Order-of-Magnitude Estimation
Earth Orbiter (Level 1)

The Question

A geostationary satellite orbits the Earth once each day. About how fast is it traveling in miles per hour?

Background

Geostationary satellites are designed to remain above the same point on the Earth's surface, orbiting the Earth exactly once per day. It is possible to expand this question to an expert-level question by determining the exact distance to a satellite with an orbital period of 24 hours. Or, as an alternative starting point, most people can be guided to estimate that a satellite orbits at "a few times the Earth's radius".

Guiding Questions

Here are some things you may need to consider:

- Always have an initial guess at the answer without any OoM estimation!
- Newton's Law of Gravity is $F = GMm/r^2$ for a body of mass (M) exerting a gravitational force on a second mass m at a distance r. G is the Gravitational Constant.
- Newton's Second Law in angular form is $F = ma = mr\omega^2 = mr(4\pi^2)/T^2$ for a mass m moving in a circle of radius r that takes a period T to orbit once.
- How can we estimate the distance to geostationary orbit?
- How can we estimate the circumference of geostationary orbit?
- How are speed and distance related to velocity?

The Solution

To determine the distance r to geostationary orbit (from the center of the Earth), equate Newton's Second Law and Newton's Law of Gravity

$$F = \frac{GMm}{r^2} = \frac{mr(4\pi^2)}{T^2} \Rightarrow r = \sqrt[3]{\frac{GM}{4\pi^2}T^2} \tag{1}$$

remembering that the time for one orbit of a geostationary satellite is 24 hours or $24 \times 60 \times 60\,\mathrm{s}$, which is about $(100/4) \times 36 \times 100 \sim 10^5$ and that $\pi^2 \sim 10$:

$$\begin{aligned} r &= \sqrt[3]{\frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{40}(10^5)^2} \sim \sqrt[3]{\frac{(40 \times 10^{13})}{40}10^{10}} \\ &\sim \sqrt[3]{10^{23}} \sim \sqrt[3]{10^{21}(100)} \sim \sqrt[3]{100}\sqrt[3]{10^{21}} \sim 4.5 \times 10^7\,\mathrm{m} \end{aligned} \tag{2}$$

Thus we have calculated that geocentric orbit is $\sim$ 45,000 km or $\sim$ 25,000 miles. The true answers are 35,786 km or 22,236 miles.

An alternate path that glosses over this more expert calculation is to use the determination of the Earth's circumference from the "Earth Runner" OoM problem to derive the Earth's radius: *There are 4 time zones in the U.S. and the U.S. is about 4000 miles across, so there are about 1000 miles per time zone. There are 24 time zones around the Earth, so the Earth's circumference is about 24,000 miles. The radius of the Earth is therefore 24000/*$2\pi \sim 4000$ *miles or* ~ 6400 *km, which is* 6.4×10^6 *m.* And then to guide the room towards the idea that typical satellites orbit at "a few times" the Earth's radius.

Now that we know that the distance to geostationary orbit is $\sim$ 25,000 miles we can derive the circumference of geostationary orbit as $\sim 25{,}000\,\text{miles} \times 2\pi \sim 150{,}000\,\text{miles}$. A geostationary satellite travels this distance in 24 hours, meaning that it travels at a speed of about:

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{150{,}000\,\text{miles}}{24\,\text{hours}} \sim \frac{15}{2.4}\frac{10^4\,\text{miles}}{10\,\text{hours}} \sim 6 \times 10^3\,\text{miles per hour} \tag{3}$$

A geostationary satellite travels at about 6000 mph. This is a speed about 600 times faster than the runner in the "Earth Runner" problem.

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS):

- Physical Sciences
  - Matter & Its Interactions
  - **Motion and Stability: Forces and Interactions**
  - Energy
  - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
  - From Molecules to Organisms: Structures and Processes
  - **Ecosystems: Interactions, Energy, and Dynamics**
  - Heredity: Inheritance and Variation of Traits
  - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
  - Earth's Place in the Universe
  - **Earth's Systems**
  - **Earth and Human Activity**
- Engineering, Technology, and Applications of Science
  - **Engineering Design**

Common Core Standards (CSS):

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- **Numbers & Operations in Base Ten**
- **Number & Operations — Fractions**
- Measurement & Data
- **Geometry**
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- Statistics & Probability