Order-of-Magnitude Estimation
Collapsing Sun (Level 3)

The Question

If the Sun wasn't supported by internal pressure, how long would it take to completely collapse?

Background

The Sun is stable because it is in a state of balance known as "hydrostatic equilibrium". Pressure is created from internal nuclear reactions as the Sun fuses hydrogen into helium to create energy. This outwards pressure perfectly balances the inwards force of gravity—hence the equilibrium. If the nuclear fusion reactions in the Sun "turned off" then there would be nothing to balance the Sun's gravitational pull on itself and the Sun would collapse. This is actually a realistic scenario. In about 5 billion years' time the Sun will have exhausted the hydrogen in its core and will begin to contract. Though the Sun will never collapse completely, this problem yields a reasonable estimate of the collapse timescale.

This problem can be reduced to a "free fall" problem. The conditions for "free fall" are that an object falls under constant acceleration caused by the gravity due to a massive body. Constant acceleration is typically a reasonable assumption when the total amount of mass that is interior to the freely falling body does not change. Such conditions would hold in this case, as the total mass of the Sun is kept interior to the surface of the Sun as it collapses. It may also be useful to read the additional background provided in the "Falling Ball" question.

Guiding Questions

Here are some things you may need to consider:

- Always have an initial guess at the answer without any OoM estimation!
- How can Newton's laws and the free fall equations be combined to derive time as a function of distance?
- How is this problem the same as or different to the falling ball question? What relevant values will change or stay the same?
- In the falling ball question we determined that the acceleration due to the Earth's gravity is $9.8\,\mathrm{m\,s^{-2}}$. Is there an equivalent value for the Sun?

The Solution

In the falling ball question we determined g, the acceleration due to the Earth's gravity. We can derive a similar quantity for the Sun (i.e. the "acceleration due to the Sun's gravity"). We can do so by equating Newton's Second Law ($F = ma = ma_\odot$; where $a_\odot$ is the acceleration due to the Sun's gravity) to the force of gravity from the Sun:

$$F = ma_\odot = \frac{GMm}{R^2} \Rightarrow a_\odot = \frac{GM}{R^2} \tag{1}$$

where R and M are the radius and the mass of the Sun, respectively.

The radius of the Sun is 4.3×10^5 miles or $\sim 7 \times 10^8$ m. We can then determine:

$$a_\odot = \frac{GM}{R^2} = \frac{(6.7 \times 10^{-11})(2 \times 10^{30})}{(7 \times 10^8)^2} \sim \frac{1.3 \times 10^{20}}{4.9 \times 10^{17}} \sim \frac{1}{4} \times 10^3 \sim 0.25 \times 10^3 \sim 250\,\mathrm{m\,s^{-2}} \tag{2}$$

Note that this acceleration is $\sim$25 times larger than that for objects near the Earth.

Now, we know the value of the free-fall acceleration due to the Sun's gravity, let's determine the time taken for collapse, which is the same as the time taken for a point on the surface of the Sun to free-fall to the Sun's center—i.e. the time taken for an object to free-fall a distance corresponding to the Sun's radius:

$$r = \frac{1}{2}at^2 = \frac{1}{2}a_\odot t^2 \Rightarrow t = \sqrt{\frac{2r}{a_\odot}} = \sqrt{\frac{2R}{a_\odot}} \tag{3}$$

We already know that the radius of the Sun is $\sim 7 \times 10^8$ m. All that is left to do is to plug in the numbers:

$$t = \sqrt{\frac{2R}{a_\odot}} = \sqrt{\frac{2(7 \times 10^8)}{2.5 \times 10^2}} \sim \sqrt{6 \times 10^6} \sim \sqrt{6} \times \sqrt{10^6} \sim 2.5 \times 10^3 \sim 2500\,\mathrm{s} \sim 40\,\text{minutes} \tag{4}$$

The Sun would take about half-an-hour to collapse under the influence of its own gravity.

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS):

- Physical Sciences
  - Matter & Its Interactions
  - **Motion and Stability: Forces and Interactions**
  - Energy
  - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
  - From Molecules to Organisms: Structures and Processes
  - **Ecosystems: Interactions, Energy, and Dynamics**
  - Heredity: Inheritance and Variation of Traits
  - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
  - **Earth's Place in the Universe**
  - Earth's Systems
  - Earth and Human Activity
- Engineering, Technology, and Applications of Science
  - **Engineering Design**

Common Core Standards (CSS):

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- **Numbers & Operations in Base Ten**
- **Number & Operations — Fractions**
- Measurement & Data
- **Geometry**
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- Statistics & Probability