Order-of-Magnitude Estimation
Falling Ball (Level 3)

The Question

How long does it take a pool ball dropped from a plane to hit the surface of the Earth?

Background

Many problems in physics can be reduced to "free fall" problems. The conditions for "free fall" are that an object falls under constant acceleration caused by the gravity due to a massive body. Constant acceleration is typically a reasonable assumption when the total amount of mass that is interior to the freely falling body does not change. Such conditions are typical for objects falling near the surface of the Earth.

Formally, the equations for acceleration under free fall conditions can be modeled by integrating the acceleration (a) and velocity (v) with respect to time (t) to determine how position (y) depends on acceleration. Typically, $v = 0$ at $t = 0$ (something is "dropped") and $y = 0$ at $t = 0$ (the reference frame is set up so that the initial position from which something is dropped is 0):

$$v = \int_0^t a\mathrm{d}t = at \quad , \quad y = \int_0^t v\mathrm{d}t = \int_0^t at\mathrm{d}t = \frac{1}{2}at^2 \tag{1}$$

A simplified, algebraic version of this derivation that is sufficient for OoM estimation is: speed = distance/time and acceleration = speed/time such that $a = v/t = (y/t)/t = y/t^2$.

The acceleration is the *effect* that gravity has on a falling object. The *cause* is the mass (M) of the body (of radius R) that the object (of mass m) is falling towards, which can be derived from Newton's Second Law and Newton's Law of Gravity:

$$F = ma = \frac{GMm}{R^2} \Rightarrow a = \frac{GM}{R^2} \tag{2}$$

where G is the Gravitational Constant.

The Solution

Education Standards

This OoM Estimation problems meets the following standards in **bold**:
Next Generation Science Standards (NGSS)*:*

- Physical Sciences
  - Matter & Its Interactions
  - **Motion and Stability: Forces and Interactions**
  - Energy
  - Waves and Their Applications in Technologies for Information Transfer
- Life Sciences
  - From Molecules to Organisms: Structures and Processes
  - **Ecosystems: Interactions, Energy, and Dynamics**
  - Heredity: Inheritance and Variation of Traits
  - Biological Evolution: Unity and Diversity
- Earth and Space Sciences
  - **Earth's Place in the Universe**
  - Earth's Systems
  - Earth and Human Activity
- Engineering, Technology, and Applications of Science
  - **Engineering Design**

Common Core Standards (CSS)*:*

- **Counting & Cardinality**
- **Operations & Algebraic Thinking**
- **Numbers & Operations in Base Ten**
- **Number & Operations — Fractions**
- Measurement & Data
- **Geometry**
- **Ratios & Proportional Relationships**
- The Number System
- **Expressions & Equations**
- Functions
- Statistics & Probability