, likewise peaks for intermediate values of $\mathcal{R}_0$.

At certain intermediate values of $\mathcal{R}_0$, free riding can be so extensive that an increase in $\mathcal{R}_0$, by reducing some of this free riding, can perversely reduce equilibrium infections. This epidemiological version of the Peltzman (1975) effect arises when consumers compensate for increased risk by increasing vaccination to such an extent that it more than offsets the direct increase in infectiousness.

Our results go beyond nonmonotonicies. Whether universal vaccination can be a viable business strategy is explored in Section 5. Previous game-theoretic analyses suggested that a perfectly effective vaccine would never be universally purchased at a positive price because, with all other consumers protected, the marginal consumer obtains no private benefit (Geoffard and Phillipson 1997, May 2000, Bauch and Earn 2004). In our model, however, universal vaccination of susceptibles with a perfectly effective vaccine can be profitable. The risk of contracting the disease from those infected before the arrival of the vaccine but not yet recovered preserves a positive willingness to pay for the marginal consumer even if all other susceptibles are protected. Universal vaccination with a perfectly effective vaccine is not just possible but guaranteed in equilibrium for sufficiently low cost and sufficiently high infectiousness.

In typical economic settings, benefit functions are concave, leading to decreasing returns; but epidemiological effects may lead to disproportionate benefits of vaccination, as folk wisdom suggests may happen around the threshold for herd immunity. Section 6 explores this issue formally by providing conditions under which vaccination exhibits increasing social returns. The key indicator is $\mathcal{R}_0\hat{S}_0$, the product of the basic reproductive number $\mathcal{R}_0$ (technically, the number of secondary cases an infectious individual would hypothetically transmit in a fully susceptible population) and

the proportion of susceptibles in the relevant population $\hat{S}_0$ (here, at date 0, when the vaccine is introduced). The product $\mathcal{R}_0\hat{S}_0$, known as the *effective reproductive number*, tells us the number of secondary cases an infectious individual transmits in the relevant population. If $\mathcal{R}_0\hat{S}_0 < 1$, primary infections lead to fewer secondary infections from the start, implying the infection rate falls throughout the epidemic even absent a vaccine. For infections to rise to a peak before falling in the classic epidemic pattern requires $\mathcal{R}_0\hat{S}_0 > 1$. If the effective reproductive number is yet higher—we show $\mathcal{R}_0\hat{S}_0 > 2$ is sufficient—then social returns to vaccination are initially increasing. With such a high effective reproductive number, the epidemic is so explosive that a small amount of vaccine does little to slow it; to make a measurable dent in the epidemic requires concentrating supplies in one region. If $\mathcal{R}_0\hat{S}_0$ exceeds a yet higher threshold, vaccination exhibits increasing social returns for all capacity levels, meaning that it is efficient to fully vaccinate one region before moving to the next.

Section 7 compares the results to a market for a drug that is similar in all ways to the vaccine except that it treats symptoms but does nothing to reduce disease transmission from treated individuals. We show that a monopoly always prefers to develop the drug but parameters exist for which social welfare is higher with the vaccine. Consistent with nonmonotonicities found elsewhere, the monopoly's bias toward a drug peaks for intermediate values of $\mathcal{R}_0$.

Section 8 calibrates the model to the Covid-19 pandemic. While too stylized for quantitative policy guidance, the calibrations provide qualitative insights into the magnitude of distortions caused by externalities and market power and allow us to assess whether the theoretical conditions behind results such as the Peltzman effect, increasing social returns, and a bias toward drugs versus vaccines are practically relevant.

Our analysis is intentionally built on a basic epidemiological model, sacrificing realism to obtain rigorous propositions involving interpretable economic conditions rather than results based on isolated simulations or structural estimates tied to current circumstances. The SIR model omits fea-

tures requiried for quantitative forcasting in a real-world epidemics such as heterogeneous agents (Ellison 2020, Acemoglu et al. 2021), transmission along networks (Newman 2002, Fajgelbaum et al. 2021), and macroeconomic dynamics (Eichenbaum, Rebelo, and Trabandt 2020). Perhaps the key omission is endogenous social distancing, which would flatten the epidemic's path relative to SIR predictions. A growing literature has advanced increasingly sophisticated models to approximate the dynamic behavior of rational, forward-looking agents, seeking to reduce risk by curtailing activity.[1] We do not address all of these omissions but address some in several pieces of additional work. Online Appendix A4 allows consumers to be heterogeneous in harm. Online Appendix A5 extends the model to allow consumers to purchase a second preventive in addition to, or instead of, the vaccine. While not capturing a continuously updating distancing decision, the extension could capture fixed investments in masks or lifestyle changes. Our companion paper (Goodkin-Gold et al. 2022) maintains the SIR framework but adopts alternative modeling assumptions suited to an endemic disease, incorporating population turnover and continuous vaccination of arriving newborn cohorts. The analysis focuses on the steady-state equilibrium, in which the effective reproductive number is always 1, which Gans (2020) suggests is a reasonable shortcut for modeling endogenous social distancing. The additional work in the online appendixes and companion paper all finds that marginal externalities and optimal subsidies are highest for intermediate values of $\mathcal{R}_0$, increasing confidence in the robustness of the results to modeling assumptions.

Our paper contributes to theoretical literature analyzing vaccine externalities.[2] Economists have long observed that vaccines may provide positive externalities that could affect consumers' and

[1] Recent theoretical advances include Acemoglu et al. (2020); Atkeson, Kopecky, and Zha (2020, 2021); Farboodi, Jarosch, and Shimer (2020); Jones, Philippon, and Venkateswaran (2020); Keppo et al. (2020); Makris and Toxvaerd (2020); McAdams (2020); Rachel (2020); Tröger (2020); Toxvaerd (2019); and Toxvaerd (2020).

[2] See Avery et al. (2020) and McAdams (2021) for recent surveys.

firms' decisions.[3] Boulier, Datta, and Goldfarb (2007) uses a standard epidemiological model alone (i.e., neither interacted with consumer decisions nor a supply-side model of firm behavior) to examine properties of vaccination externalities that arise solely due to epidemiological concerns. Geoffard and Philipson (1997) uses an epidemiological model similar to ours to show that a vaccine producer with market power will not choose to eradicate the disease in the steady-state. Galeotti and Rogers (2013) model vaccination choices in a heterogenous population, and consider the effect of network structures in determining optimal vaccine allocation. Manski (2021), building on a series of the author's earlier papers, provides guidance on optimal vaccine policies (including mandates) when the extent of externality is unknown. Avery (2021) is a recent ambitious attempt to integrate social distancing and vaccinations in a tractable model. We contribute a precise characterization of the nonmonotonicity of externalities and optimal subsidies as a function of disease infectiousness. We also contribute by formally modelling the supply side of the vaccine market, allowing firms to have market power.

Our paper is perhaps closest to two companion papers in the operations-research literature: Mamani, Adida, and Dey (2012) and Adida, Dey, and Mamani (2013). They also analyze optimal subsidies for various degrees of supplier market power. Their focus is on consumers with uniformly distributed harm. While we also examine consumer heterogeneity (see online Appendix A4), our analysis focuses on homogeneous consumers, allowing us to derive more definitive expressions for equilibrium variables, which in turn afford additional insights, most importantly allowing us to analyze the comparative-static effect of increases in $\mathcal{R}_0$. Our central result on the nonomontonicity

[3]See, among others, Brito, Sheshinski, and Intrilligator (1991); Chen and Toxvaerd (2014); Francis (1997); Geoffard and Philipson (1997); Gersovitz (2003); and Gersovitz and Hammer (2004, 2005). Work in behavioral epidemiology has begun to incorporate externalities at least implicitly, considering, for example, game-theoretic analyses of decisions around whether to vaccinate or to free ride on herd immunity (Funk et al. 2010; Manfredi and D'Onofrio 2013).

of optimal subsidies in $\mathcal{R}_0$ and other comparative statics are novel in our paper. We also provide calibrations, results on increasing social returns, and a comparison between drugs and vaccines not found in their papers.[4]

The epidemiology literature previously recognized the possibility that the nonlinear nature of epidemics may dictate optimal policy concentrating a scarce stockpile in one population rather spreading across them (Keeling and Shattock 2012, Keeling and Ross 2015, Nguyen and Carlson 2016, and Enayati and Özaltin 2020).[5] This literature has the appeal of studying increasingly rich epidemiological models but the drawback of having to simulate results in numerical examples. We contribute a formal conceptualization of initial and eventual increasing social returns and aid understanding by providing a necessary and sufficient condition for these outcomes in analytical form.

Our paper also contributes to the economic literature responding to the Covid-19 pandemic. Scholars sought to apply detailed models to forecast the course of the pandemic (Atkeson, Kopecky, and Zha 2020), to recommend lockdown and testing protocols (Alvarez, Argente, and Lippi 2021), and to recommend policies for prioritizing scarce vaccine supplies among heterogeneous consumers (Buckner, Chowell, and Springborn 2021). The focus of our Covid-19 calibration is different (on optimal subsidies in a decentralized market rather than optimal strategies for a central planner) as is

[4]Also closely related is Althouse, Bergstrom, and Bergstrom (2010), which provides a welfare analysis of vaccination, calibrating a simple model for four prominent diseases to estimate optimal subsidies under perfect competition and perfectly effective vaccination. Our paper builds on their work, allowing for imperfect vaccines, including a supply-side model of firm behavior, and generating comparative statics which allow theoretical insights into how epidemiological and economic parameters impact market outcomes and optimal policy.

[5]See also Anderson, Laxminarayan, and Salant (2012). Using an SIS model incorporating reinfection, the study finds that a planner prefers concentrating scarce supplies each period in the location with fewer infections.

our goal (obtaining general principles in a stylized model rather than quantitative results in a more complex model).

2. Model

We model the vaccine market by first specifying the disease epidemiology, which determines the probability of infection for an unvaccinated consumer. We then bring rational economic actors into the vaccine market. On the demand side, the probability of infection factors into consumers' willingness to pay for the vaccine. Profit-maximizing firms operate on the supply side under either perfect competition or monopoly.

The epidemiological model is designed to suit the current Covid-19 pandemic. We assume the epidemic is of relatively short duration, rising and falling within a human generation, against which an intensive vaccine campaign is waged to quickly mitigate the harm experienced by the current population, with future generations spared much damage by the epidemic's natural decline. For pedagogical purposes, we adopt the extreme assumption that all doses of vaccine that will ever be administered are administered in a single instant. The model contrasts the alternative analyzed in our companion paper (Goodkin-Gold et al. 2022) in which generations of newborns are continuously vaccinated against an endemic disease with a constant steady-state infection rate. The two analyses together help provide a more complete understanding of vaccine markets for different diseases.

2.1. Epidemiology

The foundation of our analysis is the standard susceptible-infected-removed (SIR) epidemiological model due to Kermack and McKendrick (1927). Continuous time indexed by t begins with the arrival of the vaccine at $t = 0$. Assume for simplicity that the disease is non-fatal and that there are no births or deaths within the short time frame considered, leaving the population size constant over time. Anticipating their role in the vaccine market, we call individuals in this population *consumers*.

Assume consumers are homogeneous in harm, disease spread, and all other dimensions (relaxed in Online Appendix A4).

Consumers are partitioned into four compartments: susceptible to infection S_t, currently infected I_t, recovered from an infection R_t, or successfully immunized Z_t. Normalizing the population mass to 1,

$$S_t + I_t + R_t + Z_t = 1. \tag{1}$$

This normalization allows compartments to be interpreted as either masses or proportions. Compartments evolve according to the following equations, where dots denote time derivatives:

$$\dot{S}_t = -\beta I_t S_t \tag{2}$$

$$\dot{I}_t = \beta I_t S_t - \alpha I_t \tag{3}$$

$$\dot{R}_t = \alpha I_t \tag{4}$$

$$\dot{Z}_t = 0. \tag{5}$$

A susceptible consumer is assumed to contract the disease from an infected consumer at rate $\beta > 0$, embodying the rate of contact between people and the rate at which a contact leads to infection. Assuming the infection rate is linear in the number of infected consumers, a single susceptible consumer is infected with probability βI_t. The mass of susceptibles §$_t$ generates $\beta I_t S_t$ new infections. Equation (2) indicates that the susceptible population falls by the number of newly infected. Equation (3) indicates that the infected population is increased by the number of newly infected and reduced by the mass αI_t of previously infected consumers who recover, where $\alpha \in (0,1)$ denotes the recovery rate. This αI_t mass flows into R_t, as indicated by equation (4). Under the assumption that recovered individuals cannot be reinfected, this is the only change to R_t. Equation (5) reflects the instantaneous nature of the vaccination campaign, with no further vaccine administered after the

initial tranche at date 0.[6] We assume that if the initial vaccine course is not effective for a person, further course will not be effective for them either. Under that assumption, administering all vaccine in the first instant is both the profit-maximizing and welfare-maximizing strategy.[7]

Let $Q \in [0, \hat{S}_0]$ denote the quantity of vaccine courses administered at date 0 to susceptibles, the only consumers who can possibly benefit from vaccination. For now, take Q as given; later, we will solve for its equilibrium value using the economic model and substitute this value back into the epidemiological model. Let $\theta \in (0,1)$ denote the efficacy of a vaccine course. Let $\hat{S}_0$, $\hat{I}_0$, and $\hat{R}_0$ denote the counterfactual value of the relevant compartments at date 0 in the absence of vaccines (so by definition, $\hat{Z}_0 = 0$). Then the initial conditions for the SIR system can be written

$$S_0 = \hat{S}_0 - Z_0 \tag{6}$$

$$I_0 = \hat{I}_0 \tag{7}$$

$$R_0 = \hat{R}_0 = 1 - \hat{I}_0 - \hat{S}_0 \tag{8}$$

$$Z_0 = \theta Q. \tag{9}$$

We treat $\hat{I}_0$ and $\hat{S}_0$ as exogenous parameters, allowing them to take on any admissible values: $\hat{I}_0 \in (0,1)$ and $\hat{S}_0 \in (0, 1-\hat{I}_0]$.

In lieu of the transmission parameter β, epidemiologists often work with a related parameter $\mathcal{R}_0$, called the basic reproductive number, equal to the expected number of secondary cases an infectious

[6] This model of vaccination is called *vaccination at recruitment* (Martcheva 2015, Section 9.2.1), distinct from *continuous vaccination* (Martcheva 2015, Section 9.2.2).

[7] Logistical constraints would prevent such rapid vaccine rollout in practice, but the model may reasonably approximate an intensive vaccine campaign against Covid-19 or other epidemic disease.

individual transmits in a fully susceptible population. In our model,

$$\mathcal{R}_0 = \frac{\beta}{\alpha}. \tag{10}$$

To understand this expression, each instant the individual remains infected, he or she infects a number of others equal to β times the size of the susceptible population, which is approximately 1 since the infected individual is introduced into a fully susceptible population. The individual remains infected for an expected duration of $1/\alpha$.[8] The subsequent analysis takes $\mathcal{R}_0$ as the key exogenous parameter, capturing the disease's infectiousness.[9]

In subsequent notation, Q is appended as an argument to equilibrium variables to emphasize their dependence on that key variable to be endogenized later. Limiting compartment values at the end of the epidemic are denoted by $S_\infty(Q)$, $I_\infty(Q)$, and $R_\infty(Q)$. For example, $S_\infty(Q) = \lim_{t\uparrow\infty} S_t(Q)$.

The following series of lemmas, which characterize $S_t(Q)$ and $I_t(Q)$ for finite and limiting values of t, help streamline the subsequent analysis. Many of the proofs are sketched in Martcheva (2015); Online Appendix A1 provides full details.

Lemma 1. *$I_t(Q) > 0$ and $S_t(Q) > 0$.*

Lemma 2. *$S_t(Q)$ is strictly decreasing in t.*

Lemma 3. *If $\mathcal{R}_0 S_0(Q) \leq 1$, then $I_t(Q)$ is strictly decreasing in t for all $t > 0$. Otherwise, $I_t(Q)$ is hump-shaped, peaking at time $T > 0$ satisfying $S_T(Q) = 1/\mathcal{R}_0$, strictly increasing for $t < T$, and*

[8] To see this, note that the sole risk of exiting the infected state is recovery, with hazard $\lambda_R(t) = \alpha$. In a Poisson duration models, the duration of a spell equals the reciprocal of the hazard, here $1/\lambda_R(t) = 1/\alpha$.

[9] Estimates of $\mathcal{R}_0$ vary considerably across diseases, from 1.1 for SARS (Chowell et al. 2003) at the low end to 16–18 for measles and pertussis at the high end (Anderson and May 1991). Estimates of $\mathcal{R}_0$ also vary across time and region.

strictly decreasing for $t > T$.

Lemma 4. *The limits $I_\infty(Q)$ and $S_\infty(Q)$ exist. In particular, $I_\infty(Q) = 0$ and $S_\infty(Q) \in (0, S_0(Q))$.*

Lemma 5. $\mathcal{R}_0 S_\infty(Q) < 1$.

Intuitively, the infection rate is always positive in finite time because, if not increasing, infections are at worst declining at a proportional rate less than 100% each instant, which can never force the infection rate to 0. The infection rate does asymptote to 0 as the stock of susceptibles is depleted and recovery takes over as the dominating force, reducing the stock of infecteds. Turning to results for the population of susceptibles, with an imperfectly effective vaccine ($\theta < 1$), even a universal vaccination campaign cannot eliminate the stock of susceptibles at date 0. The stock of susceptibles is never forced to 0 after because the proportional decline is less than 100% each instant. The stock of susceptibles strictly decreases over time since it is subject to outflows but not inflows.

According to Lemma 3, the path of infections over the epidemic has two possible shapes: monotonically decreasing or hump-shaped, expanding up to a peak and declining thereafter. The shape of the path hinges on the product $\mathcal{R}_0 S_0(Q)$. Multiplying $\mathcal{R}_0$, the expected number of secondary cases an infectious individual transmits in a fully susceptible population, by $S_0(Q)$, the proportion of susceptibles in the relevant population, yields the expected number of secondary cases an infectious individual transmits in the relevant population, called the effective reproductive number. If $\mathcal{R}_0 S_0(Q) \leq 1$, there are fewer secondary infections than primary infections in the initial population, leading the infection rate to fall initially. Otherwise, the infection rate rises initially.

Lemma 5 says that the effective reproductive number cannot exceed 1 at the end of the epidemic. If $\mathcal{R}_0 S_\infty(Q) > 1$, infections would be increasing, implying a growing, not waning, epidemic.

The term $S_\infty(Q)$—the proportion of people who remain healthy throughout the epidemic despite not being successfully immunized—plays a key role in the subsequent analysis, factoring into private and social benefits, thus determining equilibrium outcomes and the efficiency of these outcomes. While no closed-form solution exists for $S_\infty(Q)$, the next lemma expresses it as an implicit

function of other model parameters. The lemma also provides an expression for $S_\infty(Q)$ in terms of the principal branch of the Lambert W function, here denoted $\bar{L}$.[10]

Lemma 6. *$S_\infty(Q)$ satisfies*

$$\ln S_\infty(Q) - \mathcal{R}_0 S_\infty(Q) = \ln(\hat{S}_0 - \theta Q) - \mathcal{R}_0(\hat{I}_0 + \hat{S}_0 - \theta Q) \tag{11}$$

and can be written

$$S_\infty(Q) = \frac{1}{\mathcal{R}_0}\left|\bar{L}\left(-\mathcal{R}_0(\hat{S}_0 - \theta Q)e^{-\mathcal{R}_0(\hat{I}_0+\hat{S}_0-\theta Q)}\right)\right|. \tag{12}$$

Comparative-static results can be obtained by applying the Implicit Function Theorem to (11). For instance,

$$\frac{\partial S_\infty(Q)}{\partial Q} = \frac{\theta S_\infty(Q)}{S_0(Q)}\left[\frac{\mathcal{R}_0 S_0(Q) - 1}{1 - \mathcal{R}_0 S_\infty(Q)}\right]. \tag{13}$$

Since its denominator is positive by Lemma 5, the sign of (13) depends on whether the effective reproductive number, $\mathcal{R}_0 S_0(Q)$, exceeds 1 initially. The immediate effect of an increase in Q is to move an individual from the currently susceptible to the vaccinated compartment. If $\mathcal{R}_0 S_0(Q) \leq 1$, implying that the infection rate declines monotonically throughout the epidemic, this immediate effect carries through to a reduction in the final susceptible proportion, $S_\infty(Q)$. On the other hand, if $\mathcal{R}_0 S_0(Q) > 1$, implying that the infection rate initially increases, the reduction in current susceptibles has such a strong feedback effect, reducing epidemic growth, that the final susceptible proportion $S_\infty(Q)$ increases despite the immediate reduction in susceptibles.

[10]The Lambert W function L frequently arises in epidemiological applications. By definition $L(x)$ is the implicit solution to the exponential equation $L(x)e^{L(x)} = x$. The principal branch $\bar{L}$ is the sole solution to the implicit equation or, if two solutions exist, the higher of the two. The lower branch $\underline{L}$ is defined when two solutions exist as the lower of the two. Though $\bar{L}$ and $\underline{L}$ do not have a closed-form solutions, they can be computed with built-in functions included in standard software packages such as Matlab.

2.2. Consumer Demand

Consumers are homogeneous and risk neutral. Consistent with the short-run perspective adopted in this paper, assume agents do not discount the future. Let H denote the total expected harm suffered by a consumer who contracts the disease over the spell before recovery.

The $\hat{S}_0$ individuals in the susceptible compartment when the vaccine is introduced are potential consumers. They make their demand decisions by comparing the vaccine's price P to their marginal private benefit, which can be written $MPB(Q) = \theta H \Phi(Q)$, where $\Phi(Q)$ denotes the probability a susceptible contracts the disease during the epidemic.

To compute $\Phi(Q)$, note that the probability an unvaccinated individual does not contract the disease equals $S_\infty(Q)/S_0(Q)$, the number of people who remain susceptible over the model's horizon divided by the number of people who are susceptible at the start of the ex post period. The probability of infection is the complementary probability

$$\Phi(Q) = 1 - \frac{S_\infty(Q)}{S_0(Q)} = 1 - \frac{S_\infty(Q)}{\hat{S}_0 - \theta Q}, \tag{14}$$

which Lemma 4 guarantees is positive. Thus,

$$MPB(Q) = \theta H \left[1 - \frac{S_\infty(Q)}{\hat{S}_0 - \theta Q} \right]. \tag{15}$$

Differentiating, substituting from (13), and rearranging yields

$$\frac{\partial MPB(Q)}{\partial Q} = \frac{-\theta \mathcal{R}_0 S_\infty(Q) MPB(Q)}{S_0(Q)[1 - \mathcal{R}_0 S_\infty(Q)]}, \tag{16}$$

which is negative by Lemma 5, confirming the intuition that vaccinating more consumers lowers their marginal private benefit.

Proceeding to derive the demand curve, all $\hat{S}_0$ consumers purchase the vaccine if $P < MPB(\hat{S}_0)$,

and none purchase if $P > MPB(0)$. For P strictly between $MPB(\hat{S}_0)$ and $MPB(0)$, some but not all consumers purchase. Given they are homogeneous, consumers must be indifferent between purchasing and not, implying $P = MPB(Q)$. Given they are indifferent, any fraction of them are willing to purchase in equilibrium; demand is pinned down by the value of Q satisfying (15) when the right-hand side is set equal to P. Rearranging the resulting equation yields $S_\infty(Q) = (1-P/\theta H)(\hat{S}_0 - \theta Q)$. Substituting this into (11) and solving for Q gives the following expression for demand when a subset purchase:

$$d(P) = \frac{1}{\theta}\left\{\hat{S}_0 + \frac{\theta H}{P}\left[\frac{1}{\mathcal{R}_0}\ln\left(1-\frac{P}{\theta H}\right) + \hat{I}_0\right]\right\}. \tag{17}$$

Combining these facts yields the demand curve

$$D(P) = \begin{cases} 0 & P > MPB(0) \\ d(P) & P \in [MPB(\hat{S}_0), MPB(0)] \\ \hat{S}_0 & P < MPB(\hat{S}_0). \end{cases} \tag{18}$$

Equivalently, the demand curve is given by $d(P)$ unless this violates the boundary condition $d(P) \in [0, \hat{S}_0]$, in which case demand is given by the violated boundary.

2.3. Firm Supply

We analyze two different market structures in the text: perfect competition and monopoly. Online Appendix A3 provides results from a more general model of Cournot competition among n firms that nests these extremes.

Assume firms produce at constant marginal and average cost $c > 0$ per vaccine course (where a course involves multiple doses when needed to provide immunity). Under perfect competition, vaccine supply is perfectly elastic at price c. Under monopoly, the firm sets a price maximizing industry profit Π from date-0 sales.

By equation (14) and Lemma 2, $\Phi(Q) < 1$, implying $MPB(Q) < \theta H$ by (15). There are no sales

under perfect competition or indeed under any market structure if $c \geq \theta H$. To rule out trivial cases, throughout the remainder of the paper we assume

$$\frac{c}{\theta H} = \tilde{c} < 1, \tag{19}$$

introducing $\tilde{c}$ as shorthand notation to streamline subsequent expressions.

2.4. Normative Measures

Total harm experienced by consumers from the disease equals $HR_\infty(Q)$. Social benefit $SB(Q)$ is the complement of this, the harm avoided in the population who never contract the disease:

$$SB(Q) = H[1 - R_\infty(Q)] = H[S_\infty(Q) + \theta Q], \tag{20}$$

where the second equality follows from equation (1) and Lemma 4. Welfare $W(Q)$ is the difference between total social benefit and total vaccine production costs:

$$W(Q) = SB(Q) - cQ. \tag{21}$$

Marginal social benefit is the derivative $MSB(Q) = \partial SB(Q)/\partial Q$. Differentiating (20), substituting from (13)–(15), and rearranging yields

$$MSB(Q) = \frac{MPB(Q)}{1 - \mathcal{R}_0 S_\infty(Q)}. \tag{22}$$

Let $MEX(Q) = MSB(Q) - MPB(Q)$ denote the marginal externality from a vaccine course. Substituting from (22) yields

$$MEX(Q) = \frac{\mathcal{R}_0 S_\infty(Q) MPB(Q)}{1 - \mathcal{R}_0 S_\infty(Q)}. \tag{23}$$

Let Q^{**} denote the first-best quantity, maximizing $W(Q)$. If Q^{**} is not a corner solution, involving either no vaccination or universal vaccination, it is an interior solution solving the social planner's first-order condition $MSB(Q^{**}) = c$.

3. Equilibrium

3.1. Perfect Competition

Equilibrium values of variables are distinguished with stars and a subscript indicating the relevant market structure. Under perfect competition, the equilibrium price is $P_c^* = c$ and profit is $\Pi_c^* = 0$. The remaining equilibrium variables can be computed using straightforward algebra applied to the supplied equations. Table 1 reports the equilibrium values of selected variables as a function of $\mathcal{R}_0$.

The table distinguishes three relevant cases corresponding to three intervals for $\mathcal{R}_0$. In case (i), $\mathcal{R}_0$ is so low that no consumer finds it worthwhile to purchase the vaccine. The moderate values of $\mathcal{R}_0$ in case (ii) lead some but not all susceptibles to purchase. To compute the boundary value of $\mathcal{R}_0$ between cases (i) and (ii), denoted $\mathcal{R}_0'$, note that equilibrium price $P_c^* = c$ just chokes off demand at this boundary. Setting $d(c) = 0$ in (17) and solving for $\mathcal{R}_0$ yields

$$\mathcal{R}_0' = \frac{|\ln(1-\tilde{c})|}{\hat{I}_0 + \tilde{c}\hat{S}_0}. \tag{24}$$

In the remaining cases, $\mathcal{R}_0$ is so high that all susceptibles find purchasing the vaccine worthwhile. The first best is obtained in these cases: $Q_c^* = \hat{S}_0 = Q^{**}$. To compute the boundary value of $\mathcal{R}_0$ between cases (ii) and (iii), denoted $\mathcal{R}_0''$, note that equilibrium price $P_c^* = c$ just induces all susceptibles to purchase at this boundary. Setting $d(c) = \hat{S}_0$ in (17) and solving for $\mathcal{R}_0$ yields

$$\mathcal{R}_0'' = \frac{|\ln(1-\tilde{c})|}{\hat{I}_0 + (1-\theta)\tilde{c}\hat{S}_0}. \tag{25}$$

To visualize how the variables in Table 1 vary with $\mathcal{R}_0$, Figure 1 graphs a selection of them as functions of $\mathcal{R}_0$. Focus for now on the dotted curves representing equilibrium under perfect competition. Vaccine quantity Q_c^*, graphed in the first panel, rises throughout case (ii) from its value of 0 in case (i) to the first-best value Q^{**} in cases (iii) and (iv). It is unsurprising that equilibrium quantity is weakly increasing in the infectiousness of the disease measured by $\mathcal{R}_0$. Other equilibrium variables also display expected comparative statics in $\mathcal{R}_0$. MPB_c^* is weakly increasing and W_c^* is weakly decreasing in $\mathcal{R}_0$. It is noteworthy that MPB_c^* levels off at c in case (ii). Given that some but not all consumers purchase in this case, consumers must be indifferent between purchasing and not, implying that the equilibrium price $P_c^* = c$ must extract the entire marginal private benefit, implying $MPB_c^* = c$ over the whole interval.

Other variables display interesting nonmonotonicities. Cumulated infections over the epidemic $R_\infty(Q_c^*)$ initially increase in case (i) due to the epidemiological effects of the higher $\mathcal{R}_0$. In case (ii), when consumers begin purchasing vaccine, $R_\infty(Q_c^*)$ reverses course, sloping downward in $\mathcal{R}_0$. The counterintuitive downward slope can be explained by risk-compensation effect à la Peltzman (1975): the direct effect of an increase in infectiousness is more than offset by consumers' behavioral response in the form of increased vaccine purchases.[11] In cases (iii) and (iv), $R_\infty(Q_c^*)$ again rises with $\mathcal{R}_0$ because the direct effect of an increase in infectiousness cannot be offset by an increase in vaccine purchases given that all susceptibles are vaccinated. The marginal externality MEX_c^* exhibits an even more complex nonmonotonic pattern. The interplay between increasing infectiousness and increasing vaccine quantity generates two local maxima in the figure, with the global maximum occuring at the boundary between cases (i) and (ii).

The next proposition summarizes the comparative-static effects of an increase in $\mathcal{R}_0$ on the

[11] Mathematically, to maintain the constant marginal private benefit ($MPB_c^* = c$) observed throughout (ii), the increase in infectiousness $\mathcal{R}_0$ must be offset by a reduction in infections to maintain a constant probability of contracting the disease.

steady-state equilibrium under perfect competition, showing that the observations from Figure 1 are quite general. Online Appendix A1 provides proofs for results not obvious from Table 1.

Proposition 1. *Consider the comparative-static effect of $\mathcal{R}_0$ on equilibrium variables under perfect competition.*

- *Price and industry profit are constant, $P_c^* = c$ and $\Pi_c^* = 0$, respectively.*
- *Quantity Q_c^* and marginal private benefit MPB_c^* are weakly increasing in $\mathcal{R}_0$.*
- *Welfare W_c^* is weakly decreasing in $\mathcal{R}_0$.*
- *Cumulated infections, $R_\infty(Q_c^*)$, attains a single interior local maximum in $\mathcal{R}_0$, which is a global maximum if and only if $\tilde{c} \geq 1-\theta$.*
- *For the marginal social benefit MSB_c^* and marginal externality MEX_c^*, each attains no more than two interior local maxima in $\mathcal{R}_0$, one of which is a global maximum.*

For each Q_c^*, MPB_c^*, and W_c^*, there exists a nonempty interval of $\mathcal{R}_0$ such that the weak change is strict.

Note that the Peltzman effect identified for cumulated infections, whereby an increase in $\mathcal{R}_0$ leads to an equilibrium reduction in $R_\infty(Q_c^*)$, does not extend over the whole range of $\mathcal{R}_0$ but just over case (ii). Note further that the reductions in infections in case (ii) does not translate into an increase in welfare, which is weakly decreasing for all $\mathcal{R}_0$. Increased consumer spending on vaccines more than offsets the reduction in cumulated infections.

3.2. Monopoly

Since price equals cost under perfect competition, but a monopoly charges a markup above cost, price is weakly higher under monopoly and quantity weakly lower. It follows that in case (i), in which $Q_c^* = 0$, we have $Q_m^* = 0$. Case (i) is thus trivially identical across perfect competition and

monopoly. In the remaining cases, perfectly competitive firms are able to make positive sales at price c. By continuity, the monopoly can make positive sales at some small markup above c, implying $Q_m^* > 0$ for $\mathcal{R}_0$ in cases (ii) and above.

To solve for Q_m^* in these other cases, the monopoly's maximization problem can be transformed so that the choice variable is quantity rather than price. The monopoly optimally sets a price to extract the entire private benefit of the marginal consumer, leading to inverse demand $P(Q) = MPB(Q)$. The monopoly chooses Q to maximize $[MPB(Q) - c]Q$ subject to $Q \leq \hat{S}_0$, a constrained maximization problem which can be solved using the Kuhn-Tucker method. Denote the monopoly's marginal revenue by $MR(Q) = \partial[MPB(Q)Q]/\partial Q$. Using (16), one can show the preceding derivative equals

$$MR(Q) = MPB(Q)\left\{1 - \frac{\theta \mathcal{R}_0 Q[1-\Phi(Q)]}{1-\mathcal{R}_0 S_\infty(Q)}\right\}. \tag{26}$$

According to standard Kuhn-Tucker conditions, the solution is an interior optimum satisfying the textbook condition $MR(Q_m^*) = c$ unless $MR(\hat{S}_0) \geq c$, in which case the solution is the corner, $Q_m^* = \hat{S}_0$. The following proposition records this solution for equilibrium monopoly output and compares it to output under perfect competition.

Proposition 2. *For sufficiently low $\mathcal{R}_0$, namely for all $\mathcal{R}_0 \leq \mathcal{R}_0'$, there is no output in equilibrium under either monopoly or perfect competition; i.e., $Q_m^* = Q_c^* = 0$. For sufficiently high $\mathcal{R}_0$, namely for all $\mathcal{R}_0$ satisfying $MR(\hat{S}_0) \geq c$, equilibrium output attains the first best—which for these values of $\mathcal{R}_0$ involves universal vaccination of susceptibles—under both monopoly and perfect competition; i.e., $Q_m^* = Q_c^* = Q^{**} = \hat{S}_0$. Otherwise, equilibrium monopoly output is an interior value $Q_m^* \in (0, \hat{S}_0)$ satisfying $MR(Q_m^*) = c$ and is strictly lower than output under perfect competition; i.e., $Q_m^* < Q_c^*$.*

The proof in Online Appendix A1 fills in details omitted from the sketch preceding the proposition including verifying that the condition $MR(\hat{S}_0) \geq c$ is satisfied for sufficiently high $\mathcal{R}_0$.

The values of Q_m^* and other equilibrium variables are cataloged in Table 2. The entry for Q_m^* (and

by extension the other equilibrium values expressed in terms of Q_m^*) are not provided in analytic form, let alone in closed form, in cases (ii) and (iii). This need not preclude definitive comparative-statics results; one could apply the Implicit Function Theorem to the condition $MR(Q_m^*) = c$ to determine how Q_m^* changes with $\mathcal{R}_0$ in those cases. However, this approach still does not deliver a definitive sign. We can be sure that Q_m^* increases in $\mathcal{R}_0$ for some $\mathcal{R}_0$ in (ii) and (iii)—since Q_m^* must rise from 0 to the first-best quantity $\hat{S}_0$ somewhere in that set by continuity—but we have not been able to rule out the possibility that the monopoly responds to an increase in $\mathcal{R}_0$ in some subintervals by reducing output in order to extract an even larger price increase than otherwise. Despite these challenges, we are able to derive definitive comparative-statics results for some equilibrium variables, reported in the next proposition, proved in Online Appendix A1.

Proposition 3. *Consider the comparative-static effect of $\mathcal{R}_0$ on equilibrium variables under monopoly.*

- *Monopoly profit Π_m^* is weakly increasing in $\mathcal{R}_0$. For $\mathcal{R}_0 \leq \mathcal{R}_0'$, $\Pi_m^* = 0$. For $\mathcal{R}_0 > \mathcal{R}_0'$, Π_m^* is positive and strictly increasing in $\mathcal{R}_0$.*

- *Cumulative infections $R_\infty(Q_m^*)$ attains one or more interior local maxima in $\mathcal{R}_0$, one of which is a global maximum if $\tilde{c} \geq 1-\theta$.*

- *The marginal social benefit MSB_m^* and marginal externality MEX_m^* each attain an interior global maximum in $\mathcal{R}_0$.*

Equilibrium values of selected variables under monopoly are graphed as functions of $\mathcal{R}_0$ as the solid curves in Figure 1. The two market structures overlap in case (i), neither generating any vaccine output. The two market structures overlap again in (iv), both generating the first-best quantity $Q^{**} = \hat{S}_0$. In between—in (ii) and (iii)—the two market structures diverge, with monopoly generating strictly lower output, entailing more total infections over the epidemic, higher marginal private benefit, and lower welfare. The large gap between the dotted and solid curves for intermediate values of $\mathcal{R}_0$ suggests that the distortion arising from the monopoly's exercise of its market power is

worst for moderate levels of infectiousness. The marginal externality can be considerably higher under monopoly for some $\mathcal{R}_0$ but can be slightly lower for some $\mathcal{R}_0$ as the lower monopoly output generates higher marginal private benefit, leaving less residual externality.

The graph of W^* under monopoly illustrates the remarkable possibility that increasing $\mathcal{R}_0$ can increase welfare, impossible under perfect competition according to Proposition 1. Under monopoly, not only do consumers fail to consider the external benefit their vaccination provides other consumers, but the monopoly compounds this by placing negative value on consumption to the extent it reduces others' willingness to pay for a vaccine. An increase in $\mathcal{R}_0$ can mitigate this compound underconsumption problem, providing such a large indirect benefit that it swamps the direct harm from increased infectiousness, leading to an increase in social welfare.

4. Government Subsidies

We have seen that free riding can lead to inefficiently low vaccination under both perfect competition and monopoly. This naturally raises the question of whether the government can intervene to correct the market failure. In this section, we characterize the optimal government subsidy and determine its comparative-static properties.

Assume a benevolent government with the objective of maximizing social welfare commits to a per-course subsidy $G \geq 0$ at the outset of the game. According to standard public-finance results, the economic incidence is the same whether consumers or firms are the statutory target of a tax or subsidy (see, e.g., Fullerton and Metcalf 2002). We adopt the accounting convention that G is paid to firms, in which case the subsidy is equivalent to a reduction in firms' marginal cost from c to $c-G$. Since social welfare is maximized by the first-best quantity Q^{**}, the first-best subsidy G^{**} is that implementing Q^{**}. To accommodate cases in which the government is indifferent among a possibly open set of subsidies maximizing social welfare, we take G^{**} to be the infimum of the set (effectively assuming that the government has lexicographic preferences over welfare and expenditure savings).

It is straightforward to establish a set of broad results for any market structure. Since the marginal vaccine externality is nonnegative by (23), equilibrium output Q^* never exceeds the first best Q^{**}. If $Q^{**} = 0$, then $Q^* = Q^{**} = 0$ as well, implying $G^{**} = 0$ since the first best can be achieved without a subsidy. The proof of the next proposition shows that $Q^{**} = 0$ for all $\mathcal{R}_0$ in a neighborhood above 0, implying that $G^{**} = 0$ in this neighborhood for any market structure.

We can also draw broad conclusions about the optimal subsidy for high values of $\mathcal{R}_0$. By Proposition 2, $Q_m^* = \hat{S}_0$ for sufficiently high values of $\mathcal{R}_0$. For such $\mathcal{R}_0$, $\hat{S}_0 = Q_m^* \leq Q^{**} \leq \hat{S}_0$, implying $Q_m^* = Q^{**}$, in turn implying $G_m^{**} = 0$ since the first best can be achieved without a subsidy under monopoly. The result that $G^{**} = 0$ for sufficiently high $\mathcal{R}_0$ immediately extends to perfect competition or any market structure involving weakly higher output than monopoly.

Having established that $G^{**} = 0$ for intervals of low and high values of $\mathcal{R}_0$ for general market structures, if it can be shown that $G^{**} > 0$ for some intermediate value of $\mathcal{R}_0$, it is immediate that G^{**} is nonmonotonic, attaining a global maximum for some interior $\mathcal{R}_0 \in (0, \infty)$ as the next proposition states. The proof provided in Online Appendix A1 fills in this and other omitted details.

Proposition 4. *For monopoly—or any market structure involving weakly higher output including perfect competition—the optimal government subsidy equals 0 for sufficiently low and sufficiently high* $\mathcal{R}_0$ *and attains an interior global maximum in* $\mathcal{R}_0$.

As Proposition 4 indicates, the optimal subsidy is not monotonically increasing in $\mathcal{R}_0$ as might be inferred based solely on epidemiological considerations but is maximized for an interior value of $\mathcal{R}_0$. The difficulty in addressing a disease depends not only on its infectiousness but also on consumers' response to this infectiousness. Free riding on the vaccination of others disappears with extremely infectious diseases; moderately infectious diseases provide consumers more leeway to free ride, requiring a higher optimal subsidy to address.

We conclude the section with a more precise precise characterization of the optimal subsidy under perfect competition and monopoly provided by the next proposition, proved in Online Ap-

pendix A1.

Proposition 5. *The optimal government subsidies under perfect competition and monopoly depend on the first-best output, Q^{**}.*

- *If the first best involves no output, i.e., $Q^{**} = 0$, then no subsidy is needed under either market structure: $G_c^{**} = G_m^{**} = 0$.*
- *If the first best involves an interior output level, i.e., $Q^{**} \in (0, \hat{S}_0)$, then the optimal government subsidy under perfect competition corrects for the externality at the target output: $G_c^{**} = MEX(Q^{**})$. The optimal subsidy under monopoly is related but is scaled up to offset the monopoly's only partial pass-through: $G_m^{**} = MEX(Q^{**})\hat{S}_0/(\hat{S}_0 - \theta Q^{**})$.*
- *If the first best involves universal vaccination of susceptibles, i.e., $Q^{**} = \hat{S}_0$, then the optimal subsidy under perfect competition bridges the gap, if any, between consumers' marginal private benefit under universal vaccination and marginal cost, c: $G_c^{**} = \max[0, c - MPB(\hat{S}_0)]$. The optimal subsidy under monopoly is related but needs to bridge a larger gap due to the monopoly markup:*

$$G_m^{**} = \max\left[0, c - MPB(\hat{S}_0) + \left(\frac{\theta}{1-\theta}\right) MEX(\hat{S}_0)\right]. \tag{27}$$

Across all cases, the optimal subsidy is weakly higher under monopoly than perfect competition, $G_m^{**} \geq G_c^{**}$, strictly so if $Q_m^* \in (0, \hat{S}_0)$.

5. Universal Vaccination

Under both market structures, equilibrium attains universal vaccination of susceptibles for a nonempty set of parameters. Two reservoirs of infection motivate the marginal consumer to purchase at a positive price even when all other consumers also purchase. With an imperfectly effective vaccine

($\theta < 1$), some vaccinated consumers remain susceptible and able to transmit the disease to others. Even in the limit of perfect efficacy $\theta \uparrow 1$, however, the $\hat{I}_0$ initially infected remain a reservoir. Previous game-theoretic analyses finding that a perfectly effective vaccine would never be universally purchased at a positive price (Geoffard and Phillipson 1997, May 2000, Bauch and Earn 2004) omitted this feature of the SIR model.

It is obvious that equilibrium under perfect competition must attain universal vaccination if the disease is infectious enough. Even a small reservior of infecteds $\hat{I}_0$, when combined with a sufficiently high $\mathcal{R}_0$, generates high enough infection risk to motivate the marginal consumer to purchase at any fixed $c < H$. It is less obvious that universal vaccination is attained in a monopoly equilibrium. Given that its monopoly price is endogenous, not fixed, the monopoly might respond to an increase in $\mathcal{R}_0$ by raising price, keeping output short of universal vaccination. For sufficiently high $\mathcal{R}_0$, the marginal consumer is almost certain to contract the disease from the reservior of $\hat{I}_0$ infecteds irrespective of how many susceptibles are vaccinated. The monopoly serves all consumers at price approaching the marginal private benefit of being protected against certain infection, leaving little room for any further price increase without losing most customers.

The next proposition, proved in Online Appendix A1, provides a simple necessary and sufficient condition for universal vaccination with a perfectly effective vaccine to obtain in equilibrium under each market structure.

Proposition 6. *In the limit of a perfectly effective vaccine ($\theta \uparrow 1$), universal vaccination of susceptibles is attained in equilibrium under perfect competition if and only if $1-e^{-\mathcal{R}_0\hat{I}_0} > \tilde{c}$ and under monopoly if and only if $(1-e^{-\mathcal{R}_0\hat{I}_0})(1-\mathcal{R}_0\hat{S}_0e^{-\mathcal{R}_0\hat{I}_0}) > \tilde{c}$.*

It can be shown that both conditions hold for sufficiently high $\mathcal{R}_0$: the factors on the left-hand side of both conditions equal 1 in the limit $\mathcal{R}_0 \uparrow \infty$, and $1 > \tilde{c}$ by assumption (19). It is also obvious that neither condition holds for any finite $\mathcal{R}_0$ when $\hat{I}_0 = 0$, reflecting the fact that an initial stock of infecteds is required to generating demand for a perfectly effective vaccine under universal

vaccination program.

6. Increasing Social Returns

Typical products exhibit concave social benefits. The underlying logic is that initial units provide higher marginal social benefits than subsequent units since highest-value uses are served first, with subsequent units allocated to lower-value uses. Epidemiological externalities may lead this logic to fail with vaccines. Vaccinating a few individuals may do little to slow the spread of an epidemic if susceptibles are likely contract the disease from the many remaining unvaccinated people in any event. Doubling coverage may more than double the social benefit if the additional coverage is needed to make a dent in the infection rate.

In this section, we analyze conditions under which vaccines exhibit increasing rather than diminishing social returns. To this point we have assumed that any amount of vaccine can be produced at the constant marginal cost c. In reality, capacity constraints may prevent production up to the point that marginal social benefit equals production cost; rationing may be required. With the population divided into regional subunits experiencing relatively independent epidemiological processes because of restricted travel flows, it is natural to ask whether vaccine should be spread across regions in proportion to their populations (as considerations of fairness or heterogeneity in value within each region might dictate) or whether the benefits would be larger if vaccine were concentrated in fewer regions (chosen by lottery if urgency of need in certain regions does not provide sufficient reason for concentrating vaccine there).

Formally, a vaccine exhibits increasing social returns if $MSB(Q)$ is increasing in Q. Differentiating (22), substituting from (13), and rearranging yields

$$\frac{\partial MSB(Q)}{\partial Q} = \frac{\theta^2 H \mathcal{R}_0 S_\infty(Q) S_0(Q) \Phi(Q)}{[1-\mathcal{R}_0 S_\infty(Q)]^3} \{\mathcal{R}_0[S_0(Q)+S_\infty(Q)]-2\}. \tag{28}$$

All the factors on the right-hand side are definitively positive by Lemma 5 except for the last. Thus, the sign of the last factor in braces determines whether the vaccine exhibits increasing social returns. Rearranging gives the following proposition.

Proposition 7. *The Qth unit of vaccine exhibits increasing social returns if and only if*

$$\mathcal{R}_0 \left[\frac{S_0(Q) + S_\infty(Q)}{2} \right] > 1. \tag{29}$$

Earlier, we identified the inequality $\mathcal{R}_0 S_0(Q) > 1$ as necessary and sufficient for the epidemic to grow rather than decline from the start. Condition (29) is more stringent. Instead of requiring the initial value of the effective reproductive number, $\mathcal{R}_0 S_0(Q)$, to exceed 1, it requires the average of the initial value $\mathcal{R}_0 S_0(Q)$ and the final value $\mathcal{R}_0 S_\infty(Q)$ to exceed 1. By Lemma 4, $\mathcal{R}_0 S_\infty(Q) < \mathcal{R}_0 S_0(Q)$. Proposition 7 can thus be interpreted as saying that unit Q of the vaccine exhibits increasing social returns if the potential not just for immediate but for sustained epidemic expansion is sufficiently high.

The next proposition provides simpler sufficient conditions for the vaccine to exhibit increasing social returns at initial output levels and at all output levels. It is proved in Online Appendix A1 as a straightforward corollary of Proposition 7.

Proposition 8. *The vaccine exhibits initial increasing social returns—i.e., at an output level of* $Q = 0$*—if* $\mathcal{R}_0 \hat{S}_0 \geq 2$*. The vaccine exhibits everywhere increasing social returns—i.e., at all output levels* $Q \in (0, \hat{S}_0)$*—if* $\mathcal{R}_0 \hat{S}_0 \geq 2/(1-\theta)$.

According to Proposition 8, if a federal authority only has access to a small stockpile of a vaccine to allocate across several similar states with independent epidemiological processes, allocating the entire stockpile to one state would produce more social benefit than spreading it evenly across them if $\mathcal{R}_0 \hat{S}_0 > 2$. If, for example, $\hat{S}_0 = 0.8$ in each state, then concentrating the vaccine would be efficient for any $\mathcal{R}_0 > 2.5$. If the more stringent condition $\mathcal{R}_0 \hat{S}_0 > 2/(1-\theta)$ holds, then even a starker form of

concentration is efficient: not just for very small stockpiles but for any size, the federal authorities should vaccinate all susceptibles in one state before moving to the next. The starkness of the policy hinges on the modeled consumer homogeneity: if each state has some vulnerable consumers with a high benefit from vaccinating, a higher bar on $\mathcal{R}_0$ would need to be cleared for concentrating vaccines in one state to be more efficient than serving high-value consumers everywhere first.

7. Vaccines Versus Drugs

Commentators on the pharmaceutical industry frequently suggest that firms are biased in favor of developing drugs rather than vaccines. Kremer and Snyder (2015) lists a variety of reasons for this bias, ranging from vaccines' complexity relative to drug molecules, to the scale often needed for vaccine clinical trials, to the evaporation of consumers' private disease-risk information when making drug purchases (the focus of that paper).

The epidemiological externality analyzed in this paper provides another rationale. By preventing individuals from becoming infected, vaccines curtail their transmission of the disease to others. The reduction in others' disease risk is a public good that reduces others' willingness to pay for a vaccine. This public-good feature distinguishes vaccines from some drugs that treat symptoms without curing the underlying disease or inhibiting transmission. Firms would have more of incentive to develop a drug that does not have this demand-reducing public-good feature than a similarly effective vaccine.

To quantify a monopoly's bias toward a drug and against a vaccine, consider a drug that is similar in all ways to the vaccine analyzed to this point except that the drug does not reduce disease transmission. Finding the right normalization to make drug and vaccine costs equivalent is somewhat delicate since at equal marginal production costs c the total cost of serving a population with a drug is lower if it only needs to be administered to infected consumers rather than the whole population in advance as with a vaccine. We finesse this normalization issue by assuming both products are costless to produce and administer, i.e., $c = 0$. Assume the drug is effective with probability θ.

Efficacy for the drug means it eliminates any harm from the symptoms experienced by infected individuals but does not prevent them from transmitting the disease to susceptible individuals. One course of the drug is sufficient to eliminate symptoms for the rest of the consumer's life. If this first course is ineffective for an individual, further courses will be ineffective for that individual as well.

Having previously computed monopoly profit and welfare from a vaccine, respectively Π^*_{mv} and W^*_{mv}, it remains to compute the analogous variables for a drug, respectively Π^*_{md} and W^*_{md}. For all $\mathcal{R}_0 > 0$, the drug monopoly can charge $P^*_{md} = \theta H$ to the $\hat{I}_0$ individuals infected at the moment the drug is developed as well as the $\hat{S}_0 - S_\infty(0)$ individuals who become infected at some point afterwards, yielding drug profit

$$\Pi^*_{md} = \theta H \left[\hat{I}_0 + \hat{S}_0 - S_\infty(0) \right]. \tag{30}$$

To compute equilibrium welfare with a drug, the $\hat{I}_0$ individuals infected initially along with the $\hat{S}_0 - S_\infty(0)$ infected later obtain health benefit H with probability θ from the drug. The $S_\infty(0)$ remaining susceptibles are never infected and obtain health benefit H with certainty, yielding the following expression for equilibrium welfare after rearranging:

$$W^*_{md} = H \left[(1-\theta) S_\infty(0) + \theta(\hat{I}_0 + \hat{S}_0) \right]. \tag{31}$$

Comparing these expressions against the analogous entries in Table 2 for a vaccine leads to the next proposition. Details behind the proof are provided in Online Appendix A1. The proposition uses the notation $\Delta\Pi^*_m = \Pi^*_{md} - \Pi^*_{mv}$ and $\Delta W^*_m = W^*_{md} - W^*_{mv}$ for differences between equilibrium variables for the two products and $\Delta W^{**} = W^{**}_d - W^{**}_v$ for difference between first-best welfare.

Proposition 9. *Suppose $c = 0$. For all $\mathcal{R}_0 > 0$, monopoly profit is strictly higher from a drug than vaccine, i.e., $\Delta\Pi^*_m > 0$. The profit advantage from a drug $\Delta\Pi^*_m$ approaches its lowest value $\inf_{\mathcal{R}_0 > 0} \Delta\Pi^*_m = \theta H \hat{I}_0$ in the limits of extremely low and extremely high $\mathcal{R}_0$ and attains an interior global maximum in $\mathcal{R}_0$. Welfare is higher with a drug than a vaccine for extremely low and ex-*

tremely high $\mathcal{R}_0$*:* $\lim_{\mathcal{R}_0 \downarrow 0} \Delta W_m^* > 0$ *and* $\lim_{\mathcal{R}_0 \uparrow \infty} \Delta W_m^* > 0$*. However, there exist parameters for which welfare is higher with a vaccine, i.e.,* $\Delta W_m^* < 0$*.*

According to the proposition, the monopoly is biased toward the drug for all parameters, and this bias leads the firm to choose the socially inferior product for some parameters. For other parameters, the drug provides higher welfare than the vaccine. Two such cases are provided by the extremes $\mathcal{R}_0 \downarrow 0$ and $\mathcal{R}_0 \uparrow \infty$, examined in turn. Equilibrium welfare never falls below $\theta H \hat{I}_0$ for a drug monopoly, even for extreme values of $\mathcal{R}_0$. Administering a drug to the $\hat{I}_0$ initially infected provides a social benefit even if $\mathcal{R}_0$ is so low that the infection does not spread to others. A vaccine cannot provide this social benefit because it is useless unless administered prior to infection in the model. Thus, equilibrium welfare is higher with a drug than vaccine in the limit $\mathcal{R}_0 \downarrow 0$. Equilibrium welfare is also higher with a drug than vaccine in the limit $\mathcal{R}_0 \uparrow \infty$. The externality associated with vaccine disappears with an infinitely infectious disease because susceptibles are certain to contract the disease, if no one else, from the $\hat{I}_0$ initially infected. Hence, apart from the drug's remaining social benefit of treating the $\hat{I}_0$ initially infected, the drug and vaccine provide equal welfare in the limit $\mathcal{R}_0 \uparrow \infty$. The opposing welfare factors—the drug helps initially infected but the vaccine reduces subsequent spread to others—prevent many firm conclusions from being drawn about the sign of the equilibrium or first-best welfare differentials.

8. Covid Calibrations

This section provides a calibration using parameters drawn from the current Covid-19 pandemic. The calibration is meant as an illustration, not a forecast. Our present model is too stylized along many dimensions to provide accurate forecasts, abstracting from heterogeneity in infectiousness, heterogeneity in costs of prevention among consumers, and mortality effects of disease. Certain parameters are set to convenient limiting values rather than being estimated from data. A host of political-economy considerations lead real-world vaccine markets to depart from our theoretical

construct of firms selling directly to individual consumers without third-party funding.

Based on a meta-analysis of studies of the ancestral strain of Covid-19 (Liu et al. 2020), we set $\mathcal{R}_0 = 2.8$. We take estimates of needed population parameters as of October 2020, calibrating the counterfactual effect of the arrival of a vaccine when emergency use was starting to be approved for the available Covid vaccines. We use estimates from U.K. government agencies, which provide some of the best estimates for a developed country then available. Based on U.K. Office for National Statistics (2020), we take the proportion of infected at that time to be $\hat{I}_0 = 0.19\%$ and the proportion of recovered to be $\hat{R}_0 = 6.2\%$, implying $\hat{S}_0 = 1 - \hat{I}_0 - \hat{R}_0 = 93.6\%$. Based on Public Health England (2021), we set $\theta = 0.8$, the midpoint of the range of estimated efficacy of two doses of the Pfizer vaccine against Covid infection (including both symptomatic and asymptomatic). For rescaled cost, $\tilde{c} = c/\theta H$, we take the limiting case of a costless vaccine, $\tilde{c} \downarrow 0$, reflecting the low cost c for existing vaccines, especially in comparison to the potential disease harm H, as documented further in Online Appendix A4.

These parameters put us in case (iii) of Tables 1 and 2, in which perfect competition attains the first best of universal vaccination but monopoly does not. Using numerical methods to compute $S_\infty(Q)$ in (12) and to optimize monopoly profit, we find that the monopoly price is set to 49% of the harm from contracting the disease. At this price, only 51% of susceptible consumers buy, generating welfare equal to 59% of the available health benefit. The optimal subsidy required to generate the first best under monopoly is enormous, over three times the equilibrium monopoly price.

The effective reproductive number, $\mathcal{R}_0\hat{S}_0 = 2.6$, surpasses the threshold of 2 sufficient for initially increasing social returns according to Proposition 8 but not the threshold for everywhere increasing social returns, which equals $2/(1-\theta) = 10$ given calibrated efficacy. Examing equation (29) for a range of quantities, one can determine that increasing returns persists through output equal to 63% of the susceptible population. Supposing that a stockpile has to be allocated to two identical states with independent epidemiological processes, concentrating the entire stockpile in one state

generates higher welfare than dividing equally until the stockpile exceeds 81% of the population of one state. Larger stockpiles than this are more efficiently divided equally between the states. Overall, the results suggest that social returns to Covid vaccines can be strongly increasing.

For the calibrated parameters, monopoly profit and welfare are both higher with a drug than a vaccine (i.e., $\Delta\Pi_m^* > 0$ and $\Delta W_m^* > 0$). The monopoly's bias toward a drug thus does not lead to a distortion in the calibration.

To measure the sensitivity of the outcomes to infectiousness, we repeat the calibration holding all parameters constant at their October 2020 levels except $\mathcal{R}_0$, replaced with its higher value for the Delta variant: $\mathcal{R}_0 = 5.1$ according to Liu and Rocklöv's (2021) meta-analysis.[12] Monopoly quantity increases to 74% of the susceptible population, despite a 28% increase in the monopoly price. The higher vaccination rate leads to a 6% increase in equilibrium monopoly welfare in the Delta calibration compared to that for ancestral Covid despite Delta's greater infectiousness. That welfare may rise with infectiousness due to the response of economic agents appears to be not just a theoretical curiosity but may hold for realistic parameters. The increase in infectiousness reduces equilibrium welfare under perfect competition since the vaccination rate is already as high as possible when $\mathcal{R}_0 = 2.8$, so the increase to $\mathcal{R}_0 = 5.1$ results in a direct increase in disease burden.

9. Conclusion

We analyzed the market for technologies that, by protecting individuals against an infectious disease, reduce transmission to others, a positive externality. Though the analysis applies to a variety of technologies such as circumcision, bed nets, or social distancing, the discussion focused on vaccines

[12]The second calibration is a thought experiment, not meant to reflect the situation when Delta emerged, which involved several coexisting strains, a vaccine campaign already underway optimized against an earlier strain, and different proportions of susceptibles, infecteds, and recovered individuals.

for concreteness. Vaccines (and most of the other aforementioned technologies) are not pure public goods since they are physical products that exhibit rivalry and excludability in consumption, yet they share with public goods the feature that one's consumption reduces others' demand for that product, a feature that can potentially lead to large distortions in consumption and production decisions.

Such distortions and policy correctives are the focus of this paper. To study them, we constructed a theoretical model of the vaccine market involving economic agents basing their consumption and production decisions on rational expectations of the disease's evolution consistent with a standard SIR epidemiological model. Within that general framework, we made specific modeling choices to suit an intensive vaccine campaign against an epidemic disease such as Covid-19 expected to wane before generations turn over. We pursued a comprehensive account of equilibrium variables such as price, quantity, profit, and welfare across a variety of market structures ranging from perfect competition to monopoly and studied how those variables change in response to an increase in the infectiousness of the disease as measured by $\mathcal{R}_0$, among other parameters. Since our comparative-statics results are derived in a model where sales are made on a private market without government intervention, they are perforce counterfactual for real-world vaccine markets in which the potential for severe market failures and lost lives lead policymakers to intervene. Understanding how markets perform in the counterfactual absence of intervention, however, is a useful step toward characterizing optimal interventions, as we seek to do.

Perhaps the variable of most interest was the equilibrium marginal externality. Across the range of market structures studied, we found that the equilibrium marginal externality peaks for intermediate rather than extreme values of $\mathcal{R}_0$. For low levels of $\mathcal{R}_0$, one consumer's vaccination provides little benefit to others because there is little chance the consumer would have infected them. For high levels of $\mathcal{R}_0$, one consumer's vaccination provides little benefit to others since they will most likely contract it from a different source anyway. Other outcome variables also peak for intermediate values of $\mathcal{R}_0$—outcome variables including G^{**} (the minimal subsidy necessary to obtain the

first-best vaccine quantity) and $\Delta\Pi_m^*$ (extra monopoly profit from a drug that does not exert the epidemiological externality compared to a vaccine that does). Moderately infectious diseases may exhibit the greatest distortions and be prime targets for subsidy.

Across the range of market structures studied, universal vaccination of susceptible consumers is obtained in equilibrium for sufficiently high $\mathcal{R}_0$. This is true even for a perfectly effective vaccine, contrasting some impossibility results in the previous literature. The key to this result is the presence of the $\hat{I}_0$ infected individuals at vaccine rollout. Even if all other susceptibles are successfully vaccinated, the threat of contracting the disease from the $\hat{I}_0$ infected induces the marginal consumer to purchase the vaccine at a positive price. In the limit of an arbitrarily infectious disease, free riding is eliminated since the risk of contracting the disease from even a small $\hat{I}_0$ approaches 1. The presence of the $\hat{I}_0$ infecteds also raises the possibility that a vaccine with a positive epidemiological externality can be welfare-dominated by a drug without it: if the externality is small, welfare may be driven by the advantage of the drug in treating the $\hat{I}_0$ infecteds for whom the vaccine arrives too late to help (assuming the vaccine must be administered prior to infection to be effective).

We derived simple sufficient conditions under which vaccination exhibits increasing social returns: social returns are initially increasing if $\mathcal{R}_0\hat{S}_0 \geq 2$ and everywhere increasing if $\mathcal{R}_0\hat{S}_0 \geq 2/(1-\theta)$. If the first condition holds, a small supply is more efficiently concentrated in a single region; and if the second condition holds, a first region should be completely served before moving to a second regardless of the supply. These stark implications for concentrating supplies hinge on the homogeneity of consumers in the model but raise the possibility that equitable allocation can lead to inefficiency.

In our calibration to the ancestral strain of Covid-19, we found that a competitively supplied vaccine would attain the first best of universal vaccination, but a monopoly would not. A monopoly—at least one unconstrained by public repugnance (à la Roth 2007) against "profiting during a pandemic"—sets such a high price that only about half of susceptibles buy. Correcting this distortion requires an

enormous subsidy, equal to over three times the equilibrium monopoly price.

Such a subsidy is likely to be prohibitive in many practical settings, pointing to the appeal of an alternative policy—bulk purchases negotiated by the government on behalf of consumers—which could achieve the first best at a much lower expenditure level. In fact, negotiated bulk purchases were used for Covid-19 vaccines as well as vaccines for many childhood diseases. Our results for equilibrium on the private market remain relevant if, following Kremer and Snyder (2015, Section IV.C), one assumes the private market provides the threat point for Nash bargaining over the bulk purchase.

The Covid-19 calibration exhibited increasing social returns, not everywhere, but through a substantial range (63%) of the susceptible population. The presence of strongly increasing social returns argues for subsidizing aggressive investment to boost capacity beyond this point if concentrating supplies in few countries is either unpalatable or outweighed in by the benefit of vaccinating vulnerable subpopulations in every country.

Comparing the calibration for the ancestral Covid strain to a second calibration for the Delta variant, doubling infectiousness raises equilibrium welfare under monopoly by inducing more consumers to purchase even at the higher monopoly price, offsetting the direct increase in disease burden. Equilibrium welfare falls under perfect competition since the universal vaccination already attained with the less infectious ancestral strain leaves no room for a further increase in the vaccination rate to offset the increase in disease burden.

Key results—including that moderately rather than severely infectious diseases may be prime targets for subsidy—are robust to a variety of modeling alternatives. The results derived here for perfect competition and monopoly are extended in Online Appendix A3 to Cournot competition among n firms, nesting these other market structures as special cases. The assumption of homogeneous consumers is relaxed in Online Appendix A4, which allows for heterogeneity in consumer harm, H_i, as an illustrative example. Our companion paper (Goodkin-Gold et al. 2022) maintains the

SIR framework but adopts alternative modeling assumptions suited to an endemic disease against which new cohorts are continuously vaccinated in the steady state. The analysis in that paper is relevant to diseases such as measles, HIV, and even Covid-19 if continued emergence of new variants lead it to persist in the population over the long run. Despite mathematical differences—unlike here, steady states in the companion paper have simple closed-form expressions—the results are remarkably similar, down to the shape of the graphs of outcome variables against $\mathcal{R}_0$, which resemble those in Figure 1.

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TABLE 1: EQUILIBRIUM VARIABLES UNDER PERFECT COMPETITION AS FUNCTIONS OF $\mathcal{R}_0$

	Case		
	(i)	(ii)	(iii), (iv)
Variable	$\mathcal{R}_0 \in (0, \mathcal{R}_0']$	$\mathcal{R}_0 \in (\mathcal{R}_0', \mathcal{R}_0'']$	$\mathcal{R}_0 \in (\mathcal{R}_0'', \infty)$
P_c^*	c	c	c
Q_c^*	0	$\frac{1}{\theta}\left\{\hat{S}_0 + \frac{1}{\tilde{c}}\left[\frac{1}{\mathcal{R}_0}\ln(1-\tilde{c}) + \hat{I}_0\right]\right\}$	$\hat{S}_0$
Π_c^*	0	0	0
$R_\infty(Q_c^*)$	$1 - S_\infty(0)$	$1 - \hat{S}_0 - \hat{I}_0 + \frac{1}{\mathcal{R}_0}\lvert\ln(1-\tilde{c})\rvert$	$1 - S_\infty(\hat{S}_0) - \theta\hat{S}_0$
MPB_c^*	$\theta H \Phi(0)$	c	$\theta H \Phi(\hat{S}_0)$
MSB_c^*	$\frac{\theta H \Phi(0)}{1 - \mathcal{R}_0 S_\infty(0)}$	$\frac{\theta H \tilde{c}^2}{\tilde{c} + (1-\tilde{c})[\ln(1-\tilde{c}) + \mathcal{R}_0 \hat{I}_0]}$	$\frac{\theta H \Phi(\hat{S}_0)}{1 - \mathcal{R}_0 S_\infty(\hat{S}_0)}$
MEX_c^*	$\frac{\theta H \Phi(0) \mathcal{R}_0 S_\infty(0)}{1 - \mathcal{R}_0 S_\infty(0)}$	$\frac{\theta H \tilde{c}(1-\tilde{c})[\lvert\ln(1-\tilde{c})\rvert - \mathcal{R}_0 \hat{I}_0]}{\tilde{c} + (1-\tilde{c})[\ln(1-\tilde{c}) + \mathcal{R}_0 \hat{I}_0]}$	$\frac{\theta H \Phi(\hat{S}_0) \mathcal{R}_0 S_\infty(\hat{S}_0)}{1 - \mathcal{R}_0 S_\infty(\hat{S}_0)}$
W_c^*	$H S_\infty(0)$	$H(1-\tilde{c})\hat{S}_0$	$H[S_\infty(\hat{S}_0) + \theta(1-\tilde{c})\hat{S}_0]$

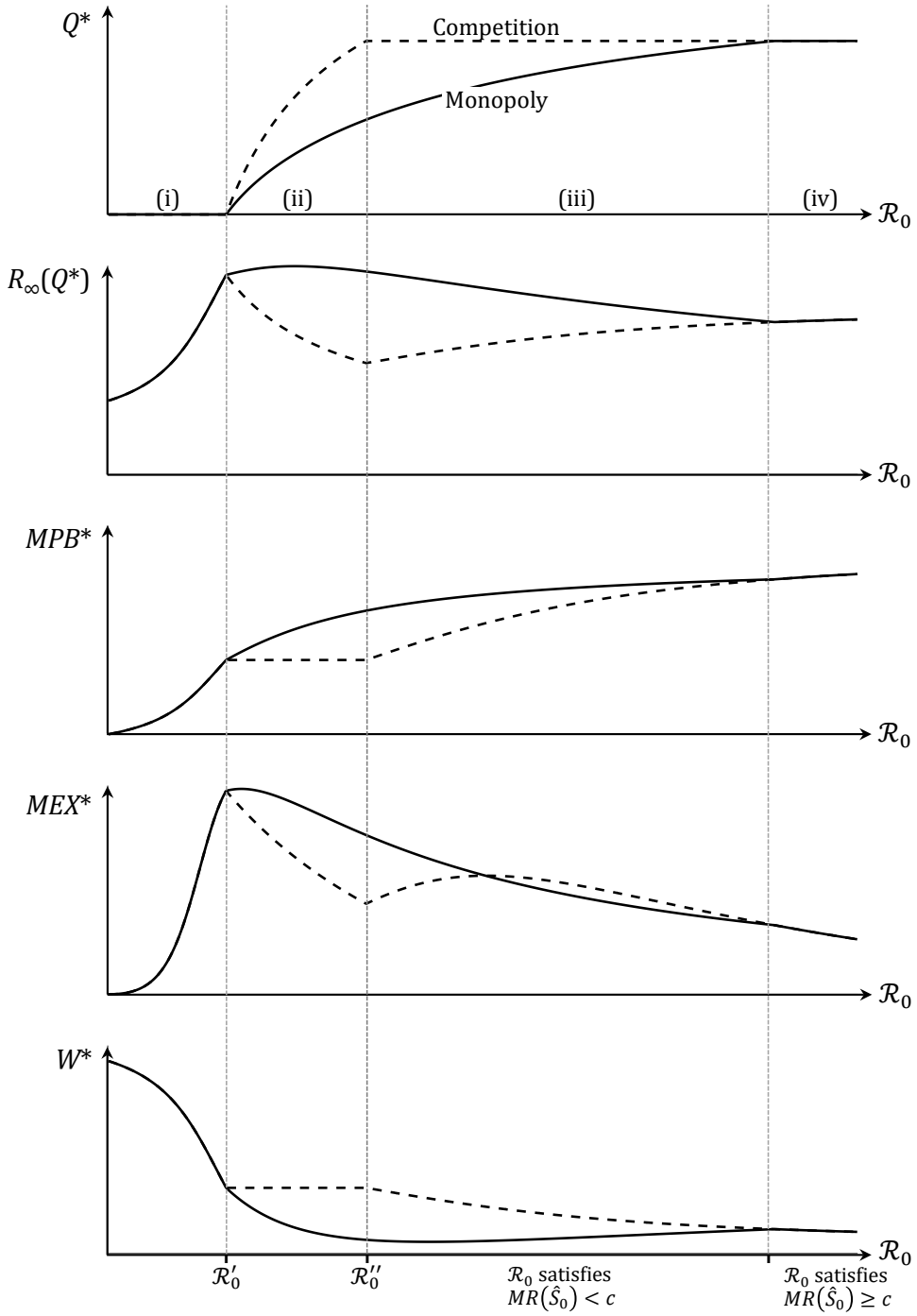
Notes: See Online Appendix A1 for verification of table entries. Computable expressions for $S_\infty(0)$ and $S_\infty(\hat{S}_0)$ can be derived from equation (12). Those expressions can be substituted into equation (14) to compute infection probabilities $\Phi(0)$ and $\Phi(\hat{S}_0)$. The distinction between cases (iii) and (iv) in the last column, relevant for monopoly in the next table, is irrelevant for perfect competition here.

TABLE 2: EQUILIBRIUM VARIABLES UNDER MONOPOLY AS FUNCTIONS OF $\mathcal{R}_0$

	Case		
	(i)	(ii), (iii)	(iv)
Variable	$\mathcal{R}_0 \in (0, \mathcal{R}_0']$	$\mathcal{R}_0 > \mathcal{R}_0'$ but $MR(\hat{S}_0) < c$	$\mathcal{R}_0$ satisfies $MR(\hat{S}_0) \geq c$
P_m^*	†	$\theta H \Phi(Q_m^*)$	$\theta H \Phi(\hat{S}_0)$
Q_m^*	0	Solution to $MR(Q_m^*) = c$	$\hat{S}_0$
Π_m^*	0	$\theta H[\Phi(Q_m^*) - \tilde{c}]Q_m^*$	$\theta H[\Phi(\hat{S}_0) - \tilde{c}]\hat{S}_0$
$R_\infty(Q_m^*)$	$1 - S_\infty(0)$	$1 - S_\infty(Q_m^*) - \theta Q_m^*$	$1 - S_\infty(\hat{S}_0) - \theta \hat{S}_0$
MPB_m^*	$\theta H \Phi(0)$	$\theta H \Phi(Q_m^*)$	$\theta H \Phi(\hat{S}_0)$
MSB_m^*	$\dfrac{\theta H \Phi(0)}{1 - \mathcal{R}_0 S_\infty(0)}$	$\dfrac{\theta H \Phi(Q_m^*)}{1 - \mathcal{R}_0 S_\infty(Q_m^*)}$	$\dfrac{\theta H \Phi(\hat{S}_0)}{1 - \mathcal{R}_0 S_\infty(\hat{S}_0)}$
MEX_m^*	$\dfrac{\theta H \Phi(0) \mathcal{R}_0 S_\infty(0)}{1 - \mathcal{R}_0 S_\infty(0)}$	$\dfrac{\theta H \Phi(Q_m^*) \mathcal{R}_0 S_\infty(Q_m^*)}{1 - \mathcal{R}_0 S_\infty(Q_m^*)}$	$\dfrac{\theta H \Phi(\hat{S}_0) \mathcal{R}_0 S_\infty(\hat{S}_0)}{1 - \mathcal{R}_0 S_\infty(\hat{S}_0)}$
W_m^*	$H S_\infty(0)$	$H[S_\infty(Q_m^*) + \theta(1 - \tilde{c})Q_m^*]$	$H[S_\infty(\hat{S}_0) + \theta(1 - \tilde{c})\hat{S}_0]$

Notes: Equation (12) provides a formula for computing $S_\infty(0)$, $S_\infty(\hat{S}_0)$, and $S_\infty(Q_m^*)$. Equation (14) provides a formulas for computing $\Phi(0)$, $\Phi(\hat{S}_0)$, and $\Phi(Q_m^*)$. The distinction between cases (ii) and (iii) in the middle column, relevant for perfect competition in previous table, is irrelevant for monopoly here. As indicated in equation (26), $MR(\hat{S}_0)$ is a function of $\mathcal{R}_0$, though for brevity, $\mathcal{R}_0$ is not included in the argument list for MR. The set of $\mathcal{R}_0$ satisfying $MR(\hat{S}_0) \geq c$ need not form an interval but does include all sufficiently high $\mathcal{R}_0$. †Any value $P_m^* \geq c$ is consistent with zero sales in equilibrium.

FIGURE 1: GRAPHS OF SELECTED EQUILIBRIUM VARIABLES AS FUNCTIONS OF $\mathcal{R}_0$



Notes: Graph of formulas provided in Tables 1 and 2, illustrated for specific parameter values ($\theta = 0.7$, $c = 0.3$, $H = 1$, $\hat{I}_0 = 0.1$, $\hat{S}_0 = 0.8$). Dashed curves represent equilibrium under perfect competition and solid curves under monopoly. Where curves overlap, solid curve represents both industry structures. Graphs produced using Matlab; for those equilibrium variables lacking closed-form expressions, numerical methods and the built-in function $\bar{L}$ are used to generate the graphs.

Online Appendixes
Optimal Vaccine Subsidies for Epidemic Diseases

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This document contains a series of online appendixes supplementing the published article. The appendixes provide analytical proofs or extensions omitted for space considerations. Appendix A1 provides proofs, omitted from the text, of lemmas and propositions. The proofs are streamlined by the inclusion of additional lemmas, stated and proved in Appendix A1. Appendix A2 provides additional documentation for the assumption made in the calibration section for taking the limit $\tilde{c} \downarrow 0$. Appendix A3 analyzes Cournot competition among n firms. This analysis nests perfect competition studied in the article in the limit $n \uparrow \infty$ and also nests monopoly studied in the article setting $n = 1$. Appendix A4 extends the analysis of homogeneous consumers to allow consumers to vary in disease harm H_i. Appendix A5 extends the model to allow for a second preventive technology, competitively supplied, possibly interpreted as social distancing. We show that the basic comparative-static results for vaccine-market equilibrium are essentially unchanged.

Appendix A1. Proofs

Proof of Lemma 1

We begin by proving the claims about $I_t(Q)$. Substituting (10) into (3) and rearranging yields

$$\frac{\dot{I}_t(Q)}{I_t(Q)} = \alpha[\mathcal{R}_0 S_t(Q) - 1]. \tag{A1}$$

Recognizing the left-hand side as $\partial \ln I_t(Q)/\partial t$ and integrating yields

$$\int_0^t \frac{\partial \ln I_\tau(Q)}{\partial \tau} d\tau = \int_0^t \alpha[\mathcal{R}_0 S_\tau(Q) - 1] d\tau. \tag{A2}$$

Invoking the Fundamental Theorem of Calculus, taking exponentials, and rearranging yields

$$I_t(Q) = I_0(Q) \exp\left(\int_0^t \alpha[\mathcal{R}_0 S_\tau(Q) - 1] d\tau \right). \tag{A3}$$

Since $I_0(Q) = \hat{I}_0 > 0$ by assumption, $I_t(Q)$ is the product of two positive factors.

Turn next to proving the claims about $S_t(Q)$. Rearranging (4), $I_t(Q) = \dot{R}_t(Q)/\alpha$. Substituting into (2) and rearranging yields $\dot{S}_t(Q)/S_t(Q) = -(\beta/\alpha)\dot{R}_t(Q) = -\mathcal{R}_0 \dot{R}_t(Q)$ by (10). Recognizing $\dot{S}_t(Q)/S_t(Q) = \partial \ln S_t(Q)/\partial t$ and integrating between $t' \geq 0$ and $t'' \geq t'$ yields

$$\int_{t'}^{t''} \frac{\partial \ln S_\tau(Q)}{\partial \tau} d\tau = -\int_{t'}^{t''} \frac{1}{\alpha} \dot{R}_\tau(Q) d\tau. \tag{A4}$$

Invoking the Fundamental Theorem of Calculus, taking exponentials, and rearranging yields

$$S_{t''}(Q) = S_{t'}(Q)e^{\mathcal{R}_0[R_{t'}(Q)-R_{t''}(Q)]}. \tag{A5}$$

Substituting $t' = 0$ and $t'' = t$ into (A5) yields

$$S_t(Q) = S_0(Q)e^{\mathcal{R}_0[R_0(Q)-R_t(Q)]}. \tag{A6}$$

Now $S_0(Q) = \hat{S}_0 - \theta Q > \hat{S}_0 - Q \geq 0$, where the first step holds by (6), the second by $\theta < 1$, and the third by $Q \in [0, \hat{S}_0]$. The right-hand side of (A6) is thus the product of two positive factors. *Q.E.D.*

Proof of Lemma 2

Substituting $I_t(Q) > 0$ into (4) yields $\dot{R}_t(Q) > 0$, implying $R_{t''}(Q) > R_{t'}(Q)$ for $t'' > t'$, implying $e^{\mathcal{R}_0[R_{t'}(Q)-R_{t''}(Q)]} < 1$. Since $S_{t'}(Q) > 0$ by Lemma 1, $S_{t''}(Q) \leq S_{t'}(Q)$ by (A5). *Q.E.D.*

Proof of Lemma 3

Since $I_t(Q) > 0$ by Lemma 1, the sign of $\dot{I}_t(Q)$ is determined by the value of $\mathcal{R}_0 S_t(Q)$ relative to 1 by (A1). First, suppose $\mathcal{R}_0 S_0(Q) \leq 1$. Consider any $t > 0$. Lemma 2 implies $S_t(Q) < S_0(Q)$, in turn implying $\mathcal{R}_0 S_t(Q) < \mathcal{R}_0 S_0(Q) \leq 1$. Substituting $\mathcal{R}_0 S_t(Q) < 1$ into (A1) implies $\dot{I}_t(Q) < 0$.

Next, suppose $\mathcal{R}_0 S_0(Q) > 1$. Substititing $t = 0$ into (A1) implies $\dot{I}_0(Q) > 0$. By Martcheva (2015, p. 13), $I_\infty(Q) = 0$. Since $I_0(Q) > 0$ by Lemma 1, $\dot{I}_t(Q) < 0$ for some $t > 0$. By continuinity, $\dot{I}_T(Q) = 0$ for some $T > 0$. Setting (A1) equal to 0 yields $\mathcal{R}_0 S_T(Q) = 1$. Since $S_t(Q)$ is strictly decreasing, $\mathcal{R}_0 S_t(Q) > \mathcal{R}_0 S_T(Q) = 1$ for all $t \in [0, T)$, implying $\dot{I}_t(Q) > 0$ for all $t \in [0, T)$ by (A1). One can similarly show $\dot{I}_t(Q) < 0$ for all $t > T$. Thus, $I_T(Q)$ is the maximum infection rate. *Q.E.D.*

Proof of Lemma 4

See Martcheva (2015, p. 13) for a proof that $I_\infty(Q) = 0$. Martcheva (2015, p. 12) argues that the fact that $S_t(Q)$ is positive and montone implies that the limit $S_\infty(Q)$ exists.

To prove the remaining claim in the lemma, take the limit $t \uparrow \infty$ in (A6):

$$S_\infty(Q) = S_0(Q)e^{\mathcal{R}_0[R_0(Q)-R_\infty(Q)]}. \tag{A7}$$

By Lemma 1, $S_0(Q) > 0$. The proof of Lemma 2 showed that $R_t(Q)$ is strictly increasing in t. Thus $R_\infty(Q) > R_0(Q)$, implying $S_\infty(Q) < S_0(Q)$ by (A7). *Q.E.D.*

Proof of Lemma 5

First, suppose $\mathcal{R}_0 S_0(Q) \leq 1$. Then $\mathcal{R}_0 S_\infty(Q) < 1$ because $S_t(Q)$ is strictly decreasing in t by Lemma 2. Next, suppose $\mathcal{R}_0 S_0(Q) > 1$. The proof of Lemma 3 established the existence of $T > 0$ such that $\mathcal{R}_0 S_T(Q) = 1$. Since $S_t(Q)$ is strictly decreasing by Lemma 2, we have $\mathcal{R}_0 S_\infty(Q) < \mathcal{R}_0 S_T(Q) = 1$. *Q.E.D.*

Proof of Lemma 6

Substituting (2) and (10) into (3) yields

$$\dot{I}_t(Q) = \frac{\dot{S}_t(Q)}{\mathcal{R}_0 S_t(Q)} - \dot{S}_t(Q). \tag{A8}$$

Integrating (A8) over $t \in [0,\infty)$ and applying the Fundamental Theorem of Calculus,

$$I_\infty(Q) - I_0(Q) = \frac{1}{\mathcal{R}_0}[\ln S_\infty(Q) - \ln S_0(Q)] - S_\infty(Q) + S_0(Q). \tag{A9}$$

Substituting $I_0(Q) = \hat{I}_0$ by (7), noting $I_\infty(Q) = 0$ by Lemma 4, and rearranging yields

$$\ln S_\infty(Q) - \mathcal{R}_0 S_\infty(Q) = \ln S_0(Q) - \mathcal{R}_0[\hat{I}_0 + S_0(Q)]. \tag{A10}$$

Further substituting $S_0(Q) = \hat{S}_0 - Q$ from (6) yields (11).

To derive (12), exponentiating both sides of (A10) and rearranging yields

$$S_\infty(Q) = \left\{ S_0(Q) e^{-\mathcal{R}_0[\hat{I}_0 + S_0(Q)]} \right\} e^{\mathcal{R}_0 S_\infty(Q)}, \tag{A11}$$

or, equivalently,

$$x = be^{ax}, \tag{A12}$$

where $x = S_\infty(Q)$, $a = \mathcal{R}_0$, and $b = S_0(Q)e^{-\mathcal{R}_0[\hat{I}_0 + S_0(Q)]}$. It is well-known that (A12) has solution $x = -\bar{L}(-ab)/a = |\bar{L}(-ab)|/a$, where the second equality holds if $a, b > 0$ implying $\bar{L}(-ab) < 0$. Substituting for x, a, and b in this solution as well as $S_0(Q) = \hat{S}_0 - Q$ from (6) yields (12).

Equation (A12) also has a solution in terms of the lower branch of the Lambert W function, $x = -\underline{L}(-ab)/a$. We reject this solution because it exceeds 1, out of bounds for $S_\infty(Q)$. *Q.E.D.*

Additional Lemmas

We state and prove two additional lemmas, which draw on previous results, which will help streamline the subsequent proofs.

Lemma 7. $\lim_{\mathcal{R}_0 \downarrow 0} S_\infty(Q) = \hat{S}_0 - \theta Q$ *and* $\lim_{\mathcal{R}_0 \downarrow 0}[\mathcal{R}_0 S_\infty(Q)] = 0$.

Proof. The first limit can be shown to hold by substituting $\mathcal{R}_0 = 0$ into (11). The second limit then follows: $\lim_{\mathcal{R}_0 \downarrow 0}[\mathcal{R}_0 S_\infty(Q)] = (\hat{S}_0 - \theta Q) \lim_{\mathcal{R}_0 \downarrow 0} \mathcal{R}_0 = 0$. *Q.E.D.*

Lemma 8. $\lim_{\mathcal{R}_0 \uparrow \infty} S_\infty(Q) = \lim_{\mathcal{R}_0 \uparrow \infty}[\mathcal{R}_0 S_\infty(Q)] = 0$.

Proof. We will verify the second limit; the first limit is an immediate consequence. We have

$$\lim_{\mathcal{R}_0 \uparrow \infty}[\mathcal{R}_0 S_\infty(Q)] = \lim_{\mathcal{R}_0 \uparrow \infty} \left| \bar{L}\left(-\mathcal{R}_0 S_0(Q) e^{-\mathcal{R}_0[\hat{I}_0 + S_0(Q)]} \right) \right| \tag{A13}$$

$$= \left| \bar{L}\left(-S_0(Q) \lim_{\mathcal{R}_0 \uparrow \infty} \frac{\mathcal{R}_0}{e^{\mathcal{R}_0(\hat{I}_0 + S_0(Q))}} \right) \right| \tag{A14}$$

$$= |\bar{L}(0)|. \tag{A15}$$

Equation (A13) follows by taking limits in (12), (A14) is a simple rearrangement, and (A15) follows from application of l'Hôpital's Rule. As is well known for the Lambert W function, $\bar{L}(0) = 0$. *Q.E.D.*

Lemma 9. $\partial S_\infty(Q)/\partial \mathcal{R}_0 < 0$.

Proof. The Implicit Function Theorem can be applied to (11) to compute the derivative

$$\frac{\partial S_\infty(Q)}{\partial \mathcal{R}_0} = \frac{-S_\infty(Q)}{1-\mathcal{R}_0 S_\infty(Q)}[\hat{I}_0 + S_0(Q) - S_\infty(Q)]. \tag{A16}$$

The first factor is negative by Lemma 5. The factor in square brackets is positive since $\hat{I}_0 + S_0(Q) - S_\infty(Q) > S_0(Q) - S_\infty(Q) > 0$, where the first inequality follows from $\hat{I}_0 > 0$ and the second by Lemma 4. *Q.E.D.*

Lemma 10. $\partial \Phi(Q)/\partial Q < 0$.

Proof. Differentiating (14) and substituting from (13) yields

$$\frac{\partial \Phi(Q)}{\partial Q} = \frac{-\theta \Phi(Q) \mathcal{R}_0 S_\infty(Q)}{S_0(Q)[1-\mathcal{R}_0 S_\infty(Q)]}, \tag{A17}$$

which is negative by Lemma 5. *Q.E.D.*

Verification of Table 1 Entries

The equilibrium condition is $P_c^* = c$. Firms earn no profit under perfect competition: $\Pi_c^* = 0$. No consumers purchase in case (i), implying $Q_c^* = 0$. All susceptibles purchase in case (iii), implying $Q_c^* = \hat{S}_0$. In case (ii), Q_c^* can be found by substituting $P_c^* = c$ in equation (17).

To find $R_\infty(Q_c^*)$, note $R_\infty(Q_c^*) = 1 - I_\infty(Q_c^*) - S_\infty(Q_c^*) - \theta Q_c^* = 1 - S_\infty(Q_c^*) - \theta Q_c^*$ since $I_\infty(Q_c^*) = 0$. Substituting $Q_c^* = 0$ gives the entry for $R_\infty(Q_c^*)$ in case (i), and substituting $Q_c^* = \hat{S}_0$ gives the entry for $R_\infty(Q_c^*)$ in case (iii). To find $R_\infty(Q_c^*)$ in case (ii), set $c = MPB(Q_c^*)$ in equation (15) and rearrange, yielding

$$S_\infty(Q_c^*) = (1-\tilde{c})(\hat{S}_0 - \theta Q_c^*) \tag{A18}$$

Substituting (A18) into $R_\infty(Q_c^*) = 1 - S_\infty(Q_c^*) - \theta Q_c^*$ and rearranging yields $R_\infty(Q_c^*) = 1 - (1-\tilde{c})\hat{S}_0 - \tilde{c}\theta Q_c^*$. Substituting from the table entry for Q_c^* yields the table entry for $R_\infty(Q_c^*)$.

Substituting $Q_c^* = 0$ in (15) gives MPB_c^* in case (i), and substituting $Q_c^* = \hat{S}_0$ in (15) gives MPB_c^* in case (iii). For some but not all consumers to purchase in case (ii) requires $MPB_c^* = c$.

Substituting $Q_c^* = 0$ in (22) gives MSB_c^* in case (i), and substituting $Q_c^* = \hat{S}_0$ in (22) gives MSB_c^* in case (iii). Substituting from (A18) into (22) yields MSB_c^* in case (ii).

The table entries for MEX_c^* can be obtained by subtracting other table entries: $MEX_c^* = MSB_c^* - MPB_c^*$. To derive the table table entries for W_c^*, by definition $W_c^* = SB_c^* - cQ_c^* = H[1 - R_\infty(Q_c^*)] - cQ_c^*$, where the second equation follows from (20). Substituting other table entries into this equation gives the table entries for W_c^*. *Q.E.D.*

Proof of Proposition 1

Results for P_c^*, Π_c^*, and W_c^*. The results for P_c^* and Π_c^* are obvious from Table 1. The comparative statics for W_c^* are also obvious from inspection of the table in view of (A16).

Results for Q_c^*. To show Q_c^* is weakly increasing, it can be verified that it is continuous at thresholds $\mathcal{R}_0'$ and $\mathcal{R}_0''$ defined in (24)–(25). In case (ii), $\partial Q_c^*/\partial \mathcal{R}_0 = -\ln(1-\tilde{c})/\theta\tilde{c}\mathcal{R}_0^2 > 0$. Hence, Q_c^* is weakly increasing in $\mathcal{R}_0$ for all $\mathcal{R}_0 > 0$ and strictly increasing for $\mathcal{R}_0$ in the interior of case (ii).

Results for MPB_c^*. To show MPB_c^* is weakly increasing, start with case (i). Differentiating the table entry,

$$\frac{\partial MPB_c^*}{\partial \mathcal{R}_0} = -\left(\frac{\theta H}{\hat{S}_0}\right)\frac{\partial S_\infty(0)}{\partial \mathcal{R}_0}. \tag{A19}$$

By Lemma 9, $\partial S_\infty(Q)/\partial \mathcal{R}_0 < 0$ for all $Q \in [0, \hat{S}_0]$, including $Q_c^* = 0$, implying (A19) is positive. In case (ii), MPB_c^* is constant. Differentiating the table entry in cases (iii) and (iv),

$$\frac{\partial MPB_c^*}{\partial \mathcal{R}_0} = -\left[\frac{\theta H}{(1-\theta)\hat{S}_0}\right]\frac{\partial S_\infty(\hat{S}_0)}{\partial \mathcal{R}_0}, \tag{A20}$$

which is negative since $\partial S_\infty(Q)/\partial \mathcal{R}_0 < 0$ by Lemma 9 for all $Q \in [0, \hat{S}_0]$, including $Q_c^* = \hat{S}_0$.

The last step in deriving comparative statics for MPB_c^* is to show it is continuous at both endpoints of case (ii). Now $MPB(Q)$ is continuous in Q because it is differentiable in Q by (16). Further, $MPB(Q)$ is continuous in $\mathcal{R}_0$ because $S_\infty(Q)$ is differentiable in $\mathcal{R}_0$ by (A16). Since Q_c^* is continuous at both endpoints of case (ii) as argued in the first paragraph of this proof, we have that MPB_c^* is continous at $\mathcal{R}_0'$ and $\mathcal{R}_0''$.

Results for $R_\infty(Q_c^*)$. To derive the comparative statics for $R_\infty(Q_c^*)$, combining the table entries with Lemma 9 shows $R_\infty(Q_c^*)$ is increasing in $\mathcal{R}_0$ in case (i) as well as cases (iii) and (iv). The table entry is obviously decreasing in $\mathcal{R}_0$ in case (ii). We thus have that $R_\infty(Q_c^*)$ attains a local maximum at $\mathcal{R}_0'$ if we can establish that $R_\infty(Q_c^*)$ is continuous at $\mathcal{R}_0'$. Using the table entry for $R_\infty(Q_c^*)$ in case (i),

$$\lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} R_\infty(Q_c^*) = 1 - \lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} S_\infty(0) = 1 - \left(1 - \lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} \frac{MPB_c^*}{\theta H}\right)\hat{S}_0 = 1 - (1-\tilde{c})\hat{S}_0. \tag{A21}$$

The second equality follows from the table entry for MPB_c^* in case (i): $MPB_c^* = \theta H \Phi(0) = \theta H[1 - S_\infty(0)/\hat{S}_0]$ by (14). The third equality follows from the continuity of MPB_c^* at $\mathcal{R}_0'$, allowing us to substitute the table entry for $MPB_c^* = c$ in case (ii). Using the table entry for $R_\infty(Q_c^*)$ in case (ii),

$$\lim_{\mathcal{R}_0 \downarrow \mathcal{R}_0'} R_\infty(Q_c^*) = 1 - \hat{S}_0 - \hat{I}_0 + \frac{1}{\mathcal{R}_0'}|\ln(1-\tilde{c})| = 1 - (1-\tilde{c})\hat{S}_0. \tag{A22}$$

The equality between (A21) and (A22) proves the continuity of $R_\infty(Q_c^*)$ at $\mathcal{R}_0'$.

Since $R_\infty(Q_c^*)$ is increasing in $\mathcal{R}_0$ in cases (iii) and (iv), the other candidate for a supremum is

$$\lim_{\mathcal{R}_0 \uparrow \infty} R_\infty(Q_c^*) = 1 - \theta\hat{S}_0. \tag{A23}$$

This equality follows from taking the limit $\mathcal{R}_0 \uparrow \infty$ of the table entry in cases (iii) and (iv) and noting that $\lim_{\mathcal{R}_0 \uparrow \infty} S_\infty(\hat{S}_0) = 0$ by Lemma 8. The local maximum is thus a global maximum if and only if $1 - (1-\tilde{c})\hat{S}_0 \geq 1 - \theta\hat{S}_0$. Rearranging gives $\tilde{c} \geq 1 - \theta$.

Results for MSB_c^*. To provide a roadmap for the analysis, we first look at cases (i) and (ii) and show that MSB_c^* has a unique local maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$. Furthermore, this restricted local maximum is the restricted global maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$. We then look at cases (iii) and (iv) and show that MSB_c^* has at most one restricted local maximum over $\mathcal{R}_0 > \mathcal{R}_0''$. If no restricted local maximum exists there, then we show that the restricted maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$ is the global maximum over all $\mathcal{R}_0 > 0$. If a restricted local maximum exists in cases (iii) and (iv), then either it or the restricted local maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$ is the global maximum. This establishes, in sum, that MSB_c^* has at most two local maxima, one of which is the global maximum.

To prove that MSB_c^* has a unique local restricted maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$, we will show that MSB_c^* is increasing in a neighborhood around 0, quasiconcave for all $\mathcal{R}_0 \in (0, \mathcal{R}_0']$, continuous at $\mathcal{R}_0'$, and decreasing for all $\mathcal{R}_0 \in (\mathcal{R}_0', \mathcal{R}_0'')$. The arguments are made in reverse order. It is clear from inspection of Table 1 that MSB_c^* is decreasing for $\mathcal{R}_0 \in (\mathcal{R}_0', \mathcal{R}_0'')$. To show MSB_c^* is continuous at $\mathcal{R}_0'$, using the table entry for MSB_c^* in case (i),

$$\lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} MSB_c^* = \frac{\lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} MPB_c^*}{1 - \mathcal{R}_0'(1 - \lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} MPB_c^*/\theta H)\hat{S}_0} = \frac{\theta H \tilde{c}(\hat{I}_0 + \tilde{c}\hat{S}_0)}{\hat{I}_0 + \tilde{c}\hat{S}_0 + (1-\tilde{c})\hat{S}_0 \ln(1-\tilde{c})}. \tag{A24}$$

The first equality follows from substituting the table entry for MPB_c^* in case (i) directly as well as substituting the implication of that table entry that $S_\infty(0) = \hat{S}_0(1 - MPB_c^*/\theta H)$. The second equality follows from $\lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} MPB_c^* = c$ by continuity and from substituting from (24). Using the table entry for MSB_c^* in case (ii),

$$\lim_{\mathcal{R}_0 \downarrow \mathcal{R}_0'} MSB_c^* = \frac{\theta H \tilde{c}^2}{\tilde{c} + (1-\tilde{c})[\ln(1-\tilde{c}) + \mathcal{R}_0'\hat{I}_0]} = \frac{\theta H \tilde{c}(\hat{I}_0 + \tilde{c}\hat{S}_0)}{\hat{I}_0 + \tilde{c}\hat{S}_0 + (1-\tilde{c})\hat{S}_0 \ln(1-\tilde{c})}. \tag{A25}$$

The equality of (A24) and (A25) proves the continuity of MSB_c^* at $\mathcal{R}_0'$.

We next show MSB_c^* is quasiconcave for all $\mathcal{R}_0$ in case (i). Differentiating the relevant table entry, substituting from (A16), and eliminating positive constants shows that $\partial MSB_c^*/\partial \mathcal{R}_0$ has the same sign as

$$\left[\hat{S}_0 - S_\infty(0)\right]\left[1 - \mathcal{R}_0 S_\infty(0)\right] + \left[\hat{I}_0 + \hat{S}_0 - S_\infty(0)\right]\left(1 - \mathcal{R}_0 \hat{S}_0\right). \tag{A26}$$

The second derivative of (A26) with respect to $\mathcal{R}_0$—after substituting from (A16), and rearranging considerably—can be shown to equal

$$2\frac{\partial S_\infty(0)}{\partial \mathcal{R}_0}\left[\hat{I}_0 + \hat{S}_0 - S_\infty(0)\right], \tag{A27}$$

which is negative—as can be shown using arguments similar to those behind Lemma 9. Hence, (A26) is concave. In the limit $\mathcal{R}_0 \downarrow 0$, (A26) approaches $2[\hat{S}_0 - S_\infty(0)] + \hat{I}_0$, which is positive by Lemma 4 and $\hat{I}_0 > 0$. Having established that (A26) is concave throughout (i) and initially positive, we have that (A26) can change sign at most once. Therefore, $\partial MSB_c^*/\partial \mathcal{R}_0$ is either nonnegative throughout case (i) or positive then negative. In either event, this proves that MSB_c^* is quasiconcave in (i). We have already established MSB_c^* is increasing in a neighborhood of $\mathcal{R}_0$ above 0, the last step needed to prove that MSB_c^* has a unique restricted local maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$, which is a global maximum on that restricted set.

We next look at the behavior of MSB_c^* in cases (iii) and (iv), showing it has a most one restricted local maximum over $\mathcal{R}_0 > \mathcal{R}_0''$. Similar calculations used in the previous paragraph can be used here

to establish the concavity of the following function,

$$\left[(1-\theta)\hat{S}_0 - S_\infty(\hat{S}_0)\right]\left[1-\mathcal{R}_0 S_\infty(\hat{S}_0)\right] + \left[\hat{I}_0 + (1-\theta)\hat{S}_0 - S_\infty(\hat{S}_0)\right]\left[1-(1-\theta)\mathcal{R}_0\hat{S}_0\right], \tag{A28}$$

which determines the sign of $\partial MSB_c^*/\partial\mathcal{R}_0$ in (iii) and (iv). Thus, (A28) has at most two roots in those cases, which cannot both be local maxima, implying that MSB_c^* has at most one local maximum over $\mathcal{R}_0 > \mathcal{R}_0''$. The limit as $\mathcal{R}_0 \uparrow \infty$ of (A28) equals

$$\hat{I}_0 + 2(1-\theta)\hat{S}_0 - (1-\theta)\hat{S}_0[\hat{I}_0 + (1-\theta)\hat{S}_0] \lim_{\mathcal{R}_0\uparrow\infty} \mathcal{R}_0 \tag{A29}$$

after substituting $\lim_{\mathcal{R}_0\uparrow\infty} S_\infty(\hat{S}_0) = \lim_{\mathcal{R}_0\uparrow\infty}[\mathcal{R}_0 S_\infty(\hat{S}_0)] = 0$ by Lemma 8. Expression (A29) approaches $-\infty$ since it involves $\mathcal{R}_0$ multiplied by negative constant. Since the limit $\mathcal{R}_0 \uparrow \infty$ cannot produce a restricted supremum over $\mathcal{R}_0 > \mathcal{R}_0''$, the restricted supremum is either the lower boundary of (iii), i.e., $\mathcal{R}_0''$, or is the interior restricted maximum. If $\mathcal{R}_0''$ provides the restricted supremum over $\mathcal{R}_0 > \mathcal{R}_0''$, this cannot be a global maximum since MSB_c^* is decreasing in (ii); the restricted maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$ must then be the global maximum.

Results for MEX_c^*. We use the same roadmap for the comparative-statics analysis of MEX_c^* as for MSB_c^*. We begin by proving that MEX_c^* has a unique local restricted maximum over $\mathcal{R}_0 \leq \mathcal{R}_0''$. We do this by showing that MEX_c^* is increasing in a neighborhood around 0, quasiconcave for all $\mathcal{R}_0 \in (0, \mathcal{R}_0']$, continuous at $\mathcal{R}_0'$, and decreasing for all $\mathcal{R}_0 \in (\mathcal{R}_0', \mathcal{R}_0'')$. The arguments are made in reverse order. Differentiating the table entry for case (ii) yields

$$\frac{\partial MEX_c^*}{\partial\mathcal{R}_0} = \frac{-\theta H\tilde{c}^2(1-\tilde{c})\hat{I}_0}{\left\{\tilde{c} + (1-\tilde{c})[\ln(1-\tilde{c}) + \mathcal{R}_0\hat{I}_0]\right\}^2}, \tag{A30}$$

which is negative. The proof that MEX_c^* is continuous at $\mathcal{R}_0'$ is similar to that for MSB_c^* and omitted.

We next show MEX_c^* is quasiconcave for all $\mathcal{R}_0$ in case (i). Differentiating the relevant table entry, substituting from equation (A16), and eliminating positive constants shows that $\partial MEX_c^*/\partial\mathcal{R}_0$ has the same sign as

$$\left[\hat{S}_0 - S_\infty(0)\right]\left[1-\mathcal{R}_0(\hat{I}_0+\hat{S}_0)\right] + \mathcal{R}_0 S_\infty(0)\left[1-\mathcal{R}_0 S_\infty(0)\right]\left[\hat{I}_0 + \hat{S}_0 - S_\infty(0)\right]. \tag{A31}$$

All of the factors in (A31) are definitively positive except for $1-\mathcal{R}_0(\hat{I}_0+\hat{S}_0)$. If this is also nonnegative, then $\partial MEX_c^*/\partial\mathcal{R}_0$ is positive in (i), implying MEX_c^* is quasiconcave in (i), as desired.

So suppose instead that

$$\mathcal{R}_0(\hat{I}_0+\hat{S}_0) > 1. \tag{A32}$$

We will show that (A32) implies that (A31) is concave. The second derivative of (A31) with respect to $\mathcal{R}_0$—after substituting from (A16), rearranging considerably, and removing positive factors—can be shown to have the same sign as

$$\begin{aligned} S_\infty(0)\left[1-\mathcal{R}_0(\hat{I}_0+\hat{S}_0)\right] - \left[1-\mathcal{R}_0 S_\infty(0)\right](\hat{I}_0+\hat{S}_0) - S_\infty(0)\left[2(\hat{I}_0+\hat{S}_0) - S_\infty(0)\right] \\ - S_\infty(0)\left\{1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]\right\}\left[\hat{I}_0+\hat{S}_0-2S_\infty(0)\right]. \end{aligned} \tag{A33}$$

By (A32) and familiar arguments, all the terms in (A33) are negative except possibly the last. If the last term is also nonpositive, the whole expression is negative, establishing (A31) is concave. So

suppose instead that the last term is positive. For this to be the case, one of its last two factors must be positive and the other negative. That is, one of the following two sets of conditions must hold:

$$1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]>0, \qquad \hat{I}_0+\hat{S}_0-2S_\infty(0)<0 \tag{A34}$$

$$1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]<0, \qquad \hat{I}_0+\hat{S}_0-2S_\infty(0)>0 \tag{A35}$$

Suppose (A34) holds. Then (A33) is strictly less than

$$-S_\infty(0)\left[2(\hat{I}_0+\hat{S}_0)-S_\infty(0)\right]-S_\infty(0)\left\{1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]\right\}\left[\hat{I}_0+\hat{S}_0-2S_\infty(0)\right] \tag{A36}$$

$$<-S_\infty(0)\left[2(\hat{I}_0+\hat{S}_0)-S_\infty(0)\right]-S_\infty(0)\left[\hat{I}_0+\hat{S}_0-2S_\infty(0)\right] \tag{A37}$$

$$=-3S_\infty(0)\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]. \tag{A38}$$

Equation (A36) follows from eliminating the first two negative terms of (A33). Equation (A37) follows from substituting 1, which is greater than the factor in braces, for the factor in braces. The fact that this substitution results in an increase in (A37) follows from (A34). Straightforward algebra yields (A38), which is negative by familiar arguments.

Suppose (A35) holds. Then (A33) is strictly less than

$$S_\infty(0)\left[1-\mathcal{R}_0(\hat{I}_0+\hat{S}_0)\right]-S_\infty(0)\left\{1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]\right\}\left[\hat{I}_0+\hat{S}_0-2S_\infty(0)\right] \tag{A39}$$

$$<S_\infty(0)\left[1-\mathcal{R}_0(\hat{I}_0+\hat{S}_0)\right]-S_\infty(0)\left\{1-\mathcal{R}_0\left[\hat{I}_0+\hat{S}_0-S_\infty(0)\right]\right\} \tag{A40}$$

$$=-\mathcal{R}_0 S_\infty(0)^2. \tag{A41}$$

Equation (A39) follows from eliminating the second and third two negative terms from (A33). Equation (A40) follows from substituting 1 for the last factor, $\hat{I}_0+\hat{S}_0-2S_\infty(0)$. To see that this increases the expression, note $\hat{I}_0+\hat{S}_0-2S_\infty(0)<\hat{I}_0+\hat{S}_0\leq 1$, where the last inequality holds since the size of the infected and susceptible subpopulations at date 0, $\hat{I}_0+\hat{S}_0$, cannot exceed the size of the entire population, normalized to 1. The fact that substituting 1 for $\hat{I}_0+\hat{S}_0-2S_\infty(0)$ increases (A39) follows from (A35). Straightforward algebra yields (A41), which is obviously negative.

In sum, we have shown (A33) is negative for $\mathcal{R}_0<\mathcal{R}_0'$, implying (A31) is concave. In the limit $\mathcal{R}_0\downarrow 0$, (A31) approaches $\hat{S}_0-S_\infty(0)$, which is positive by Lemma 4. These facts are sufficient to establish that MEX_c^* is quasiconcave in case (i) by the same arguments used for MSB_c^* above. These are all the facts needed to prove that MEX_c^* has a unique restricted local maximum over $\mathcal{R}_0\leq\mathcal{R}_0''$, which is a global maximum on that restricted set.

We next investigate the behavior of MEX_c^* in (iii) and (iv). Calculations similar to those used above can be used to show a function determining the sign of $\partial MEX_c^*/\partial\mathcal{R}_0$ in cases (iii) and (iv),

$$\left[(1-\theta)\hat{S}_0-S_\infty(0)\right]\left\{1-\mathcal{R}_0[\hat{I}_0+(1-\theta)\hat{S}_0]\right\}+\mathcal{R}_0 S_\infty(\hat{S}_0)\left[1-\mathcal{R}_0 S_\infty(\hat{S}_0)\right]\left[\hat{I}_0+(1-\theta)\hat{S}_0-S_\infty(\hat{S}_0)\right]. \tag{A42}$$

is concave. Thus, (A42) has at most two roots in cases (iii) and (iv), at most one of which is a local maximum for MEX_c^*. Taking the limit of the table entry for MEX_c^* in cases (iii) and (iv) and substituting the limit $\lim_{\mathcal{R}_0\uparrow\infty}S_\infty(\hat{S}_0)=\lim_{\mathcal{R}_0\uparrow\infty}[\mathcal{R}_0 S_\infty(\hat{S}_0)]=0$ by Lemma 8 yields $\lim_{\mathcal{R}_0\uparrow\infty}MEX_c^*=0$. Since the limit $\mathcal{R}_0\uparrow\infty$ produces an infimum for MEX_c^*, not a supremum, the restricted supremum of MEX_c^* over $\mathcal{R}_0>\mathcal{R}_0''$ is either the lower boundary of case (iii), i.e., $\mathcal{R}_0''$, or the interior restricted maximum. If $\mathcal{R}_0''$ provides the restricted supremum over $\mathcal{R}_0>\mathcal{R}_0''$, this cannot be a global maximum since MSB_c^* is decreasing in case (ii); the restricted maximum over $\mathcal{R}_0\leq\mathcal{R}_0''$ must then be the global maximum. *Q.E.D.*

Proof of Proposition 2

We first show that the condition for monopoly to deliver the first best involving universal vaccination of susceptibles, $MR(\hat{S}_0) \geq c$, holds for sufficiently high $\mathcal{R}_0$. Substituting $\hat{S}_0$ for Q in (26) shows that $MR(\hat{S}_0) \geq c$ holds if and only if

$$\Phi(\hat{S}_0)\left[1-\left(\frac{\theta}{1-\theta}\right)\frac{\mathcal{R}_0 S_\infty(\hat{S}_0)}{1-\mathcal{R}_0 S_\infty(\hat{S}_0)}\right] \geq \tilde{c}. \tag{A43}$$

The left-hand side equals 1 in the limit $\mathcal{R}_0 \uparrow \infty$. To see this, note that $\lim_{\mathcal{R}_0\uparrow\infty} S_\infty(Q) = 0$ by Lemma 8, implying $\lim_{\mathcal{R}_0\uparrow\infty} \Phi(\hat{S}_0) = 1$ by (14). Also by Lemma 8, $\lim_{\mathcal{R}_0\uparrow\infty} \mathcal{R}_0 S_\infty(Q) = 0$, implying the factor in square brackets in (A43) equals 1 in the limit $\mathcal{R}_0 \uparrow \infty$. The left-hand side exceeds the right-hand side in the limit since $1 > \tilde{c}$ by assumption (19).

We next verify that when monopoly output is an interior solution, i.e., $Q_m^* \in (0, \hat{S}_0)$, we have $Q_m^* < Q_c^*$. Given $Q_m^* > 0$, as argued in the text, $\mathcal{R}_0 > \mathcal{R}_0'$, ruling out case (i). As shown in Table 1 $Q_c^* = \hat{S}_0$ in cases (iii) and (iv), so it is immediate that $Q_m^* < Q_c^*$ for interior Q_m^*. This leaves case (ii). We have $MPB(Q_c^*) = MPB_c^* = c = MR(Q_m^*) < \theta H \Phi(Q_m^*) = MPB(Q_m^*)$, where the first step is definitional, the second step follows from the relevant entry in Table 1 in case (ii), the third step follows from the Kuhn-Tucker conditions for an interior solution, the fourth step follows from the fact that the factor in braces in (26) is less than 1, and the fifth step follows from (15). Since (16) is negative, $MPB(Q_c^*) < MPB(Q_m^*)$ implies $Q_c^* > Q_m^*$. *Q.E.D.*

Proof of Proposition 3

Results for Π_m^*. The result is a consequence of the Envelope Theorem. Monopoly profit can be written

$$\Pi_m^* = \theta H\left[1-\frac{S_\infty(Q_m^*)}{S_0(Q_m^*)}-\tilde{c}\right]Q_m^*. \tag{A44}$$

This is a function of $\mathcal{R}_0$ indirectly through its dependence on Q_m^*, which in turns depends on $\mathcal{R}_0$. It also depends on $\mathcal{R}_0$ because $S(Q)$ is a function of $\mathcal{R}_0$ (although the argument is omitted for brevity). If Q_m^* is an interior solution, as in case (ii) and (iii), the first-order condition ensures that the indirect effect of $\mathcal{R}_0$ on Π_m^* through Q_m^* equals 0. Only the direct effect remains. Hence,

$$\frac{\partial \Pi_m^*}{\partial \mathcal{R}_0} = \left[\frac{-\theta H Q_m^*}{S_0(Q_m^*)}\right]\frac{\partial S_\infty(Q_m^*)}{\partial \mathcal{R}_0}, \tag{A45}$$

which is positive since the derivative on the right-hand side is negative by Lemma 9.

Results for $R_\infty(Q_m^*)$. We first show that $R_\infty(Q_m^*)$ has at least one interior local maximum in $\mathcal{R}_0$. By Tables 1 and 2, $Q_m^* = Q_c^*$ in case (i), implying $R_\infty(Q_m^*) = R_\infty(Q_c^*)$. The proof of Proposition 1 showed $R_\infty(Q_c^*)$ is increasing in $\mathcal{R}_0$ in case (i), implying $R_\infty(Q_m^*)$ is increasing in case (i).

According to Table 2, $R_\infty(Q_m^*) = 1 - S_\infty(Q_m^*) - \theta Q_m^*$ in cases (ii) and (iii). Differentiating, substituting from (13), and rearranging yields

$$\frac{\partial R_\infty(Q_m^*)}{\partial \mathcal{R}_0} = -\left[\frac{\theta\Phi(Q_m^*)}{1-\mathcal{R}_0 S_\infty(Q_m^*)}\right]\frac{\partial Q_m^*}{\partial \mathcal{R}_0}. \tag{A46}$$

Since Q_m^* increases from 0 at the threshold $\mathcal{R}_0$ below case (ii) to $\hat{S}_0$ at the threshold above case (iii), we must have $\partial Q_m^*/\partial \mathcal{R}_0 > 0$ on a set of $\mathcal{R}_0$ in (ii) and (iii) of positive measure. Thus, $\partial R_\infty(Q_m^*)/\partial \mathcal{R}_0 < 0$ on a set of $\mathcal{R}_0$ of positive measure by (A46) since the factor in square brackets in (A46) is positive by Lemma 5. If $R_\infty(Q_m^*)$ is decreasing for $\mathcal{R}_0$ in a neighborhood above $\mathcal{R}_0'$ at the threshold between cases (i) and (ii), then $R_\infty(Q_m^*)$ attains a local maximum at $\mathcal{R}_0'$. Otherwise, the lower bound of the first set of positive measure for which $\partial R_\infty(Q_m^*)/\partial \mathcal{R}_0 < 0$ is a local maximum.

Suppose for the remainder of the proof that $\tilde{c} \geq 1-\theta$. We will show $R_\infty(Q_m^*)$ has an interior global maximum. Since $R_\infty(Q_m^*) = R_\infty(Q_c^*)$ for all $\mathcal{R}_0$ in case (i),

$$\lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} R_\infty(Q_m^*) = \lim_{\mathcal{R}_0 \uparrow \mathcal{R}_0'} R_\infty(Q_c^*) = 1-(1-\tilde{c})\hat{S}_0, \tag{A47}$$

where the first equality follows by continuity since $\mathcal{R}_0'$ is the upper bound on case (i) by (24) and the second equality follows from (A21).

We proceed to compare (A47) to the limits of $R_\infty(Q_m^*)$ for extreme values of $\mathcal{R}_0$. We have

$$\lim_{\mathcal{R}_0 \downarrow 0} R_\infty(Q_m^*) = \lim_{\mathcal{R}_0 \downarrow 0} R_\infty(Q_c^*) = 1 - \lim_{\mathcal{R}_0 \downarrow 0} S_\infty(0) = 1-\hat{S}_0, \tag{A48}$$

where the first equality follows since $R_\infty(Q_m^*) = R_\infty(Q_c^*)$ for all $\mathcal{R}_0$ in case (i), the second equality follows from the entry for $R_\infty(Q_c^*)$ in case (i) in Table 1, and the third equality follows from Lemma 7. Equation (A48) is less than (A47). At the other extreme,

$$\lim_{\mathcal{R}_0 \uparrow \infty} R_\infty(Q_m^*) = \lim_{\mathcal{R}_0 \uparrow \infty} R_\infty(Q_c^*) = 1-\theta\hat{S}_0, \tag{A49}$$

where the first equality follows by continuity since $R_\infty(Q_m^*) = R_\infty(Q_c^*)$ for all $\mathcal{R}_0$ in case (iv), the second equality follows from the entry for $R_\infty(Q_c^*)$ in cases (iii) and (iv) in Table 1, and the third equality follows from (A23). Since $\tilde{c} \geq 1-\theta$, (A49) is weakly less than (A47). We have shown that $R_\infty(Q_m^*)$ is greater at the interior $\mathcal{R}_0'$ than at extreme values of $\mathcal{R}_0$, implying that $R_\infty(Q_m^*)$ has an interior global maximum.

Results for MSB_m^*. We show that the limits of MSB_m^* for extreme values of $\mathcal{R}_0$ are exceed by interior values. We have

$$\lim_{\mathcal{R}_0 \downarrow 0} MSB_m^* = \lim_{\mathcal{R}_0 \downarrow 0} MSB_c^* = \theta H\left[1-\frac{1}{\hat{S}_0}\lim_{\mathcal{R}_0 \downarrow 0} S_\infty(0)\right] = \theta H\left(1-\frac{\hat{S}_0}{\hat{S}_0}\right) = 0, \tag{A50}$$

where the first equality follows since $MSB_m^* = MSB_c^*$ for all $\mathcal{R}_0$ in case (i), the second equality follows from the entry for MSB_c^* in case (i) in Table 1, and the third equality follows from Lemma 7. To examine the upper limit, the proof of Proposition 1 showed that MSB_c^* asymptotes downward toward $\lim_{\mathcal{R}_0 \uparrow \infty} MSB_c^* = 1$. Since $MSB_m^* = MSB_c^*$ in case (iv), and all $\mathcal{R}_0$ above a sufficiently high value are contained in case (iv), MSB_m^* must also slope downward toward its asymptote. Thus MSB_m^* is higher at interior values of $\mathcal{R}_0$ than the extremes.

Results for MEX_m^*. Arguments similar to those just used for MSB_m^* can be used to show $\lim_{\mathcal{R}_0 \downarrow 0} MEX_m^* = \lim_{\mathcal{R}_0 \uparrow \infty} MEX_c^* = 0$. Hence, MEX_m^* is higher for interior values of $\mathcal{R}_0$ than extreme values and thus attains an interior maximum. *Q.E.D.*

Proof of Proposition 4

The sketch of the proof in the text omitted two details filled in here. We first prove Q^{**} for $\mathcal{R}_0$ in a neighborhood above 0. Taking limits in (22),

$$\lim_{\mathcal{R}_0 \downarrow 0} MSB(Q) = \theta H \left[1 - \frac{S_0(Q)}{S_0(Q)}\right] = 0. \tag{A51}$$

Hence, there exists $\mathcal{R}_0$ in a neighborhood above 0 and $\epsilon \in (0,c)$ such that $MSB(Q) < \epsilon$. For $\mathcal{R}_0$ in this neighborhood, $W(Q) = \int_0^Q [MSB(x) - c]dx < (\epsilon - c)Q < 0 = W(0)$. Thus, $Q^{**} = 0$ for $\mathcal{R}_0$ in this neighborhood.

We next prove $G^{**} > 0$ for some $\mathcal{R}_0 \in (0,\infty)$. Since $Q^{**} = 0$ for all $\mathcal{R}_0$ in neighborhood of 0, $Q^* \leq Q^{**} = 0$ implies $Q^* = 0$ for all $\mathcal{R}_0$ in a neighborhood of 0. The text argued $Q_m^* = \hat{S}_0$ for sufficiently high $\mathcal{R}_0$, implying $\hat{S}_0 = Q_m^* \leq Q^{**} \leq hS_0$, implying $Q^{**} = \hat{S}_0$ for sufficiently high $\mathcal{R}_0$. By the Theorem of the Maximum, since Q^{**} is a maximizer of continuous function $W(Q)$, Q^{**} is continuous, implying the existence of $\mathcal{R}_0 \in (0,\infty)$ such that $Q^* \in (0,\hat{S}_0)$. This Q^{**} must satisfy the first-order condition $MSB(Q^{**}) = c$, implying $MPB(Q^{**}) + MEX(Q^{**}) = c$, implying $MPB(Q^{**}) < c$ since $MEX(Q) > 0$ for all $Q \in (0,\hat{S}_0)$ by (23). *Q.E.D.*

Proof of Proposition 5

Start with the analysis of perfect competition. To derive G_c^{**} for various values of Q^{**}, first suppose $Q^{**} = 0$. Arguments in the text preceding Proposition 4 can be used to show $G_c^{**} = 0$.

Next, suppose $Q^{**} \in (0,\hat{S}_0)$. Then Q^{**} must satisfy the first-order condition for welfare maximization $MSB(Q^{**}) = c$, implying $MPB(Q^{**}) + MEX(Q^{**}) = c$, in turn implying $P_c^{**} = MPB(Q^{**}) = c - MEX(Q^{**})$. Since competitive firms pass the subsidy through to consumers, $P_c^{**} = c - G_c^{**}$. Combining the preceding equations yields $G_c^{**} = MEX(Q^{**})$.

Next, suppose $Q^{**} = \hat{S}_0 > Q_c^*$. Then the highest price at which output $\hat{S}_0$ is purchased satisfies $P_c^{**} = MPB(\hat{S}_0)$. Combined with competitive pass through, $P_c^{**} = c - G_c^{**}$, we have $G_c^{**} = c - MPB(\hat{S}_0)$.

Finally, suppose $Q_c^* = Q^{**} = \hat{S}_0$. Arguments in the text preceding Proposition 4 can be used to show $G_c^{**} = 0$. The various results for $Q^{**} = \hat{S}_0$ can be nested as $G_c^{**} = \max[0, c - MPB(\hat{S}_0)]$.

Turn next to the analysis of monopoly. To derive G_m^{**} for various values of Q^{**}, first suppose $Q^{**} = 0$. Arguments in the text preceding Proposition 4 can be used to show $G_m^{**} = 0$.

Next, suppose $Q^{**} \in (0,\hat{S}_0)$. The monopoly regards the subsidy as a reduction in marginal cost, maximizing $[MPB(Q) - c + G]Q$. To generate the first best, the optimal subsidy G_m^{**} must force the monopoly's first-order condition to be satisfied by Q^{**}:

$$MR(Q^{**}) = c - G_m^{**}. \tag{A52}$$

For general Q, (23) and (26) can be combined to show

$$MR(Q) = MPB(Q) - \frac{MEX(Q)\theta Q}{\hat{S}_0 - \theta Q}. \tag{A53}$$

Evaluting (A53) at $Q = Q^{**}$ yields

$$MR(Q^{**}) = MPB(Q^{**}) - \frac{MEX(Q^{**})\theta Q^{**}}{\hat{S}_0 - \theta Q^{**}} = c - MEX(Q^{**}) - \frac{MEX(Q^{**})\theta Q^{**}}{\hat{S}_0 - \theta Q^{**}}, \tag{A54}$$

where the second step follows from $MPB(Q^{**}) = c - MEX(Q^{**})$, which was shown in the analysis of perfect competition above when $Q^{**} \in (0, \hat{S}_0)$. Combining (A52) and (A54) and rearranging yields $G_m^{**} = MEX(Q^{**})\hat{S}_0/(\hat{S}_0 - \theta Q^{**})$.

Next, suppose $Q^{**} = \hat{S}_0 > Q_m^*$. According to standard Kuhn-Tucker conditions, for a subsidy G to induce the monopoly to produce at the corner $\hat{S}_0$, G must satisfy $MR(\hat{S}_0) \geq c - G$. This condition holds with equality at the lowest such subsidy, which is the optimal subsidy under monopoly, implying

$$MR(\hat{S}_0) = c - G_m^{**}. \tag{A55}$$

Evaluating (A53) at $Q = \hat{S}_0$ yields

$$MR(\hat{S}_0) = MPB(\hat{S}_0) - \frac{MEX(Q)\theta}{1-\theta}. \tag{A56}$$

Combining (A55) and (A56) yields

$$G_m^{**} = c - MPB(\hat{S}_0) + \left(\frac{\theta}{1-\theta}\right) MEX(\hat{S}_0). \tag{A57}$$

Finally, suppose $Q_c^* = Q^{**} = \hat{S}_0$. Arguments in the text preceding Proposition 4 can be used to show $G_m^{**} = 0$. The various results for $Q^{**} = \hat{S}_0$ can be nested as stated in (27). *Q.E.D.*

Proof of Proposition 6

Section 3.1 argued that universal vaccination is attained under perfect competition if $\mathcal{R}_0 > \mathcal{R}_0''$. Substituting $\theta = 1$ into the expression for $\mathcal{R}_0''$ in (25) yields $\mathcal{R}_0 > |\ln(1-\tilde{c})|/\hat{I}_0$. Rearranging and exponentiating yields $1 - e^{-\mathcal{R}_0 \hat{I}_0} > \tilde{c}$.

Turning next to the analysis of monopoly, according to Proposition 2, monopoly attains universal vaccination if and only if $MR(\hat{S}_0) \geq c$. Using (15) and (26) and rearranging, this inequality can be written

$$\Phi(\hat{S}_0)\left\{1 - \frac{\theta \mathcal{R}_0 \hat{S}_0[1-\Phi(\hat{S}_0)]}{1 - \mathcal{R}_0 S_\infty(\hat{S}_0)}\right\} \geq \tilde{c}. \tag{A58}$$

To determine whether (A58) holds with a perfectly effective vaccine, we need to take limits as $\theta \uparrow 1$, requiring us to compute limits $\lim_{\theta\uparrow 1} S_\infty(\hat{S}_0)$ and $\lim_{\theta\uparrow 1} \Phi(\hat{S}_0)$. To compute the first limit, by (12), $\lim_{\theta\uparrow 1} S_\infty(\hat{S}_0) = |\bar{L}(0)|/\mathcal{R}_0 = 0$, where the second step follows from the well-known fact that $\bar{L}(0) = 0$. Computing the second limit is more delicate since it involves a $0/0$ form. Manipulating (11), we have

$$\frac{S_\infty(Q)}{S_0(Q)} = e^{-\mathcal{R}_0[\hat{I}_0 + \hat{S}_0 - \theta Q - S_\infty(Q)]}, \tag{A59}$$

implying

$$\lim_{\theta\uparrow 1}\left[\frac{S_\infty(\hat{S}_0)}{S_0(\hat{S}_0)}\right] = e^{-\mathcal{R}_0 \hat{I}_0}, \tag{A60}$$

using $\lim_{\theta\uparrow 1} S_\infty(\hat{S}_0) = 0$. Hence, $\lim_{\theta\uparrow 1} \Phi(\hat{S}_0) = 1 - e^{-\mathcal{R}_0 \hat{I}_0}$. Substituting these limits into (A58) and recognizing that the inequality must be strict to hold for $\theta < 1$ yields $(1 - e^{-\mathcal{R}_0 \hat{I}_0})(1 - \mathcal{R}_0 \hat{S}_0 e^{-\mathcal{R}_0 \hat{I}_0}) > \tilde{c}$. *Q.E.D.*

Proof of Proposition 8

Suppose $\mathcal{R}_0\hat{S}_0 > 2$. Then $1 < \mathcal{R}_0\hat{S}_0/2 < \mathcal{R}_0[\hat{S}_0 + S_\infty(0)]/2 = \mathcal{R}_0[S_0(0) + S_\infty(0)]/2$, where the second step follows from $S_\infty(0) > 0$ by Lemma 4. This chain of inequalities implies that (29) holds at $Q = 0$ and thus that the vaccine exhibits initially increasing social returns.

At a general output level $Q \in (0, \hat{S}_0)$,

$$\mathcal{R}_0\left[\frac{S_0(Q) + S_\infty(Q)}{2}\right] > \mathcal{R}_0\left(\frac{S_0(Q)}{2}\right) = \mathcal{R}_0\left(\frac{\hat{S}_0 - \theta Q}{2}\right) \geq \mathcal{R}_0\left(\frac{(1-\theta)\hat{S}_0}{2}\right). \tag{A61}$$

If $\mathcal{R}_0\hat{S}_0 \geq 2/(1-\theta)$, then the last expression weakly exceeds 1, implying (29) holds for all feasible Q, implying the vaccine exhibits everywhere increasing social returns. *Q.E.D.*

Proof of Proposition 9

The assumption $c = 0$ implies $\tilde{c} = 0$, leaving two cases in Table 1: (ii)–(iii) and (iv). Nesting those cases, we can write

$$\Delta\Pi_m^* = \theta H\left\{\hat{I}_0 + \hat{S}_0\Phi(0) - Q_{mv}^*\Phi(Q_{mv}^*)\right\}, \tag{A62}$$

where Q_{mv}^* solves $\max_{Q\in[0,\hat{S}_0]} Q\Phi(Q)$. Since $Q_{mv}^* > 0$, we have $Q_{mv}^*\Phi(Q_{mv}^*) < Q_{mv}^*\Phi(0) \leq \hat{S}_0\Phi(0)$, where the first inquality follows from Lemma 10 and the second inequality from $Q_{mv}^* \in [0, \hat{S}_0]$. Substituting the preceding inequality into (A62) yields $\Delta\Pi_m^* > \theta H\hat{I}_0$. Thus, $\Delta\Pi_m^* > 0$ for all $\mathcal{R}_0 > 0$.

To derive the results on limits of $\Delta\Pi_m^*$, we have that $\lim_{\mathcal{R}_0\downarrow 0}\Phi(Q) = \lim_{\mathcal{R}_0\downarrow 0}[1 - S_\infty(Q)/S_0(Q)] = 1$ for all $Q \in [0, \hat{S}_0]$ since $\lim_{\mathcal{R}_0\downarrow 0} S_\infty(Q) = \hat{S}_0 - \theta Q = S_0(Q)$ by Lemma 7. Hence, $\lim_{\mathcal{R}_0\downarrow 0}\Delta\Pi_m^* = \theta H\hat{I}_0$. For all $Q \in [0, \hat{S}_0]$, $\lim_{\mathcal{R}_0\uparrow\infty}\Phi(Q) = 1$ since $\lim_{\mathcal{R}_0\uparrow\infty} S_\infty(Q) = 0$ by Lemma 8. Therefore,

$$\lim_{\mathcal{R}_0\uparrow\infty} Q_{mv}^*\Phi(Q_{mv}^*) = \lim_{\mathcal{R}_0\uparrow\infty}\left\{\max_{Q\in[0,\hat{S}_0]} Q\Phi(Q)\right\} = \max_{Q\in[0,\hat{S}_0]}\left[Q\lim_{\mathcal{R}_0\uparrow\infty}\Phi(Q)\right] = \hat{S}_0 \cdot 1. \tag{A63}$$

Substituting from (A63) into (A62) along with $\lim_{\mathcal{R}_0\uparrow\infty}\Phi(0) = 1$ yields $\lim_{\mathcal{R}_0\uparrow\infty}\Delta\Pi_m^* = \theta H\hat{I}_0$. Now $\Delta\Pi_m^* > \theta H\hat{I}_0$ for all $\mathcal{R}_0 > 0$ implies $\theta H\hat{I}_0 \leq \inf_{\mathcal{R}_0>0}\Delta\Pi_m^* \leq \lim_{\mathcal{R}_0\downarrow 0}\Delta\Pi_m^* = \theta H\hat{I}_0$, which in turn implies $\inf_{\mathcal{R}_0>0}\Delta\Pi_m^* = \theta H\hat{I}_0$.

Combining the results from the previous paragraph, $\lim_{\mathcal{R}_0\downarrow 0}\Delta\Pi_m^* = \lim_{\mathcal{R}_0\uparrow\infty}\Delta\Pi_m^* = \inf_{\mathcal{R}_0>0} = \theta H\hat{I}_0$. But the first paragraph showed $\Delta\Pi_m^* > \theta H\hat{I}_0$. Hence, $\Delta\Pi_m^*$ must be nonmonotonic in $\mathcal{R}_0$, higher in the interior than for either limiting value of $\mathcal{R}_0$.

Turning to limiting values of ΔW_m^* as $\mathcal{R}_0 \downarrow 0$ and $\mathcal{R}_0 \uparrow \infty$, one can show that (A43) holds in these limits. Thus, the relevant case for computing W_{mv}^* is (iv). Substituting $\tilde{c} = 0$ into the relevant entry of Table 2 and multiplying by $\theta H\hat{S}_0$ to reverse the rescaling yields $W_{mv}^* = H[S_\infty(\hat{S}_0) + \theta\hat{S}_0]$. Subtracting from (31) and rearranging yields

$$\Delta W_m^* = H\left[\theta\hat{I}_0 + (1-\theta)S_\infty(0) - S_\infty(\hat{S}_0)\right]. \tag{A64}$$

By Lemma 7, $\lim_{\mathcal{R}_0\downarrow 0}[(1-\theta)S_\infty(0)] = (1-\theta)\hat{S}_0$. The lemma also implies $\lim_{\mathcal{R}_0\downarrow 0} S_\infty(\hat{S}_0) = (1-\theta)\hat{S}_0$. Substituting these limits into (A64) yields $\lim_{\mathcal{R}_0\downarrow 0}\Delta W_m^* = \theta H\hat{I}_0$. By Lemma 8, $\lim_{\mathcal{R}_0\uparrow\infty} S_\infty(0) = \lim_{\mathcal{R}_0\uparrow\infty} S_\infty(\hat{S}_0) = 0$. Substituting these limits into (A64) yields $\lim_{\mathcal{R}_0\uparrow 0}\Delta W_m^* = \theta H\hat{I}_0$.

The final step is to provide parameters for which $\Delta W_m^* < 0$. Using Matlab, we verified that for $\mathcal{R}_0 = 2$, $\theta = 0.5$, $\hat{I}_0 = 0.1$, $\hat{S}_0 = 0.8$, (A43) holds, implying that the vaccine monopoly supplies first-best quantity $\hat{S}_0$, putting us in case (iv). Subtracting the relevant Table 2 entry from (31) and simplifying yields $\Delta W_m^* = H[(1-\theta)S_\infty(0) + \theta\hat{I}_0 - S_\infty(\hat{S}_0)]$, which Matlab calculations show equals –0.09 for the specified parameters. *Q.E.D.*

Appendix A2. Calibration Details

The calibration considers the limiting case in which rescaled cost, $\tilde{c} = c/\theta H$, is set to 0. This appendix provides additional documentation justifying that limiting value.

Castillo et al. (2021) reports that prices for available Covid vaccines were no greater than $40 per course. Health losses can be computed following Snyder et al. (2020). Hanlon et al. (2021) estimates 12 years of lost life (YLL) per death. Since this estimate already allocates shorter lifespans to people with comorbidities, we assume one YLL translates into one disability adjusted life year (DALY) without need for further downward adjustment to reflect a proportion of years lived with a disability. To convert DALYs into monetary values, we multiply DALYs lost in a country by three times that country's 2019 GDP per capita, reflecting World Health Organization (WHO) standards for a cost-effective health intervention in a country stated in Marseille et al. (2015). According to this standard, a health intervention is cost effective if the cost per DALY saved is less than three times that country's per-capita GDP ($65,253 in the U.S. in 2019). Putting these estimates together yields an estimate of $H = 12 \times 3 \times \$65,253 = \$2.35$ million. Using the calibrated value of $\theta = 0.8$ yields $\tilde{c} = 40/(0.8 \times 2.35 \times 10^6) = 2.13 \times 10^{-5}$.

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Appendix A3. Cournot Competition

This appendix extends the analysis to Cournot competition, which nests the perfectly competitive and monopoly market structures studied in the text. Under Cournot competition, the vaccine is manufactured by $n \geq 1$ homogeneous firms, which choose quantities each period simultaneously. We will look for the symmetric Nash equilibrium, denoting a firm's equilibrium output by q_n^* and market output by $Q_n^* = nq_n^*$.

Case (i) from Table 1, in which $\mathcal{R}_0 \leq \mathcal{R}_0'$, which involves no sales under perfect competition, also involves no sales under Cournot since firms mark up marginal costs. Thus the entries in case (i) from both Tables 1 and 2 will also apply to Cournot. For the remainder of this appendix, suppose $\mathcal{R}_0 > \mathcal{R}_0'$.

Suppose market equilibrium output is an interior value: $Q_n^* \in (0, \hat{S}_0)$. Since some but not all consumers purchase, consumers must be indifferent between purchaing and not, implying the price must extract marginal private benefit: $P(Q) = MPB(Q)$. Thus, firm i's profit equals

$$[P(q_i + Q_{-i}) - c]q_i = [MPB(q_i + Q_{-i}) - c]q_i. \tag{A65}$$

Consider the following generalization of a firm's marginal revenue when n symmetric firms together produce Q units:

$$MR(Q,n) = MR(Q) = MPB(Q)\left\{1 - \frac{\theta \mathcal{R}_0 Q[1-\Phi(Q)]}{n[1-\mathcal{R}_0 S_\infty(Q)]}\right\}. \tag{A66}$$

The only difference from marginal revenue defined for a monopoly in (26) is the appearance of n in the denominator of the term in braces. It is obvious that (A66) reduces to (26) when $n = 1$. Taking the first-order condition of (A65) with respect to q_i, imposing symmetry by substituting $q_i^* = Q_n^*/n$, and rearranging, one can show that an interior equilibrium satisfies $MR(Q_n^*, n) = c$.

This interior solution is the equilibrium market output under Cournot if $Q_n^* < \hat{S}_0$. Otherwise, $Q_n^* = \hat{S}_0$, and all firms produce an equal share $q_n^* = \hat{S}_0/n$ in the symmetric equilibrium. A necessary and sufficient condition for this corner solution is $MR(\hat{S}_0, n) \geq c$.

Appendix A4. Consumer Heterogeneity

The model in the text assumes consumers are homogeneous. This appendix introduces consumer heterogeneity and shows that the key result regarding the nonmonotonicities of the marginal externality continues to hold in this extension.

For concreteness, assume consumers, indexed by i, differ in disease harm, H_i. Similar analysis applies if consumers experience different efficacies θ_i or have different lifespans. We conjecture that the analysis is also similar if consumers contract the disease at different rates, but modeling heterogeneity in that dimension requires delicacy to avoid changing the epidemiological process.

Denote the probability density function (pdf) by $f(H_i)$, the cumulative distribution function (cdf) by $F(H_i)$, and the complementary cdf by $\bar{F}(H_i) = 1 - F(H_i)$, and the expected value by $E(H_i) = \int_0^\infty H_i f(H_i) dH_i$. Assume H_i has full support on $(0, \infty)$.

Assume further that the population distribution of H_i is common knowledge but the specific realization of H_i is consumer i's private information. The model requires consumers to be aware of

their heterogeneity (for example, differences in income leading to different willingnesses to pay to avoid harm, or a family history of disease). Undiagnosed conditions that lead harm to vary but are unknown to the consumer are better accommodated in the homogeneous-harm model.

With homogeneous consumers, we showed the marginal private benefit can be written $MPB(Q) = \theta H \Phi(Q)$, the product of efficacy, harm, and probability of contracting the disease. With consumer heterogeneity, consumer i's marginal private benefit becomes $MPB_i(Q) = \theta H_i \Phi(Q)$.

Incorporating heterogeneity in some of the normative measures requires additional work to keep track of the high-value consumers who end up purchasing. We have

$$SB(Q) = \left\{ \left[1-\Phi(Q)\right] \int_0^{\hat{H}} H_i f(H_i) dH_i + \left[1-\Phi(Q)-\theta\Phi(Q)\right] \int_{\hat{H}}^{\infty} H_i f(H_i) dH_i \right\} \hat{S}_0. \tag{A67}$$

The first integral reflects the expected health experienced by those whose harm is below the threshold $\hat{H}$ for purchase. With no vaccine to protect them, consumer i in this group obtains H_i with probability $1-\Phi(Q)$. The second integral reflects the expected health experienced by those who purchase. Consumer i in this group obtains H_i if either they would not have been infected anyway (probability $1-\Phi(Q)$) or would have been infected without a vaccine but receive the vaccine protection (probability $\theta\Phi(Q)$). The final factor $\hat{S}_0$ allows the per-consumer surplus given by the integrals to be scaled up to the population of potential consumers. Differentiating (A67) yields

$$MSB(Q) = \left\{ -\frac{\partial\Phi(Q)}{\partial Q} \left[E(H_i) - \theta \int_{\hat{H}}^{\infty} H_i f(H_i) dH_i \right] + \theta\Phi(Q)\hat{H} f(\hat{H}) \frac{\partial \hat{H}}{\partial Q} \right\} \hat{S}_0. \tag{A68}$$

To compute $\partial\hat{H}/\partial Q$, note threshold consumer type $\hat{H}$ is given as an implicit function of Q by $Q = \bar{F}(\hat{H})\hat{S}_0$. Totally differentiating this identity with respect to Q and rearranging yields $\partial\hat{H}/\partial Q = 1/f(\hat{H})\hat{S}_0$. Substituting this derivative into (A68) shows that the last term equals $\theta\hat{H}\Phi(Q)$. This is the private benefit of the threshold consumer, equal to MPB^* when evaluated at the equilibrium Q^*. Subtracting to compute $MEX^* = MSB^* - MPB^*$ leaves just the first term of (A68), as stated in the following lemma.

Lemma 11. *In the model with heterogeneity in consumer harm H_i, the marginal externality in both the long- and short-run analyses equals*

$$MEX^* = -\frac{\partial\Phi(Q^*)}{\partial Q} \left[E(H_i) - \theta \int_{\hat{H}(Q^*)}^{\infty} H_i f(H_i) dH_i \right] \hat{S}_0. \tag{A69}$$

Intuitively, Lemma 11 says that the marginal externality is proportional to $-\partial\Phi(Q^*)/\partial Q$, the decline in the equilibrium probability of infection for an unvaccinated individual when one additional susceptible is vaccinated. The proof of the next proposition shows that that leading factor approaches 0 as $\mathcal{R}_0 \downarrow 0$ since a noninfectious disease presents no danger of infection in either analysis. The factor also approaches 0 as $\mathcal{R}_0 \uparrow \infty$ in both analyses since the individual will almost certainly contract the infinitely infectious disease in any event—from someone who was vaccinated but for whom the vaccine was ineffective if no one else. The remaining factors are obviously positive and finite for all $\mathcal{R}_0$. Thus, MEX^* approaches 0 for extreme values of $\mathcal{R}_0$, implying it peaks for an interior value of $\mathcal{R}_0 \in (0,\infty)$, as the following proposition states.

Proposition 10. *In the model with heterogeneity in consumer harm H_i, MEX^* peaks for an interior value of $\mathcal{R}_0 \in (0,\infty)$ under both perfect competition and monopoly.*

Proof. It remains to analyze the limits of $\partial\Phi(Q^*)/\partial Q$ as $\mathcal{R}_0 \downarrow 0$ and $\mathcal{R}_0 \uparrow \infty$, showing that the limits equal 0 for both market structures. Lemma 7 states $\lim_{\mathcal{R}_0\downarrow 0}[\mathcal{R}_0 S_\infty(Q)] = 0$, implying $\lim_{\mathcal{R}_0\downarrow 0}\partial\Phi(Q)/\partial Q = 0$ by (A17). Lemma 8 states $\lim_{\mathcal{R}_0\uparrow\infty}[\mathcal{R}_0 S_\infty(Q)] = 0$, implying $\lim_{\mathcal{R}_0\uparrow\infty}\partial\Phi(Q)/\partial Q = 0$ by (A17). These limits both hold for all Q, including $Q = Q_c^*$ and $Q = Q_m^*$. *Q.E.D.*

Appendix A5. Additional Preventive

The model in the text assumes that the vaccine is the only preventive technology available to consumers. In practice, consumers may pursue other preventive technologies instead of or in addition to vaccines, including social distancing. The extension in this appendix extends the model to allow for a second technology having efficacy δ against the disease. For simplicity, we initially derive results supposing the second technology is competitively supplied at a zero price, so that all consumers adopt it. We then show the results generalize to the case in which the price of the competitively supplied second product is nonnegative, as long as the price is sufficiently low.

Generalizing Model

We will generalize the model so that the successfully immunized compartment Z_0 now covers all successfully protected from the disease by any product. The epidemiological model remains the same as in the text, governed by equations (1)–(8). The only change is that Z_0 can have a more general form than in (9). To accommodate that more general form, we will rederive expressions for epidemiological outcomes as functions not of the vaccine quantity Q but of the proportion protected Z_0. We will write the susceptible, infected, and recovered compartments as $S_t(Z_0)$, $I_t(Z_0)$, and $R_t(Z_0)$, respectively, and write the probability of infection as

$$\Phi(Z_0) = 1 - \frac{S_\infty(Z_0)}{S_0(Z_0)} = 1 - \frac{S_\infty(Z_0)}{\hat{S}_0 - Z_0}. \tag{A70}$$

Since only an initial condition has changed, not one of the laws of motion, most of the epidemiological outcomes remain the same as in the text. In particular, Lemmas 1–5 remain unchanged substituting $S_t(Z_0)$ and $I_t(Z_0)$ for $S_t(Q)$ and $I_t(Q)$. Lemma 6 would remain unchanged had it been written in terms of $S_0(Q)$, but given it is written in terms of the exogenous parameters, the equations characterizing $S_\infty(Z_0)$, (11) and (12), need to be replaced by

$$\ln S_\infty(Z_0) - \mathcal{R}_0 S_\infty(Z_0) = \ln(\hat{S}_0 - Z_0) - \mathcal{R}_0(\hat{I}_0 + \hat{S}_0 - Z_0) \tag{A71}$$

$$S_\infty(Q) = \frac{1}{\mathcal{R}_0}\left|\bar{L}\left(-\mathcal{R}_0(\hat{S}_0 - Z_0)e^{-\mathcal{R}_0(\hat{I}_0+\hat{S}_0-Z_0)}\right)\right|. \tag{A72}$$

Applying the Implicit Function Theorem to (A71) yields

$$\frac{\partial S_\infty(Z_0)}{\partial Z_0} = \frac{S_\infty(Z_0)}{S_0(Z_0)}\left[\frac{\mathcal{R}_0 S_0(Z_0) - 1}{1 - \mathcal{R}_0 S_\infty(Z_0)}\right] \tag{A73}$$

$$\frac{\partial S_\infty(Z_0)}{\partial \mathcal{R}_0} = \frac{-S_\infty(Z_0)}{1 - \mathcal{R}_0 S_\infty(Z_0)}[\hat{I}_0 + S_0(Z_0) - S_\infty(Z_0)]. \tag{A74}$$

Note the similarity of these derivatives to their analogs, (13) and (A16), in the model without the second preventive.

Additional Preventive Freely Available

Assume that whether technologies protect an individual are independent draws across consumers and across technologies. Assume further that consumers do not learn about the success or failure of one technology before buying the other, so make a simultaneous buying decision in period $t = 0$. Given that all $\hat{S}_0$ initial susceptibles consume the second technology with efficacy δ and Q susceptibles buy the vaccine with efficacy θ, the number of initial susceptibles who are protected is

$$Z_0(Q) = \delta\hat{S}_0 + (1-\delta)\theta Q, \tag{A75}$$

replacing equation (9) in the epidemiological model.

We will continue the notational convention of writing variables related to demand, supply, and social welfare in terms of the quantity Q of the vaccine, still the product of key interest since it alone is the potential target of subsidy. The marginal private benefit from the vaccine is

$$MPB(Q) = (1-\delta)\theta H\Phi(Z_0(Q)), \tag{A76}$$

and its derivative is

$$\frac{\partial MPB(Q)}{\partial Q} = \frac{-(1-\delta)\theta\mathcal{R}_0 S_\infty(Z_0(Q))MPB(Z_0(Q))}{S_0(Z_0(Q))[1-\mathcal{R}_0 S_\infty(Z_0(Q))]}. \tag{A77}$$

The demand curve remains the same as in (18), substituting the following expression for demand when a subset purchase for (17):

$$d(P) = \frac{1}{(1-\delta)\theta}\left\{(1-\delta)\hat{S}_0 + \frac{(1-\delta)\theta H}{P}\left[\frac{1}{\mathcal{R}_0}\ln\left(1-\frac{P}{(1-\delta)\theta H}\right)+\hat{I}_0\right]\right\}. \tag{A78}$$

Regarding normative measures, social benefit becomes

$$SB(Q) = H[1-R_\infty(Z_0(Q))] = H[S_\infty(Z_0(Q))+Z_0(Q)]. \tag{A79}$$

The expressions for welfare, marginal social benefit, and marginal externality in (21)–(23) remain unchanged since they were already written in sufficient generality.

On the supply side, redefine normalized unit cost as

$$\tilde{c} = \frac{c}{(1-\delta)\theta H} = \tilde{c}. \tag{A80}$$

Marginal revenue for a monopoly becomes

$$MR(Q) = MPB(Q)\left\{1-\frac{(1-\delta)\theta\mathcal{R}_0 Q[1-\Phi(Z_0(Q))]}{1-\mathcal{R}_0 S_\infty(Z_0(Q))}\right\}. \tag{A81}$$

The preceding expressions can be used to derive equilibrium variables under perfect competition and monopoly shown in Tables A1 and A2. The threshold values of $\mathcal{R}_0$ become

$$\mathcal{R}_0' = \frac{|\ln(1-\tilde{c})|}{\hat{I}_0 + (1-\delta)\tilde{c}\hat{S}_0}. \tag{A82}$$

$$\mathcal{R}_0'' = \frac{|\ln(1-\tilde{c})|}{\hat{I}_0 + (1-\delta)(1-\theta)\tilde{c}\hat{S}_0}. \tag{A83}$$

The entries in Table A1 can be used to show Proposition 1 holds without modification in this generalization. More specifically, for all variables except $R_\infty(Z_0(Q_c^*))$ and W_c^*, the entries are identical in the two tables after transforming two constants: $\breve{\theta} = (1-\delta)\theta$ and $\breve{S}_0 = (1-\delta)\hat{S}_0$. The entries for the remaining two variables just add a constant that does not affect the derivative with respect to $\mathcal{R}_0$, since $\partial S_\infty(Q)/\partial\mathcal{R}_0$ is invariant to the generalization as shown in (A74).

A similar argument can be used to show that Proposition 3 holds without modification in this generalization. It is immediate that Proposition 2 holds in the generalization because the expressions are provided in a general enough way that they remain unchanged in the generalization.

Turn next to an analysis of the comparative-statics effects of an increase in δ. The top panel of Figure A1 shows how Q^* varies with $\mathcal{R}_0$ for a given value of δ. An increase in δ effectively stretches the solid and dotted black curves for vaccine quantity rightward. Formally, one can show that an increase in δ increases the threshold values of $\mathcal{R}_0$ in (A82)–(A83) determining the regions in which some but not all consumers purchase a competitively supplied vaccines can be shown to increase in δ (taking into account the fact that an increase in δ increases $\tilde{c}$ in (A80)). The rightward stretch means that Q^* weakly declines in δ for a given $\mathcal{R}_0$. The reduction in vaccine quantity is not enough to offset the increase in population protection from the second preventive. Population protection increases in δ since each consumer has more options for protection, so can arrange weakly lower cost personal protection for any given level of population protection.

Additional Preventive Sold at Low Price

The top panel of Figure A1 illustrates the comparative-static effect of $\mathcal{R}_0$ on Q^* in the presence of a second preventive that is freely available. The picture is similar even when the second preventive is sold for a positive price if that price is sufficiently low that all consumers purchase the second preventive for any $\mathcal{R}_0$ such that any purchase the vaccine. The new situation is shown in the lower panel in Figure A1. While the gray curve representing the quantity of the second preventive looks different from the upper panel, it is only different in a region that is irrelevant for vaccine purchase; they are identical in the region of $\mathcal{R}_0$ labeled (d).

While it is intuitive that the strict results from the previous subsection, which hold when the price of the second preventive is zero, should hold in a neighborhood of strictly positive prices by continuity, we proceed to verify this formally. In particular, if the following condition holds,

$$c_2 < \frac{\delta c_1[1-\max(\delta,\theta)]}{(1-\delta)\theta}. \tag{A84}$$

then, for each $\mathcal{R}_0$ in regions (a)–(c) in Figure A1, there exists an equilibrium in which no vaccine is purchased and the quantity indicated purchase the second preventive; furthermore, for each $\mathcal{R}_0$ in region (d), there exists an equilibrium in which all consumers purchase the second preventive and the quantity indicated purchase the vaccine as well.

Case (a) is defined as that region of $\mathcal{R}_0$ for which no consumer purchases the second preventive when no vaccine is purchased either. For no consumer to purchase the second preventive in equilibrium, consumer surplus must be negative:

$$\delta H\Phi(0) - P_2 \leq 0. \tag{A85}$$

We will show that no consumer deviates to purchasing the vaccine either. We have

$$\theta H\Phi(0)-P_1 \leq \theta H\Phi(0)-c_1 \tag{A86}$$

$$< \theta H\Phi(0)-\frac{\theta c_2}{\delta} \tag{A87}$$

$$= \frac{\theta}{\delta}[\delta H\Phi(0)-c_2]. \tag{A88}$$

Condition (A86) follows from a nonnegative markup on vaccines, (A87) from $c_1 > \theta c_2/\delta$ by (A84), and (A88) from algebra. Substituting (A85) into (A88) implies $\theta H\Phi(0)-P_1 < 0$, implying that purchasing a vaccine provides negative consumer surplus for all $\mathcal{R}_0$ in case (a). The incremental consumer surplus from buying the vaccine in addition to the second preventive,

$$(1-\delta)\theta H\Phi(0)-P_1, \tag{A89}$$

is yet lower, so no vaccine is purchased in region (a).

In case (b), some but not all consumers purchase the second preventive, implying that the equilibrium quantity of the second preventive Q_2 is such that they are indifferent between buying the second preventive and not:

$$\delta H\Phi(\delta Q_2)-P_2 = 0. \tag{A90}$$

Conditions (A86)–(A88) continue to apply. Substituting (A90) into (A88) implies $\theta H\Phi(0)-P_1 < 0$, implying that purchasing a vaccine provides negative consumer surplus in case (b). As argued in the previous paragraph, the incremental consumer surplus from buying the vaccine in addition to the second preventive is yet lower, so no vaccine is purchased in region (b).

The upper threshold of case (c) is given by the $\mathcal{R}_0$ such that consumers first start to buy the vaccine in addition to the second preventive when all other consumers buy the second preventive and only that, when the vaccine is supplied under perfect competition. The marginal vaccine consumer obtains zero incremental surplus from a vaccine sold at marginal cost:

$$(1-\delta)\theta H\Phi(\delta\hat{S}_0)-c_1 = 0. \tag{A91}$$

We will show that consumers do not prefer buying just the vaccine to buying just the second preventive. The consumer surplus from buying just the vaccine is

$$\theta H\Phi(\delta\hat{S}_0)-P_1 \leq \theta H\Phi(\delta\hat{S}_0)-c_1 \tag{A92}$$

$$= \delta H\Phi(\delta\hat{S}_0)+(\theta-\delta)H\Phi(\delta\hat{S}_0)-c_1 \tag{A93}$$

$$= \delta H\Phi(\delta\hat{S}_0)-\frac{\delta(1-\theta)c_1}{(1-\delta)\theta} \tag{A94}$$

$$< \delta H\Phi(\delta\hat{S}_0)-c_2. \tag{A95}$$

Condition (A92) follows from a nonnegative markup on vaccines, (A87) from rearranging, (A94) from substituting from (A91) for $H\Phi(\delta\hat{S}_0)$ and rearranging, and (A95) from

$$c_1 > \frac{(1-\delta)\theta c_2}{\delta(1-\theta)}, \tag{A96}$$

which follows from (A84).

In case (d), consumers start to add the vaccine when all consumers purchase the second preventive. For each $\mathcal{R}_0$, there exists an equilibrium in which the analysis from the previous subsection characterizes equilibrium quantities of both products.

TABLE A1: Equilibrium Variables under Perfect Competition Adding Second Preventive Technology

	Case		
	(i)	(ii)	(iii), (iv)
Variable	$\mathcal{R}_0 \in (0, \mathcal{R}_0']$	$\mathcal{R}_0 \in (\mathcal{R}_0', \mathcal{R}_0'']$	$\mathcal{R}_0 \in (\mathcal{R}_0'', \infty)$
P_c^*	c	c	c
Q_c^*	0	$\frac{1}{(1-\delta)\theta}\left\{(1-\delta)\hat{S}_0 + \frac{1}{\tilde{c}}\left[\frac{1}{\mathcal{R}_0}\ln(1-\tilde{c}) + \hat{I}_0\right]\right\}$	$\hat{S}_0$
Π_c^*	0	0	0
$R_\infty(Z_0(Q_c^*))$	$1 - S_\infty(0) - \delta\hat{S}_0$	$1 - \hat{S}_0 - \hat{I}_0 + \frac{1}{\mathcal{R}_0}\lvert\ln(1-\tilde{c})\rvert$	$1 - S_\infty(\hat{S}_0) - [\delta + (1-\delta)\theta]\hat{S}_0$
MPB_c^*	$(1-\delta)\theta H\Phi(0)$	c	$(1-\delta)\theta H\Phi(\hat{S}_0)$
MSB_c^*	$\frac{(1-\delta)\theta H\Phi(0)}{1-\mathcal{R}_0 S_\infty(0)}$	$\frac{(1-\delta)\theta H\tilde{c}^2}{\tilde{c}+(1-\tilde{c})[\ln(1-\tilde{c})+\mathcal{R}_0\hat{I}_0]}$	$\frac{(1-\delta)\theta H\Phi(\hat{S}_0)}{1-\mathcal{R}_0 S_\infty(\hat{S}_0)}$
MEX_c^*	$\frac{(1-\delta)\theta H\Phi(0)\mathcal{R}_0 S_\infty(0)}{1-\mathcal{R}_0 S_\infty(0)}$	$\frac{(1-\delta)\theta H\tilde{c}(1-\tilde{c})[\lvert\ln(1-\tilde{c})\rvert-\mathcal{R}_0\hat{I}_0]}{\tilde{c}+(1-\tilde{c})[\ln(1-\tilde{c})+\mathcal{R}_0\hat{I}_0]}$	$\frac{(1-\delta)\theta H\Phi(\hat{S}_0)\mathcal{R}_0 S_\infty(\hat{S}_0)}{1-\mathcal{R}_0 S_\infty(\hat{S}_0)}$
W_c^*	$H[S_\infty(0) + \delta\hat{S}_0]$	$H[1-(1-\delta)\tilde{c}]\hat{S}_0$	$H\{S_\infty(\hat{S}_0) + [\delta + (1-\delta)(1-\tilde{c})\theta]\hat{S}_0\}$

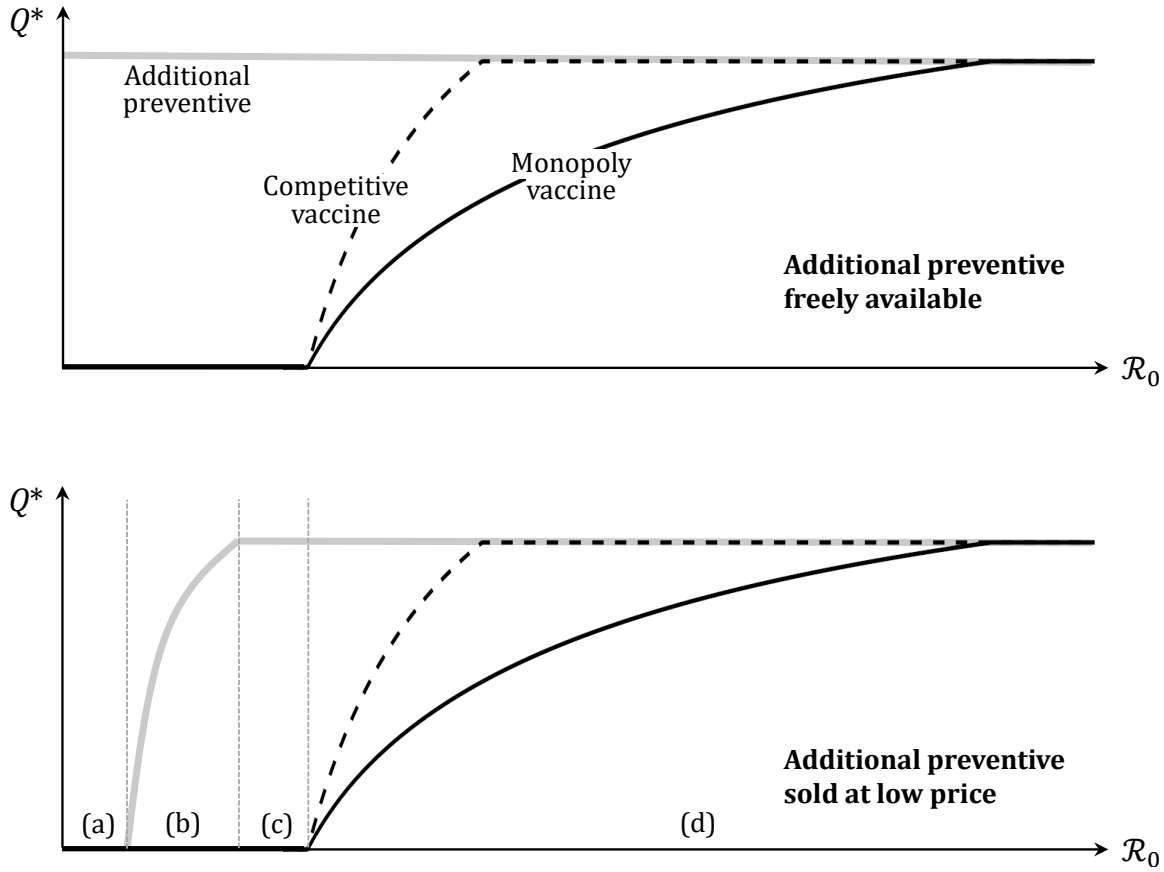
Notes: Generalizes Table 1, allowing for a second technology with efficacy δ against the disease. Entries in Table 1 can be recovered by setting $\delta = 0$. See that table for additional explanatory notes.

TABLE A2: Equilibrium Variables under Monopoly Adding Second Preventive Technology

	Case		
	(i)	(ii), (iii)	(iv)
Variable	$\mathcal{R}_0 \in (0, \mathcal{R}_0']$	$\mathcal{R}_0 > \mathcal{R}_0'$ but $MR(\hat{S}_0) < c$	$\mathcal{R}_0$ satisfies $MR(\hat{S}_0) \geq c$
P_m^*	†	$(1-\delta)\theta H \Phi(Q_m^*)$	$(1-\delta)\theta H \Phi(\hat{S}_0)$
Q_m^*	0	Solution to $MR(Q_m^*) = c$	$\hat{S}_0$
Π_m^*	0	$(1-\delta)\theta H[\Phi(Q_m^*) - \tilde{c}]Q_m^*$	$(1-\delta)\theta H[\Phi(\hat{S}_0) - \tilde{c}]\hat{S}_0$
$R_\infty(Z_0(Q_m^*))$	$1 - S_\infty(0) - \delta\hat{S}_0$	$1 - S_\infty(Q_m^*) - \delta\hat{S}_0 - (1-\delta)\theta Q_m^*$	$1 - S_\infty(\hat{S}_0) - [\delta + (1-\delta)\theta]\hat{S}_0$
MPB_m^*	$(1-\delta)\theta H \Phi(0)$	$(1-\delta)\theta H \Phi(Q_m^*)$	$(1-\delta)\theta H \Phi(\hat{S}_0)$
MSB_m^*	$\frac{(1-\delta)\theta H \Phi(0)}{1-\mathcal{R}_0 S_\infty(0)}$	$\frac{(1-\delta)\theta H \Phi(Q_m^*)}{1-\mathcal{R}_0 S_\infty(Q_m^*)}$	$\frac{(1-\delta)\theta H \Phi(\hat{S}_0)}{1-\mathcal{R}_0 S_\infty(\hat{S}_0)}$
MEX_m^*	$\frac{(1-\delta)\theta H \Phi(0)\mathcal{R}_0 S_\infty(0)}{1-\mathcal{R}_0 S_\infty(0)}$	$\frac{(1-\delta)\theta H \Phi(Q_m^*)\mathcal{R}_0 S_\infty(Q_m^*)}{1-\mathcal{R}_0 S_\infty(Q_m^*)}$	$\frac{(1-\delta)\theta H \Phi(\hat{S}_0)\mathcal{R}_0 S_\infty(\hat{S}_0)}{1-\mathcal{R}_0 S_\infty(\hat{S}_0)}$
W_m^*	$H[S_\infty(0) + \delta\hat{S}_0]$	$H[S_\infty(Q_m^*) + \delta\hat{S}_0 + (1-\delta)(1-\tilde{c})\theta Q_m^*]$	$H\{S_\infty(\hat{S}_0) + [\delta + (1-\delta)(1-\tilde{c})\theta]\hat{S}_0\}$

Notes: Generalizes Table 2, allowing for a second technology with efficacy δ against the disease. Entries in Table 2 can be recovered by setting $\delta = 0$. See that table for additional explanatory notes.

FIGURE A1: Equilibrium Quantities at Various Prices for Second Preventive



Notes: Schematic diagram of comparative statics of Q^* in $\mathcal{R}_0$, analogous to top panel of Figure 1, but here for model with additional preventive. Panels illustrate two different prices for the second preventive. Dotted black curve represents Q_c^*, and solid black curve represents Q_m^*. Gray curve represents quantity of second preventive. Where curves overlap, solid black curve represents all overlapping curves.