Estimation Workshop Cheat Sheet

1 Scientific Notation

Describing the Universe requires some very big (and some very small) numbers. Such numbers are tedious to write in long decimal notation, so we use *scientific notation*. Scientific notation is written as a power of 10 in the form:

$$m \times 10^e$$

where m is the *mantissa* and e is the *exponent*. The mantissa is a decimal number between 1.0 and 9.999 and the exponent is an integer. To write numbers in scientific notation, move the decimal until only one digit appears to the left of the decimal. Count the number of places the decimal was moved and place that number in the exponent. For example,

$$540,000. = 5.4 \times 10^5$$

$$314.15 = 3.1415 \times 10^2$$

$$234.5 \times 10^2 = 2.345 \times 10^4$$

If you move the decimal to the right instead of left, the exponent gets a negative sign:

$$0.00042 = 4.2 \times 10^{-4}$$

Arithmetic in Scientific Notation

To multiply numbers in scientific notation, first *multiply* the mantissas and then *add* the exponents:

$$2.5 \times 10^6 \ \times \ 2.0 \times 10^4 = (2.5 \times 2.0) \ \times 10^{6+4} = 5.0 \times 10^{10}.$$

To divide, *divide* the mantissas and then *subtract* the exponents:

$$\frac{6.4 \times 10^5}{3.2 \times 10^2} = (6.4/3.2) \ \times 10^{5-2} = 2.0 \times 10^3.$$

This is even more straightforward for *order-of-magnitude* estimation, because you only have to deal with the exponents. For example, what is ten million times one billion?

$$\text{ten million} = 10,000,000 = 1 \times 10^7 = 10^7$$

$$\text{one billion} = 1,000,000,000 = 1 \times 10^9 = 10^9$$

$$10^7 \times 10^9 = 10^{7+9} = 10^{16}$$

Be careful if you need to add or subtract numbers in scientific notation:

$$4.0 \times 10^6 \ + \ 2.0 \times 10^5$$

$$\begin{array}{rcl} 4.0 \times 10^6 & = & 4{,}000{,}000 \\ + \quad 2.0 \times 10^5 & = & + \quad 200{,}000 \\ \hline 4.2 \times 10^6 & = & 4{,}200{,}000 \end{array}$$

2 Converting Units

Converting between units generally just involves setting up an equation so that the units you want to get rid of are canceled by division, and you are left with the units you want. For example, how many kilometers is one hundred million (10^8) centimeters? We know that there are 100 centimeters in a meter, and 1000 meters in a kilometer.

$$10^8 \cancel{\text{cm}} \times \frac{1 \cancel{\text{m}}}{100 \cancel{\text{cm}}} \times \frac{1 \text{ km}}{1000 \cancel{\text{m}}} = 10^{8-2-3} \text{ km} = 1000 \text{ km}$$

Always make sure your answer makes qualitative sense! A kilometer is much larger than a centimeter, so a given distance should be smaller when measured in kilometers. Our result (1000) is smaller than what we started with (10^8), so our answer checks out.

3 Areas and Volumes

Circumference of a circle: $2\pi r$

Area of a circle: πr^2

Surface area of a sphere: $4\pi r^2$

Volume of a sphere: $\frac{4}{3}\pi r^3$

Area of a square/rectangle: $l \times w$

Volume of a cube: $l \times w \times h$

Volume of a cylinder: $\pi r^2 \times h$

4 Metric Prefixes

Prefix	abbreviation	factor
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

5 Some constants and conversions

(Note that some of these are approximations)

$$\text{Speed of light} = c = 3 \times 10^8 \text{ m/s}$$

$$\text{Astronomical unit (mean Earth-Sun distance)} = \text{AU} = 1.5 \times 10^8 \text{ km}$$

$$\text{Light year} = \text{ly} = 9.5 \times 10^{15} \text{ m}$$

$$\text{Mass of a proton} = m_p = 1.7 \times 10^{-27} \text{ kg}$$

$$\text{Radius of Milky Way Galaxy} = 1 \times 10^5 \text{ ly}$$

$$\text{Thickness of Milky Way Galaxy} = 1 \times 10^4 \text{ ly}$$

$$\text{Mass of Sun} = 2 \times 10^{30} \text{ kg}$$

$$\text{Radius of Sun} = 7 \times 10^8 \text{ m}$$

$$\text{Mass of Earth} = 6 \times 10^{24} \text{ kg}$$

$$\text{Radius of Earth} = 6,400 \text{ km}$$

$$\text{Gravitational Constant} = G = 6.7 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

$$\text{Graviational acceleration on Earth} = g = 9.8 \text{ m/s}^2$$

$$1 \text{ m} = 3.3 \text{ feet}$$

$$1 \text{ mile} = 1.6 \text{ kilometer}$$