

# Quantum Entanglement From Theory to New Technology

by Anura Abeyesinghe '01

The micro world as revealed by quantum theory is a very strange place where our intuitions formed by direct experience with the macro world are no longer valid. Among the many strange features of this theory is a phenomenon known as quantum entanglement, a correlation between particles that cannot be explained classically. Predicted by the superposition principle and the mathematics of Hilbert space in which quantum particles live, they imply that nature may act non-locally, a “spooky action at a distance” that led to Einstein’s dissatisfaction with quantum mechanics (Bernstein, 1991). For decades after Schrödinger discovered this quantum property in 1926, physicists have been worrying about these philosophical consequences. This was mainly after Bell’s work in the 1960s; before then not many people paid much attention to these issues due to the ever-increasing practical success of the theory. However, recently physicists stopped worrying and are taking a positive approach. They are finding ways to manipulate entanglement and do remarkable things with it. It is exploited in the newly developing fields of quantum computation, quantum cryptography and is the foundation of quantum information theory and quantum teleportation. Entanglement, once a curiosity, is now a valuable resource. This paper will introduce entanglement, show how entanglement leads to correlations that cannot be explained classically, and show how those correlations are exploited in quantum cryptography and quantum teleportation.

Two particles are said to be entangled if their combined state cannot be written as a direct product of complete states of each individual particle. If the two subsystems are denoted by A and B and if  $|i\rangle$  and  $|j\rangle$  are a basis for A and B then a combined state is entangled if it cannot be written in the form:

$$\left(\sum_i C_i^{(A)} |i\rangle_A\right) \left(\sum_j C_j^{(B)} |j\rangle_B\right)$$

Given a state in the general form

$$|\psi\rangle_{AB} = \sum_{i,j} C_{ij} |i\rangle_A |j\rangle_B$$

it is not easy to look at it and determine if it is entangled or not. However it can be shown that it is entangled if and only if each subsystem is in a mixed state. This is good news because making a local measurement on a subsystem of an entangled state puts the other subsystem in a mixed state, and if it weren’t in a mixed state beforehand this difference could be noted and used to send superluminal information, violating relativity theory.<sup>1</sup>

Looking for entanglement in Hilbert space is one thing but creating it in a lab is quite another. In 1997 experimental observation of entanglement had been restricted to pairs of photons, electrons, protons and atoms (Bouwmeester & Zeilinger, 1997). I will describe briefly an experiment in which he obtained entangled rubidium atoms (Bouwmeester & Zeilinger, 1997). First an excited rubidium atom was entered into a microwave cavity and kept there long enough that there was a 50% chance of emitting a photon and falling to its ground state. The cavity and atom are

$$\frac{1}{\sqrt{2}} (|1\rangle|G\rangle + |0\rangle|E\rangle)$$

- |1⟩ = photon in cavity
- |0⟩ = no photon in cavity
- |G⟩ = atom in ground state
- |E⟩ = atom in excited state

now in the entangled state

The cavity was made from superconducting niobium operating at a very low temperature to isolate

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<sup>1</sup> That is  $\rho_A^2 \neq \rho_A$  where  $\rho_A$  is the reduced density matrix for A. This is equivalent to showing that  $|\psi\rangle_{AB}$  is a product state if and only if  $\rho_A^2 = \rho_A$ . Now, if  $|\psi\rangle_{AB} = |\psi^{(A)}\rangle |\psi^{(B)}\rangle$ , then

$$\rho_A = \text{tr}_B(|\psi\rangle_{AB}\langle\psi|) = \text{tr}_B(|\psi^{(A)}\rangle\langle\psi^{(A)}| \otimes |\psi^{(B)}\rangle\langle\psi^{(B)}|) = |\psi^{(A)}\rangle\langle\psi^{(A)}|$$

and so  $\rho_A^2 = \rho_A$ . Now for the converse statement, suppose  $\rho_A^2 = \rho_A$ .

First diagonalize  $\rho_A$  and let  $|\phi_i\rangle$  be the set of eigenvectors

$$|\psi\rangle_{AB} = \sum_{i,j} C_{ij} |\phi_i\rangle_A |j\rangle_B$$

Since  $\text{tr} \rho_A = 1$  there can be only one nonzero term

on the diagonal (i.e. 1). Let  $|\phi_1\rangle$  be the eigenvector corresponding to it. Then since  $\text{tr}_A(|\phi_i\rangle\langle\phi_i| |\phi_1\rangle\langle\phi_1|) = 0$  for  $i \neq 1$ , the expectation value of

$$|\phi_i\rangle\langle\phi_i| = A \text{ is zero. Therefore } \sum_{i,j} C_{ij} |\phi_i\rangle\langle\phi_i| |j\rangle = \sum_j |C_{1j}|^2 |j\rangle = 0 \text{ and so } C_{ij} =$$

0 for  $i \neq 1$ . Therefore  $|\psi\rangle_{AB}$  is a product state.

it from its environment and prevent decoherence of the superposed state. A second rubidium atom in its ground state was then entered into the cavity and kept there for a sufficiently long time that if a photon was present, it would be absorbed by the atom moving it to an excited state. The combined state of the two atom is,

$$1/\sqrt{2} ( |E\rangle|G\rangle + |G\rangle|E\rangle )$$

and so the atoms are entangled.

In 1935, Einstein, Podolsky, and Rosen presented their famous "EPR" paper. They made the very reasonable assumption that measurements of one particle cannot affect the results of measurements of a second, distant particle. However since entangled pairs of particles seem to do exactly this, they argued that the quantum mechanical description of states cannot be complete. There must be some "hidden variables" associated with each particle that determine beforehand the outcome of measurements that seem to violate locality. However, as John Bell showed in 1969, "the reasonable thing just doesn't work" (Bernstein, 1991). His result was generalized by Clauser, Horne, Shimony, and Holt (CHSH). They showed that if a local hidden variable theory exists, a certain quantity that can be measured must satisfy the CHSH inequality (Shimony lecture, 2000). They then demonstrated that there are certain systems in which the same quantity predicted by quantum theory violates the CHSH inequality. Therefore, no local hidden variable theory is consistent with quantum mechanics. Furthermore, other experiments suggested that the quantum mechanical prediction triumphed (Tittel et al., 1998). For these systems, their correlations cannot be explained classically. It can also be shown that, in general, for any n number of entangled particles (in pure states), there is always a measurement that yields a result inconsistent with the EPR assumption of local hidden variables (Propescu & Rohrlich, 1992). Detectors have a limited efficiency, and it has been shown that the above measured quantity as predicted by quantum mechanics has to be multiplied by a factor of efficiency squared. To violate Bell inequalities, efficiency has to be at least 67% (Shimony lecture, 2000). However, no experiment has yet achieved this efficiency. To get around this problem an untestable supplementary assumption known as the "fair sampling assumption" is made (Hardy, 1998). The detectors sample the ensemble in a fair way so that those events in which both particles are detected are representative of the whole ensemble. The

good news is that there are experiments currently using entangled  $^{199}\text{Hg}$  atoms formed by dissociating  $^{199}\text{Hg}_2$  dimers using a weak laser, in which the detection efficiency is high enough to not require the fair sampling assumption (Shimony lecture, 2000; Bouwmeester & Zeilinger, 1997).

I will not discuss the details of the CHSH inequality here. Rather I will describe a more direct, simple, and beautiful proof of non-locality using three photons presented by Greenberger, Horne and Zeilinger (GHZ) (Lo et al., 1998; Hardy, 1998). This scheme has not been implemented experimentally due to problems faced in producing these GHZ triplets. I will use the notation  $|H\rangle$  and  $|V\rangle$  for horizontal and vertically polarized photons and  $|45\rangle$  and  $|-45\rangle$  for photons polarized at  $45^\circ$  and  $-45^\circ$  to the vertical, respectively. The three photons are initially prepared in the state,

$$1/2\sqrt{2} ( |H\rangle|H\rangle|H\rangle - |H\rangle|V\rangle|V\rangle - |V\rangle|H\rangle|V\rangle - |V\rangle|V\rangle|H\rangle ) \dots (i)$$

They are then made to propagate to three separate locations where their polarizations can be measured using birefringent calcite crystals (these crystals distinguish polarization in two perpendicular directions). Define  $A(\theta) = +1$  if, when measured at  $\theta$  to horizontal, polarization is along direction and  $A(\theta) = -1$  polarization is in  $90-\theta$  direction. Similarly define B and C for photons 2 and 3 respectively. Now by looking at expression (1) it is seen that the values  $(A(0), B(0), C(0))$  can take are  $(+1, +1, +1)$ ,  $(+1, -1, +1)$ ,  $(-1, +1, -1)$  and  $(-1, -1, +1)$ . In all cases the product  $A(0)B(0)C(0) = 1$ . Now consider the case where  $A(0)$ ,  $B(45)$  and  $C(45)$  are measured. (i) can be written as

$$\frac{1}{\sqrt{2}} |H\rangle [ \frac{1}{\sqrt{2}} ( |H\rangle|H\rangle - |V\rangle|V\rangle ) ] - \frac{1}{\sqrt{2}} |V\rangle [ \frac{1}{\sqrt{2}} ( |H\rangle|V\rangle + |V\rangle|H\rangle ) ]$$

which can then be written as initial state=

$$\frac{1}{2\sqrt{2}} ( |H\rangle|45\rangle|-45\rangle + |H\rangle|-45\rangle|45\rangle - |V\rangle|45\rangle|45\rangle + |V\rangle|-45\rangle|-45\rangle ) \dots (ii)$$

Looking at this expression, the possible values of  $(A(0), B(45), C(45))$  are  $(+1, +1, -1)$ ,  $(+1, -1, +1)$ ,  $(-1, +1, +1)$ , and  $(-1, -1, -1)$ . In each case the product  $A(0)B(45)C(45) = -1$ . By symmetry we also have,  $A(45)B(0)C(45) = -1$  and  $A(45)B(45)C(0) = -1$ . Note here that in these expressions the A's, B's, and C's are non-local. That is  $A(45)$  in  $A(0)B(45)C(45) = -1$  is not the same as  $A(45)$  in  $A(45)B(0)C(45) = -1$ . They depend on the results of the B and C measurements of photons 2 and 3. The only way to get around this prediction of non-locality is to assume that quantum mechanics is not complete, there is some other variable associated with each photon that determines beforehand what a measurement

of A, B, or C would yield, and this variable doesn't depend on the results of measurements on the other photons. Let the hidden variable be denoted by  $\lambda$  and let  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  be its values for photons 1, 2 and 3 respectively. Then, for the results of the hidden variable theory to be consistent with quantum mechanics, we need that,

$$A(0, \lambda_1)B(0, \lambda_2)C(0, \lambda_3) = +1 \text{-----(1)}$$

$$A(0, \lambda_1)B(45, \lambda_2)C(45, \lambda_3) = -1 \text{-----(2)}$$

$$A(45, \lambda_1)B(0, \lambda_2)C(45, \lambda_3) = -1 \text{-----(3)}$$

$$A(45, \lambda_1)B(45, \lambda_2)C(0, \lambda_3) = -1 \text{-----(4)}$$

Note that now the four A's, B's, and C's are the same, as they are determined completely by  $\theta$  and the hidden variable, and are independent of the B and C measurements (this is exactly the EPR assumption which we are attempting to refute).

Now, multiplying equations (1) - (4) we get +1 on the left side since each quantity which is  $\pm 1$  appears exactly twice, but on the right side we get -1! Therefore we can conclude that no hidden variable theory can be consistent with the predictions of quantum mechanics in this case, and this completes the proof of quantum theory being inconsistent with local realism.

During the last decade there has been rapid progress in the theoretical study of quantum technologies based on these non-local correlations caused by entanglement. However, the experimental realizations of these proposals have not kept up with this pace due to the difficulties in handling fragile quantum systems. The first of the various schemes implemented was quantum cryptography, a method by which the security of the communication is based not on the supposed computational difficulty of certain problems (such as the factorization of large prime numbers, as in RSA cryptography), but rather on fundamental quantum principles. This is good news since the above difficult computations would be easy if quantum computers were realized, which would lead to a retroactive security break with catastrophic consequences! (Lo et al., 1998). The first such scheme was proposed in 1984 by Bennet and Brassard for quantum key distribution and is known as BB84 (Lo et al., 1998). BB84 uses transmitted photons but does not employ any entanglement. Later, in 1991 Eckert proposed another scheme which does employ entanglement and in fact can be implemented on any experimental setup used to test Bell's inequalities (Eckert, 1991). A simplified version of the Eckert protocol is as follows: Alice and Bob, the universal sender and receiver in information theory, need to establish a secret key which can subsequently be used for a one-time

pad communication between them (Lo et al., 1998). They each have an analyzer that can measure polarization and a source that can emit entangled photons in the state

$$1/\sqrt{2}(|H\rangle|H\rangle + |V\rangle|V\rangle)$$

sending one photon of each pair to Alice and the other to Bob. Alice and Bob measure polarizations of their photons randomly in the H-V or 45-135 basis. After measurement they publicly announce what basis was used for each measurement. They can then analyze those measurements where they each used a different basis and make a Bell inequality measurement on the data. If an eavesdropper (Eve) were present disturbing the photons, it would be equivalent to introducing an element of physical reality to the measurements and hence Bell inequalities will be seen to hold and the eavesdropper's presence will be detected (Eckert, 1991). On the other hand, if Bell's inequality is seen to be violated then Alice and Bob can be sure that the results of the measurements in which they used the same basis must be exactly correlated and they can each use this to independently create a common secret key between them. This method is superior over BB84 in that it solves the key storage problem. Alice and Bob can hold on to their photons until they need to use a key, and only then start making measurements. In BB84, Alice has to note down the polarizations of the photons that she transmits, and Eve can always break into Alice's lab and steal this information. With the knowledge of the basis used (which was announced publicly), Eve could derive the key for herself. With this key, Eve could perform more sophisticated attacks. However, it has been shown that, together with privacy amplification schemes, the protocol is indeed secure against all such attacks (Lo et al., 1998; Naik et al., 1998).

Another clever use of entanglement is a scheme devised in 1992 by C.H Bennett and Steven Wiesner known as quantum superdense coding (Lo et al., 1998). In dense coding, Alice can send 2 bits of information to Bob by transmitting only one qbit (two-state quantum system) provided that they share an entangled state. Suppose the entangled state they share is the EPR (singlet) state

$$|\psi^-\rangle = 1/\sqrt{2}(|0\rangle|1\rangle - |1\rangle|0\rangle)$$

Now, Alice can transform this state into any one of the 4 Bell bases,

$$00 \quad |\psi^-\rangle = 1/\sqrt{2}(|0\rangle|1\rangle - |1\rangle|0\rangle) \text{--- (iii)}$$

$$01 \quad |\psi^+\rangle = 1/\sqrt{2}(|0\rangle|1\rangle + |1\rangle|0\rangle) \text{--- (iv)}$$

$$10 \quad |\psi^-\rangle = 1/\sqrt{2}(|0\rangle|0\rangle - |1\rangle|1\rangle) \text{--- (v)}$$

$$|11\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad (vi)$$

by performing the following appropriate unitary transformations locally to her particle.

$$U_{00} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad U_{10} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$U_{01} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad U_{11} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Now to send any two bits she simply performs the appropriate unitary transformation to her particle and transmits it to Bob. Upon receiving it, Bob can perform a Bell measurement on the collective state of the two particles and discern which Bell basis it is in, thereby gaining knowledge of the 2 bits sent by Alice.

Finally I will discuss quantum teleportation, an important ingredient in quantum information tasks. It is, as Samuel Braunstein puts it, "the disembodied transport" of the quantum state of a system from one site to another (Caves, 1998). Alice has a particle (C) in the state

$$\frac{1}{\sqrt{2}}(a|0\rangle + b|1\rangle)$$

which she needs to teleport to Bob (i.e. send to Bob without actually sending the particle). She cannot do this by classically transmitting all the information needed to specify the state due to the inaccessibility of quantum information (Fuchs, 1996). Performing a measurement to determine the state will destroy the state. If the particle was prepared in a specified state so that Alice knows the state, classically communicating this information would not be very efficient. Once again, entanglement comes to the rescue. Suppose Alice and Bob share a pair of maximally entangled qubits B and C (a state that can be obtained from the singlet state by local unitary transformations). They can then by definition turn this into a singlet state by local operations. Now the overall state of the 3 qubits is,

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|\psi^+\rangle_{AB}(|0\rangle_C + |1\rangle_C) + |\psi^-\rangle_{AB}(|0\rangle_C - |1\rangle_C))$$

By a simple algebraic rearrangement this can be written in terms of the Bell states of B and C, equations (vii) - (vi) as,

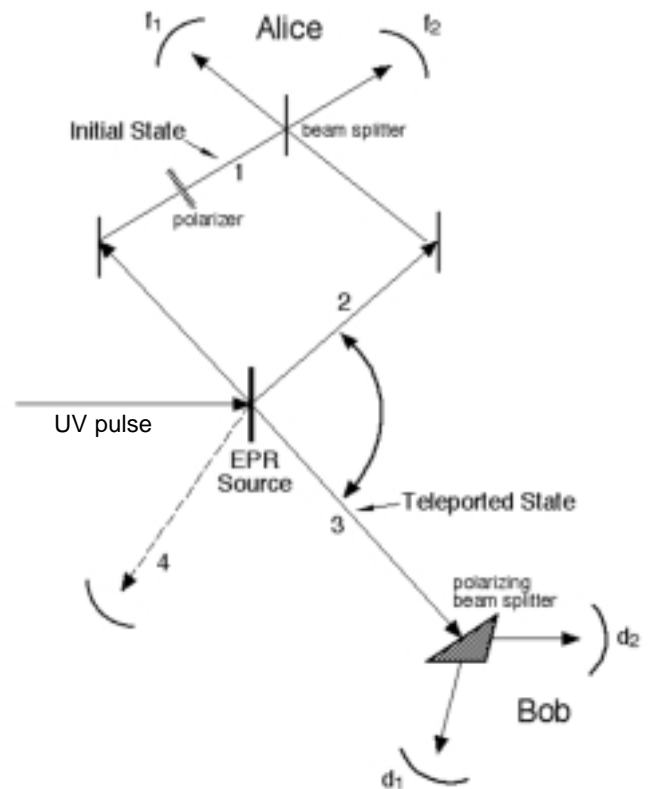
$$|\psi\rangle_{ABC} = \frac{1}{2} [ |\psi^+\rangle_{BC} (a|0\rangle_C + b|1\rangle_C) + |\psi^-\rangle_{BC} (a|0\rangle_C - b|1\rangle_C) + |\psi^0\rangle_{BC} (b|0\rangle_C + a|1\rangle_C) + |\psi^1\rangle_{BC} (b|0\rangle_C - a|1\rangle_C) ]$$

Now Alice performs a Bell measurement on her two particles. Since each outcome occurs with an equal probability of 25% she gains no information about the state of her particle. However, it is seen that in each case the state of Bob's particle is related to  $|\psi\rangle_C$  by a fixed unitary transformation  $(-1)^{ij} U_{ij}$  regardless of the identity of  $|\psi\rangle_C$ . (The  $U_{ij}$ s are the same as in dense coding, and so is the

assignment of two bits to each Bell basis). Therefore all Alice has to do is classically transmit these two bits (the results of her Bell measurement) to Bob who by performing  $(-1)^{ij} U_{ij}$  on his particle obtains the exact state of Alice's qbit! It is seen that with this protocol the original state of the particle C is destroyed. This will be true for any teleportation protocol and follows from the general "no-cloning" theorem (a consequence of the linearity of quantum mechanics and also derives from the inaccessibility of quantum information) (Wooters & Zurek, 1982). Another feature of this protocol is that the initial entanglement in the quantum channel has been destroyed. This again is a general result for any conceivable teleportation protocol and follows from the fundamental law of quantum information processing, which will be discussed shortly.

First, I will briefly sketch the details of the first experimental realization of the above protocol for the polarization state of a photon by Bouwmeester et al. at the University of Innsbruck, Austria (Hardy, 1998). It is represented diagrammatically below,

A UV pulse is sent through a non-linear crystal which generates maximally entangled photons



tons (2) and (3). After retroreflection during its second passage through the crystal, the UV pulse creates another pair of photons (1), which is sent through a polarizer and thus prepared in an arbitrary state for teleportation. The other photon (4) serves as a trigger indicating that a photon to be teleported is under way. To make a Bell measurement on (1) and (2) they are incident one on each side at a beam splitter. They then use the fact that it is only for the antisymmetric Bell state  $|\psi\rangle$  that either both photons either reflect or transmit. Therefore, simultaneous clicks at f1 and f2 record a Bell measurement on the antisymmetric state and this occurs 25% of the time. In these cases it is seen by (\*) that photon (3) would be in the initial state of (1) and so teleportation would have occurred. To check if this indeed does occur, photon (1) is sent through a polarizer at  $45^\circ$  which is equivalent to it being prepared in the state

$$2^{-1/2} (|H\rangle + |V\rangle)$$

and (3) is sent through a polarizing beam splitter which distinguishes polarizations of  $45^\circ$  (detection at d2) and  $-45^\circ$  (detection at d1). Therefore, if teleportation is successful in the cases where f1 and f2 click simultaneously, then d2 should also click and d1 shouldn't. This has been observed. There are many technical details in the experiment that I have left out and one of them is that photons 1 and 2 are initially sent through a narrow bandwidth filter which serves to pinpoint the frequency (energy) of the photons and hence smear them out in time. The uncertainty in time is arranged to be larger than the pulse time so that the timing of the photons cannot be used even in principle to distinguish between the photons (Berglund, personal communication).

The key ingredient in the teleportation protocol is the maximally entangled qbit shared by Alice and Bob. How do Alice and Bob obtain such an entangled qbit? They cannot, with no matter how small a probability, by local operations or classical communication, turn a disentangled state that they share into an entangled one. This is the fundamental law of quantum information processing (Plenio & Vedral, 1998). Therefore either Alice or Bob will have to create an entangled pair and send one of the entangled particles over to the other. However in doing so the singlet state will be disturbed and Alice and Bob will end up sharing some mixed state that is no longer maximally entangled. However Alice and Bob do not have to lose hope since it has been shown that there are

“purification protocols” which can be used to “concentrate” the entanglement, that is, create a lesser number of maximally entangled pairs from a large number of weakly entangled pairs (Plenio, M., Vedral, V., 1998).

Creating and maintaining entangled qbits is difficult, and so it would be nice if one maximally entangled pair could be used to teleport a large number of qbits. However, as Plenio and Vedral put it, “There is no free lunch,” and it can be shown that to teleport  $n$  qbits Alice and Bob need to share  $n$  maximally entangled pairs. The reasoning is as follows. If one can teleport an unknown pure state then one can obviously teleport an unknown mixed quantum state as well. Now, as was proved earlier, any mixed state of a single qbit can be thought of as part of a pure state of two entangled qbits. Now, if this qbit is teleported to Bob, and if the initial entanglement channel is not destroyed, Alice and Bob will end up sharing an additional entangled pair. This is because the teleported mixed state was part of an entangled pair, its partner held by Alice. Therefore once this state is teleported to Bob, its partner is still with Alice and so they get to share another entangled state. However, a stronger version of the fundamental law states that, by local operations and classical communications alone, Alice and Bob cannot increase the total amount of entanglement they share (Plenio & Vedral, 1998). For this law to make sense there must be a measure of entanglement so that it can be known that entanglement has increased (Plenio & Vedral, 1998). However any reasonable measure should pronounce that entanglement has increased in the above scheme, and so it is impossible.

As Michael Berry discusses (Lo et al., 1998), civilizations are transformed by technology which is in turn driven by science. In the nineteenth century, life was transformed by the conscious application of classical mechanics and thermodynamics to the engines of the industrial revolution. In the last century, it was electromagnetism that revolutionized our lives by enabling the generation and distribution of electric power and the communication of words and pictures across the world at the speed of light. He predicts that in the 21<sup>st</sup> century it will be quantum mechanics that will dramatically influence our lives with the advent of these new quantum technologies which have the deliberate manipulation of entangled states at the heart of their operations. With very recent advances in experimental techniques to create and transport entanglement, Seth Lloyd of MIT predicts

that entanglement will in the future become a very valuable commodity, more valuable perhaps than even gold or silver (Mullins, 2000). He has plans to build a "quantum internet" that could be used for creating, storing and distributing entanglement. In the future, anyone needing entanglement for their quantum computations can simply download it from this network. It can also be used to connect quantum computers in parallel and enable the calculation of massive computations. In fact, connecting many of the already existing simple 7-qbit quantum computers at Los Alamos in parallel would allow useful computations to be carried out. The network could also be used to send quantum information, teleport atoms and maybe eventually larger systems (Mullins, 2000). All this sounds like a "kooky" (as Feynmann would have called it) idea. The question is, is it "kooky" enough to be inevitable? ❖

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### Aside:

In this paper I failed to talk about quantum computing, one of the most exciting of the emerging quantum technologies. For a good introduction see Andy Berglund '00's article in DUIS Vol. 1, Spring 1999 titled "Quantum Computing," or Gerald Milburn's very accessible book *The Feynman Processor* which also contains an introduction to the lay reader on quantum mechanics, entanglement and teleportation.

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Anura Abeyesinghe '01 is a physics and mathematics major with a primary interest in theoretical physics. He is currently working on general relativity with Prof. Marcelo Gleiser for his senior honors thesis. After Dartmouth, Anura plans to go on to graduate study.