Order-of-Magnitude Estimation
Colonizing the Galaxy (Level 3)

The Question

How long will it take an intelligent civilization to colonize the Galaxy?

Background

Our Galaxy is a disk made up of a few hundred billion stars, many of them like our Sun (roughly a "typical" star, but not quite). Many, if not most of these, have planetary systems of their own that might be colonized (or already host life!). If an intelligent species develops the means to travel at 10% of the speed of light, how long will it take to colonize systems on the other side of the Galactic disk?

We won't do it here, but one can prove that the "root-mean-squared" distance traveled when taking random steps is proportional to the step size and the square-root of the number of steps. Mathematically, $d_{\text{tot}} \sim d_{\text{step}}\sqrt{N}$.

Guiding Questions

Here are some things you may need to consider:

- How many stars are in the Galaxy?
- What fraction are Sun-like with planets?
- How far apart are planetary systems?
- How long does it take to colonize a system and travel to the next?

The Solution

Assuming that there are 200 billion stars in the Galaxy, and 10% of them are Sun-like, and 10% of these have planets, we have

$$N_{\text{stars}} = 2 \times 10^{11} \times 0.1 \times 0.1 = 2 \times 10^{9} \text{ stars} \tag{1}$$

The volume of the Milky way is $\pi R_{\text{MW}}^2 h$, where h is the thickness of the disk. Thus the volume is:

$$V = \pi(1 \times 10^5)^2 \times 1 \times 10^4 = 3 \times 10^{14} \text{ ly}^3 \tag{2}$$

Therefore the number density of Sun-like stars with planets is:

$$n = \frac{2 \times 10^9}{1 \times 10^{14}} = 2 \times 10^{-5} \text{ stars/ly}^3 \tag{3}$$

The typical distance between stars is proportional to the cubed-root of the number density:

$$d_{\text{mean}} = \frac{1}{n^{(1/3)}} = \frac{1}{(2 \times 10^{-5})^{(1/3)}} = 40 \text{ ly} \tag{4}$$

Traveling at the speed of light, it would take 40 years between each system. Assuming the civilization only travels at 10% of the speed of light, it takes 400 years.

Under the assumption that the nearest planetary system is in a roughly random direction from each stop, the number of stops it takes to make it a linear distance d is proportional to $(d/d_{\text{mean}})^2$. This means that to reach the other side of the galaxy, a civilization will colonize:

$$N = \frac{1 \times 10^5}{40} = 5 \times 10^4 \tag{5}$$

With travel time and additional 600 years of preparation to move on the the next system, each colonization takes 1000 years. Therefore the time it takes to have bases set up across the Galaxy is:

$$t_{\text{tot}} = N \times t_{\text{step}} = 5 \times 10^4 \times 1000 = 5 \times 10^7 \text{ yr} \tag{6}$$

This is quite short on astronomical timescales!

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS):

- Physical Sciences
  - **Matter & Its Interactions**
  - Motion and Stability: Forces and Interactions
  - **Energy**
  - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
  - From Molecules to Organisms: Structures and Processes
  - **Ecosystems: Interactions, Energy, and Dynamics**
  - Heredity: Inheritance and Variation of Traits
  - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
  - **Earth's Place in the Universe**
  - Earth's Systems
  - Earth and Human Activity
- Engineering, Technology, and Applications of Science
  - Engineering Design

Common Core Standards (CSS):

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- **Numbers & Operations in Base Ten**
- **Number & Operations — Fractions**
- Measurement & Data
- **Geometry**
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- **Statistics & Probability**