Order-of-Magnitude Estimation
Solar Escape (Level 3)

The Question

How long does it take a photon generated in the core of the Sun to escape the surface?

Background

The Sun's energy is generated via nuclear fusion (smashing Hydrogen atoms together to make Helium). The energy is released as light, which we can treat as packets called photons. However, these photons can't stream directly out of the Sun's core, because they are scattered by the protons and electrons that make up the Sun's mass. If a photon is scattered in a random direction each time it runs into a proton, how long does it take to make it to the surface of the Sun where it can stream freely into space?

Guiding Questions

Here are some things you may need to consider:

- How far is it from the Sun's core to the surface?
- How many particles are there to interact with photons?
- How fast does a photon move?
- What is the relationship between (linear) distance traveled, step size, and number of steps in a "random walk"?

The Solution

First we need to determine the density of particles that the photon can interact with, on average (note that the density is not constant throughout the sun). The mass of the Sun is 2×10^{30} kg and the mass of a proton is 2×10^{-27} kg (protons are far more massive than electrons, so we'll ignore the latter). So:

$$N_{\mathrm{p}} = \frac{2 \times 10^{30}}{2 \times 10^{-27}} = 10^{57} \text{ protons} \tag{1}$$

The volume of the Sun is $(4/3)\pi R_{\text{Sun}}^3 = 10^{33}$ cm^3. Therefore the number density of protons is:

$$n_p = \frac{10^{57}}{10^{33}} = 10^{24} \text{ protons/cm}^3 \tag{2}$$

Now, photons don't scatter off *every* proton they encounter. Maybe it's more like one in every 10,000 or so. So the *effective* number density is more like $n_p = 10^{20}$ protons/cm^3. You can neglect this step if you like, you'll just get a larger answer at the end.

The typical distance between interactions, or the mean free path (mfp) of a photon is proportional to the cubed-root of the number density of the thing it's interacting with. Therefore:

$$d_{mfp} = \frac{1}{n_p^{(1/3)}} = \frac{1}{10^6} = 10^{-6} \text{ cm} \tag{3}$$

The number of steps it takes to cover a distance d (the radius of the Sun in our case) in a random walk is approximately the square of the distance over the mean free path:

$$N_{\text{steps}} = \left(\frac{d}{d_{mfp}}\right)^2 = \left(\frac{7 \times 10^{10}}{10^{-6}}\right)^2 = 10^{32} \text{ steps} \tag{4}$$

The time it takes to complete a step depends on how fast the photon moves, which is always the speed of light $c = 3 \times 10^{10}$ cm/s:

$$t_{\text{step}} = \frac{d_{\text{step}}}{c} = \frac{10^{-6}}{3 \times 10^{10}} = 10^{-16} \text{ s} \tag{5}$$

Finally, the total time to escape the sun is the number of steps multiplied by the time it takes to make each step:

$$t_{\text{tot}} = N_{\text{steps}} \times t_{\text{step}} = 10^{32} \times 10^{-16} = 10^{16} \text{ s} \tag{6}$$

This is a big number, so converting to years gives approximately:

$$t_{\text{tot}} = 10^8 \text{ years} \tag{7}$$

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS):

- Physical Sciences
  - **Matter & Its Interactions**
  - Motion and Stability: Forces and Interactions
  - **Energy**
  - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
  - From Molecules to Organisms: Structures and Processes
  - **Ecosystems: Interactions, Energy, and Dynamics**
  - Heredity: Inheritance and Variation of Traits
  - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
  - **Earth's Place in the Universe**
  - Earth's Systems
  - Earth and Human Activity
- Engineering, Technology, and Applications of Science
  - Engineering Design

Common Core Standards (CSS):

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- **Numbers & Operations in Base Ten**
- **Number & Operations — Fractions**
- Measurement & Data
- **Geometry**
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- **Statistics & Probability**