Mathematics: The Stray Cat

Then our minds engage in the joys of academia and the great disciplines of mankind, we naturally group the scholarly subjects in a sensible, orderly fashion, leaving mathematics as the sole area absent from our mental conversations. We push the humanities together, easily seeing the relevance between literary genre, historical movements, and the importance of language in conveying history and literature. We likewise attribute other acts of this human struggle to the humanities, such as economics, law, philosophy, logic and anthropology. These subjects are admired for their noble attempt to explain the social world and to answer the perennial questions. Then our brains will section the humanities off; opposite the humanities will be the sciences. Biology, chemistry, and physics will all perfectly interrelate, intricately explaining the same natural processes from various points of view. Also on this side, without much ado, we place a new friend, computer science. Our minds will praise these sciences for their practical applications, their proficiencies in the bettering of life, their abounding relevance, and their general acceptance as important, scholarly topics. As we work with our neat organization, a stray cat appears: mathematics. Where will we put it? Surely not in the humanities--any senseless fool can see why. And should the rest of the sciences be forced to share space with what is viewed by many as an irrelevant, useless discipline in emotionless drill? But in spite of our preconceived notions of mathematics, the question is not where to put mathematics, but where not to put it.

Mathematics always seems to be the outcast of academia. The humanist traditionally criticizes mathematics as a pointless (and endless) line of numbers and calculations, while the scientist demeans the discipline as merely a subject subservient to his or her own field of work. Alfred Nobel, while creating the Nobel prizes, scoffed at the idea of creating a mathematics prize. However, our intuitions to label mathematics as an esoteric subject could not be more inaccurate. Rather, mathematics underlies virtually every academic discipline, from poetry to chemistry. The lure and curiosity of mathematics escapes not one subject,

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and, although viewed as a dry, irrelevant field of study, mathematics may prove to be the most fundamental and exciting area of study after all.

Although not completely in conjunction with the work of the sciences, mathematics is their backbone. Without differential geometry, Einstein's relativity theory would have no relevance; without basic calculus, physical mechanics could only exist in one dimension; and without the notions of statistics and variance, chemistry would have little sense of itself. Mathematics penetrates computer science through the notions of algorithm and software programming logic. In fact, most universities require linear algebra before a student even can embark upon the computer science major. Mathematics is part of the fundamental fabric of the sciences.

As we stray from the sciences, though, the influence of mathematics is unclear. Mathematics is routinely linked to economics, but only because the subject involves numbers. Yet, economics is not only an analysis of world trade and number systems; economics can be interpreted as a branch of applied mathematics called game theory. Coined by John Von Neumann, game theory has existed as a respected topic in mathematics for less than a century. However, game theory's application to practical situations has won mathematicians Nobel prizes for economics, sparked military interest, and forced students of economics to take a more mathematical approach to their endeavors. After Von Neumann pioneered the subject, he co-authored, with Oskar Morganstern, a proof of the minimax theorem. The theorem concerns two-person zerosum games of strategy, that is, any strategic situation in which, given two players in a game, one player's win is the other's loss. The minimax theorem uses topology and advanced calculus to prove that, given a two-person zero-sum game, there is exactly one optimum strategy for both players. John Forbes Nash Jr. expounded upon this idea, proving that for every non-zero-sum game, there also exists a strategic equilibrium and program for optimum outcome. The applications of non-zerosum games go far beyond economics; in fact, through organizations like the Rand Corporation,

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the U.S. military has pioneered a large amount of game theory research. Even beyond the military, game theory pervades almost every situation that involves some strategy of play between two or more sides.

Mathematics has applications to international relations besides the military and economics. Cartography has been a subject of interest for mathematicians for centuries. The Four-Color Theorem, which states that any map can be colored adequately with no more than four colors, had gone unproven for centuries. Finally, in the 1970's, two mathematicians, Appel and Haken, "proved" the theorem named using multiple computers logged onto the problem for hundreds of hours. The computers checked every possible case of map design, proving by exhaustion the Four-Color Theorem. Doubted by some mathematicians as a cop-out, the proof is theoretically impossible to check. As the computers far out-did humans in the methods of this proof, the proof of the Four-Color Theorem may provide the surest link between computer science and mathematics in the future.

Most surprising is mathematics' strong link to language and literature. Venturing beyond cartography, the military, and economics, we reach the frontier of language, the story, poetry; everything mathematics typically is not. But there are certainly more connections than meet the eye. Take the notions of story, for example. According to John Allen Paulos, author of Once Upon a Number, we are all amateur mathematicians by the time we can read. Mathematical notions, such as statistics and probability, pervade our language and storytelling through words like chance, few, possibly, about, likely, and unlikely, among others. Mathematics also dominates the logic of storytelling; logically persuading a reader one way takes mathematical forms such as modus ponens and universal instantiation. Although these words might sound like foreign tongue, most every person employs these models in conveying a point, telling a story, or relaying gossip. For example, the form modus ponens says that, given an if-then statement such as, "If it is raining, then the sidewalks are wet," then, given the antecedent, which in this case is "it is raining," we can conclude that the sidewalks must be wet, also. Syllogism is the equivalent of modus ponens in the humanities. Universal instantiation dictates that, given a set of numbers, letters, peoples, objects, etc., if something is true for the whole set, then the same is true for any individual

in that set. For example, suppose that there is a requirement at the school that you attend that every student must take three semesters of physical education. The deduction that you must also take three semesters of physical education is obvious; however, this small deduction is mathematics at work. Making deductions, concluding from evidence, and understanding language are tools rooted in logical mathematics. Taking the use of evidence and deduction one step further, mathematics is essential in interpreting literature and the written word.

Although logic is the most readily available link mathematics has with the humanities, various scholars took this connection a few steps further. Andrei Andreevich Markov, a Russian mathematician, developed the idea of Markov chains, which use matrices to illustrate probabilities. Although used initially by Markov to prove other theorems, the Markov chains were also used in analyzing vowel-consonant interchanges in 20,000 letters of Pushkin's poetry. Another Russian, Yevgeny Zamyatin, used mathematics as a symbol in his book WE. Describing the square root of -1, Zamyatin compares the impossibility of the number with the impossibility of the narrator's expression of emotion under communist rule. Dostoyevsky also addresses the impossibility of mathematics as compared to the impossibility of life in The Brothers Karamozov when Ivan says,

If God exists and if he really did create the earth then, as common knowledge tells us, he created it according to Euclidean geometry, while he created the human mind with an awareness of only three spatial dimensions. Even so, there have been and still are even today geometers and philosophers of the most remarkable kind who doubt that the entire universe or, even more broadly, the entirety of being was created solely according to Euclidean geometry, and who even make so bold as to dream that the two parallel lines which according to Euclid can on no account converge upon earth may yet do so somewhere in infinity. And so, my lad, I've decided that if I can't even understand that, then how am I to understand about God?...Even if the parallel lines converge and I actually witness it, I shall witness it and say they have converged, but all the same I shall not accept it.

As illustrated by mathematicians, authors, and scientists, mathematics is not a self-contained subject. Coined the Universal Language, mathematics does not sell its nickname short. Mathematics is a beautiful language with which we describe our world. It encompasses fields as diverse as number theory, differential equations, and probability in order to explain the world. A friend of mine once told me that his definition of math appeared a contradiction, but was really a tautology: "mathematics is philosophy." Mathematics isn't different than any other kind of philosophy; mathematics is a distinct way of looking of looking at the world. Though counter-intuitive, this philosophy penetrates our daily lives in more ways than simply counting change in the supermarket. As Dostovevsky suggests, two parallels may converge, and it seems that this distinct type of counter-intuition drives our world, at least when dealing with mathematics. And so, I find that two apparently parallel lines, that is, mathematics and what may seem to be nonmathematics, converge infinitely in the most usual--and unusual--ways.

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