

Order-of-Magnitude Estimation

Earth Runner (Level 1)

The Question

How long would it take to run around the world?

Background

This is a straightforward OoM question that requires very little background knowledge beyond the fact that the time taken to travel some distance is that distance divided by the speed of travel.

Guiding Questions

Here are some things you may need to consider:

- Always have an initial guess at the answer without any OoM estimation!
- How can we estimate the circumference of the Earth (even without knowing its radius)?
- How can we estimate the running pace of a human being?

The Solution

Rearranging the equation that $speed = distance/time$ (e.g. “miles per hour”) we obtain the relationship $time = distance/speed$. To determine the time taken to run around the Earth, then, we need to know the distance that corresponds to the circumference of the Earth and the typical speed at which a human can run.

How can we determine the circumference of the Earth? There are 4 time zones in the U.S. and the U.S. is about 4000 miles across, so there are about 1000 miles per time zone. There are 24 time zones around the Earth, so the Earth’s circumference is about 24,000 miles.

About how fast does a human being run? This can be estimated in a number of ways, but let’s take the example of a marathon runner. Marathons are about 25 miles long and a decent runner can complete one in several hours. Taking 25 miles in 2.5 hours, at least some humans can run at $\sim 10 \text{ miles hour}^{-1}$.

All that is left to do is to plug in the numbers:

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{24,000 \text{ miles}}{10 \text{ miles hour}^{-1}} = 2400 \text{ hours} \quad (1)$$

Note that as these values aren't in standard "SI" units of meters and seconds, we left the units explicitly in the equation to guard against scaling mistakes. There are 24 hours in a day, so 2400 hours is about 100 days or about 3.5 months. A reasonably fast runner could circumnavigate the Earth in about 3-4 months, if they ran constantly!

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS):

- Physical Sciences
 - Matter & Its Interactions
 - **Motion and Stability: Forces and Interactions**
 - Energy
 - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
 - From Molecules to Organisms: Structures and Processes
 - **Ecosystems: Interactions, Energy, and Dynamics**
 - Heredity: Inheritance and Variation of Traits
 - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
 - **Earth's Place in the Universe**
 - Earth's Systems
 - Earth and Human Activity
- Engineering, Technology, and Applications of Science
 - Engineering Design

Common Core Standards (CSS):

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- Numbers & Operations in Base Ten
- **Number & Operations — Fractions**
- Measurement & Data
- Geometry
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- Statistics & Probability