

# Figure of Merit for Resonant Wireless Power Transfer

Aaron L.F. Stein

Phyo Aung Kyaw

Charles R. Sullivan

Thayer School of Engineering

Dartmouth College

Hanover, NH 03755 USA

Email: {Aaron.L.Stein, Phyo.A.Kyaw.TH, Charles.R.Sullivan}@dartmouth.edu

**Abstract**—Improvements in performance of resonant coils are important for the range and efficiency of wireless power transfer (WPT); however, these improvements are difficult to measure using the conventional figure-of-merit (FoM), which is the product of quality factor and coupling factor. The conventional FoM does not account for important WPT system performance parameters such as coil size and range of power transfer; furthermore, it requires information which is not commonly reported. In this paper, we propose a new FoM that is the ratio of the loss fraction of a reference system and the system-under-test. The reference system models two-loops of solid wire that are scaled to the same overall size as the system-under-test. The FoM is an indication of the performance of a WPT coil technology independent of a specific implementation, and can be calculated using commonly reported parameters: coil diameter, transmission range, and coil-to-coil efficiency. We computed the FoM for a few WPT systems in the literature, and found the highest FoM to be 3.4, achieved using a self-resonant structure.

## I. INTRODUCTION

Magnetically-coupled resonant wireless power transfer (WPT) is an increasingly popular method for both charging and powering electrical devices. It has been effectively used in many applications including biomedical, automotive, and consumer hand-held electronics [1]–[4]. The performance of a wireless power transfer system is measured by its efficiency, range, and size. These metrics are limited by the quality factor  $Q$  of the resonant coils and the magnetic coupling coefficient  $k$  between the resonant coils [3], [5].

The conventional figure-of-merit (FoM) for resonant WPT systems is  $Q \cdot k$ , which can be used to derive the maximum achievable efficiency between two coils [5], [6]. Therefore, the conventional FoM describes the efficiency of a WPT system, and can be useful in the design of a particular WPT system. For example, given a coil diameter  $d$  and wireless transmission distance  $x$ , maximizing the conventional FoM will maximize system performance. However, the conventional FoM does not consider important WPT performance metrics such as coil diameter and wireless range, and, therefore, has limited usefulness in comparing WPT systems with different coil diameters or transmission ranges.

The components of the conventional FoM ( $Q, k$ ) are significantly impacted by coil size and transmission distance. The scaling laws discussed in [7] illuminate the challenge of creating small and efficient magnetic components; including small high- $Q$  WPT coils. The relationship between  $Q$  and

coil diameter  $d$  in skin-depth-limited conductors led to a FoM proposed in [8]  $Q/d$ . This FoM incorporates the impact of coil size on the quality factor into a FoM, which allows for a more fair comparison between systems; however, it does not consider the impact of size and transmission distance on magnetic coupling. This relationship is demonstrated in [6] for two loops of wire. Other work models magnetic coupling as a function of coil size [9], which further highlights the importance of considering size and transmission distance on magnetic coupling; however, to the authors' knowledge, the impacts of these important factors on magnetic coupling have yet to be incorporated into a FoM.

We present a new FoM that accounts for the diameter and range of wireless power transfer coils. The proposed FoM is the ratio of the loss fraction of a reference system to the loss fraction of the system-under-test (SUT). The reference system is inspired by two loops of solid wire that are scaled to the same coil diameter and transmission distance as the system-under-test. The proposed FoM allows for a direct comparison between WPT systems with the same ratio of transmission range to coil diameter ( $x/d$ ) despite variations in size and transmission range. Furthermore, a system with a large FoM is likely to perform well at any  $x/d$ ; therefore, the FoM also provides general insights into a system's performance. The proposed FoM is expressed in terms of parameters which are commonly reported: coil-to-coil efficiency  $\eta_{sut}$ , coil diameter  $d$ , and air space between the coils  $x$ . This tool provides a simple method for assessing and comparing WPT coil designs.

## II. USER'S GUIDE TO THE FoM

This section provides a quick reference for calculating the FoM, an example calculation, and a brief discussion of how to interpret the results. The theoretical basis of the FoM is derived in Section III, and the reasons that the FoM can be used as a tool for comparing WPT systems of different sizes is described in Section IV. Finally, in Section V, the FoM is used to compare the performance of a few WPT systems reported in the literature.

### A. Calculating the FoM

In order to calculate the FoM of a system, three parameters of the system-under-test (SUT) are required: the diameter of the resonant coil  $d$ , the distance between the resonant coils

$x$ , and the efficiency of power transfer  $\eta_{sut}$ . The efficiency is measured from the input of the transmit coil to the output of the receive coil, and does not include circuit losses. The FoM compares the loss fraction of a reference system to the SUT, and is

$$\text{FoM} = \frac{(1 - \eta_{ref}) \eta_{sut}}{\eta_{ref} (1 - \eta_{sut})}, \quad (1)$$

where

$$\eta_{ref} = \frac{\gamma^2}{(1 + \sqrt{1 + \gamma^2})^2}, \quad (2)$$

and

$$\gamma = \frac{(3.26 \text{ mm}^{-1}) d}{1 + 10.2 \left(\frac{x}{d}\right) + 16 \left(\frac{x}{d}\right)^3}. \quad (3)$$

Consider an example system with a coil diameter  $d = 75$  mm, range  $x = 100$  mm, and WPT efficiency  $\eta_{sut} = 0.75$ . The efficiency of the reference system  $\eta_{ref}$  is calculated from (2), and is  $\eta_{ref} = 0.704$ . Substituting  $\eta_{ref}$  and  $\eta_{sut}$  into (1) yields the FoM of the example system  $\text{FoM} = 1.25$ .

### B. Interpreting the Result

The FoM provides a metric for comparing the performance of WPT systems that have different diameters, and are operated at different wireless ranges. The FoM compares the loss fraction of a reference system to the loss fraction of the system-under-test (SUT). The reference system is an equation which models two loops of solid wire that are scaled to the same outer diameter and range as the SUT. The scaled system provides a standard for comparison that accounts for the physical dimensions of the SUT. This allows the FoM to be a metric of resonant coil performance that is independent of the dimensions a particular implementation. Therefore, a system that earns a large FoM is an indication of good technology that should be considered for other WPT implementations.

For the reader's reference, we calculate the FoM of a few systems in Section V. A general rule-of-thumb based on the current state-of-the-art is that a system with  $\text{FoM} > 0.9$  is a high-quality system. To the authors' knowledge, the highest performing system earned a FoM of 3.4 at an  $x/d$  of 0.45. These guidelines are based on the current literature, and are likely to change as WPT technology continues to improve.

### III. FIGURE OF MERIT DERIVATION

The proposed FoM is defined as

$$\text{FoM} := \frac{\lambda_{ref}}{\lambda_{sut}}, \quad (4)$$

where  $\lambda_{sut}$  is the resonant-coil-to-resonant-coil loss fraction, including inductor and capacitor loss, of the system-under-test, and  $\lambda_{ref}$  is the resonant-coil-to-resonant-coil loss fraction of a reference system with the same outer diameter and transmission distance as the system-under-test. The proposed FoM provides a metric for comparing WPT systems that accounts for both the size of the coils and the range of power transfer. The reference system is selected, not as the best

TABLE I: The baseline resonant coil variables and values

Variable	Description	Value
$d_o$	Baseline coil diameter	100 mm
$d_w$	Baseline wire diameter	3 mm
$Q_o$	Baseline quality factor	407
$\delta$	Skin depth	25 $\mu\text{m}$

possible design, but instead as a benchmark WPT system that can be used for comparison.

The reference system is defined by an equation which approximately models a simple physical system. The simple physical system, called the baseline design, is two resonant coils made from loops of solid wire that are scaled to match the outer diameter of the SUT, are separated by the same distance as the SUT, and whose currents are distributed uniformly across the area that is less than one skin depth from the surface of the conductors. The baseline design, described in Table I, is chosen to have an outer loop diameter  $d_o$  of 100 mm, a wire diameter  $d_w$  of 3 mm, and a skin depth  $\delta$  of 25  $\mu\text{m}$ . An illustration of the current distribution in the baseline coil and the diagram of the dimensions of the coil is shown in Fig. 1. The physical dimensions of the baseline coil design ( $d_o$ ,  $d_w$ ) are scaled by a linear scaling factor  $\epsilon$  in order to match the outer diameter of the SUT. The skin depth  $\delta$  is unaffected by  $\epsilon$  because the resonant capacitor can be chosen such that the resonant frequency is constant. The resonant capacitor in the baseline design is loss-less, which is a good approximation for most WPT systems because the winding loss is typically much larger than the capacitor loss. The current distribution was chosen for simplicity and to resemble the behavior of a skin-depth-limited resonant coil that is operated at a relatively large transmission distance, so that current is distributed uniformly around the surface of the conductor.

The definition of the FoM was chosen as the ratio of loss fractions, as opposed to efficiencies, in order to make the FoM

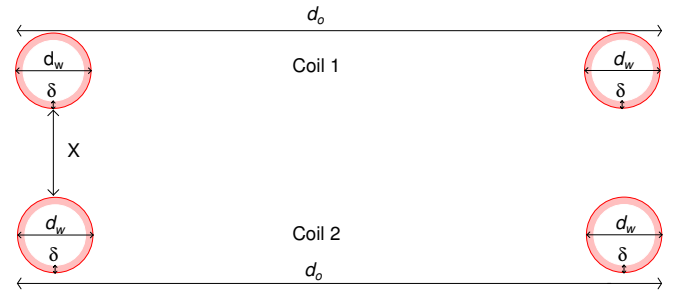


Fig. 1: A cross section of a pair of resonant coils similar to the baseline design, but not to scale, to illustrate coil diameter  $d_o$ , wire diameter  $d_w$ , skin depth  $\delta$ , and transmission distance  $x$ . Based on the current distribution constraint of the baseline design, the current in the coil is equally distributed throughout the area that is less than one skin depth from the surface (shaded region).

correlate to the ratio of energy dissipated in the reference system to that dissipated in the SUT. Often the loss fraction is not reported, but it can be calculated from the efficiency  $\eta$ ,

$$\lambda = \frac{P_{diss}}{P_{out}} = \frac{1 - \eta}{\eta}. \quad (5)$$

Using (5), the loss fraction of the SUT is calculated based on the reported efficiency  $\eta_{sut}$ , and the loss fraction of the reference system  $\lambda_{ref}$  is calculated from the theoretical maximum efficiency of the reference system  $\eta_{ref}$ . The theoretical maximum efficiency of a WPT system, derived in [6], is

$$\eta_{ref} = \frac{(Qk)^2}{\left(1 + \sqrt{1 + (Qk)^2}\right)^2}. \quad (6)$$

Therefore, we can define the reference system through closed-form expressions for the quality factor  $Q_{ref}$  and the coupling factor  $k_{ref}$  that model the scaled baseline design as a function of  $d$  and  $x$ .

#### A. Reference System: Modeling Quality Factor

The quality factor of the scaled baseline design is  $Q = \frac{\omega_o L}{R_{wind}}$ , where  $\omega_o$  is the angular resonant frequency of the coil,  $L$  is the inductance of the coil, and  $R_{wind}$  is the winding resistance of resonant coil. Both the inductance and the ac resistance must be computed based on uniform current distribution in the region one skin depth into the conductor as shown in Fig. 1.

The resistance included in the  $Q$  calculation is dependent only on the winding resistance because the definition of the baseline case establishes that the resonant capacitance is lossless. The winding resistance can be approximated as

$$R_{wind} = \frac{\epsilon \rho \pi (d_o - d_w)}{\pi (\epsilon d_w - \delta) \delta}. \quad (7)$$

An approximate model for the inductance is derived from an expression for the inductance of a loop of solid wire assuming uniform current distribution throughout the conductor and subtracting the inductance due to the magnetic field inside the solid conductor. The magnetic field inside the solid conductor is not present in the baseline design where the current is confined to the conductor surface. The model for the inductance is

$$L = \mu_o \epsilon \frac{d_o}{2} \left( \ln \left( \frac{8d_o}{d_w} \right) - 2 \right) - \frac{\mu_o \epsilon d_o}{8}. \quad (8)$$

Despite the impact of the scaling factor on the inductance, the resonant frequency  $\omega_o$  is chosen to be constant. This is possible because there is no restriction on the resonant capacitance. Through substitution, we find

$$Q = \frac{\omega_o \epsilon \left[ \mu_o \frac{d_o}{2} \left( \ln \left( \frac{8d_o}{d_w} \right) - 2 \right) - \left( \frac{\mu_o d_o}{8} \right) \right] (\epsilon d_w \delta - \delta^2)}{\epsilon \rho (d_o - d_w)}. \quad (9)$$

If the conductor diameter  $d_w$  is much larger than the skin depth  $\delta$ , then the quality factor is approximately proportional

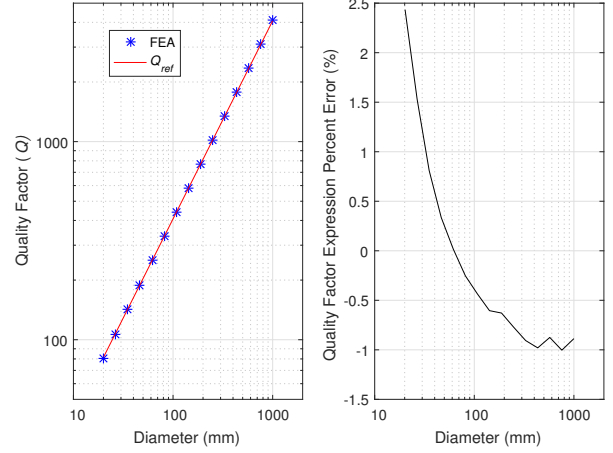


Fig. 2: The quality factor of the reference coil  $Q_{ref}$  is compared to FEA results of the scaled baseline coil over a range of coil diameters  $d$ .  $Q_{ref}$  closely approximates the  $Q$  of the scaled baseline coil, and it has less than 2.5% error over the range shown. The error increases as the diameter of the wire approaches the skin depth.

to the scaling factor

$$Q \propto \epsilon = \frac{d}{d_o}. \quad (10)$$

Therefore, we define  $Q_{ref}$  as

$$Q_{ref} := \frac{Q_o d}{d_o}, \quad (11)$$

where  $Q_o$  is (9) evaluated for the baseline coil, and as listed in Table I, is  $Q_o = 407$ .

The expression for  $Q_{ref}$  is compared to finite element analysis (FEA) results of the scaled baseline coil in Fig. 2. The FEA was done using a magnetostatic simulation on a pipe with an outer diameter of  $\epsilon d_w$ , and an inner diameter of  $\epsilon d_w - 2\delta$  in order to enforce the current distribution constraint of the scaled baseline design. The proposed model provides a simple expression for  $Q_{ref}$  that has less than 2.5% error compared to the FEA of the scaled baseline design when  $d_w > 24\delta$  (e.g.  $d_w > 4.9$  mm at 100 kHz and  $d_w > 0.60$  mm at 6.78 MHz).

#### B. Reference System: Magnetic Coupling Factor

The magnetic coupling factor  $k$  between the scaled baseline coils can be calculated from the mutual inductance  $L_m$  and leakage inductance  $L_l$ ,

$$k = \frac{L_m}{L_l + L_m} \quad (12)$$

However, the complexity of inductance calculations, especially with the current distribution shown in Fig. 1, means that deriving a closed-form expression for the coupling factor of the reference system  $k_{ref}$  is challenging. Even in the simple case of magnetic coupling between two single-turn loops of round wire with uniform current distribution through the entire

conductor, the magnetic coupling is not in a closed-form and needs to be expressed in terms of an integral [6]. In [9], an approximate formula is derived by combining the limits of the weakly coupled regime ( $x \gg d$ ) and the strongly coupled regime ( $x \ll d$ ). However, this formula is inaccurate and can overestimate  $k$  by as much as a factor of 2.2 between these two limits, namely when  $x$  and  $d$  are comparable, and this is for uniform current distribution in the conductor. Thus, a more accurate closed-form formula for magnetic coupling factor of the baseline design is desired for developing the FoM.

Such an expression is developed from the combination of two different models. First, a scalable model of strongly coupled coils is derived from the reluctance of the leakage path  $\mathcal{R}_l$  and the mutual path  $\mathcal{R}_m$ . For the leakage inductance  $L_l$ , the magnetic path area and length of the reference coil are proportional to  $d^2$  and  $d$  respectively giving

$$\mathcal{R}_l \propto \frac{d}{d^2} \propto \frac{1}{d} \implies L_l \propto d. \quad (13)$$

For the mutual inductance  $L_m$ , the magnetic path area still varies with  $d^2$  whereas the path length now varies with the transmission distance  $x$ , resulting in  $L_m \propto \frac{d^2}{x}$ . From this and (12), it can be shown that  $k \propto \frac{1}{1+\frac{x}{d}}$ . Second, according to [9], the magnetic coupling factor for weakly coupled coils is  $k \propto \left(\frac{d}{x}\right)^3$ . So a combination of these two limits, with an appropriate transition, is a closed-form approximation for the magnetic coupling factor for all transmission distances:

$$k \approx \frac{k_0}{1 + c_1 \left(\frac{x}{d}\right) + c_2 \left(\frac{x}{d}\right)^3}, \quad (14)$$

where  $k_0$ ,  $c_1$  and  $c_2$  are constants which can be obtained by fitting the equation to the magnetic coupling factor to FEA results of the scaled baseline coils. The FEA was done using a magnetostatic simulation on a pipe similar to the one used for verification of the  $Q$  expression. This curve-fitting gives  $k_0 = 0.8$ ,  $c_1 = 10.2$  and  $c_2 = 16$ . As  $x/d$  approaches 0, the magnetic coupling converges to  $k_0 = 0.8$ . The constant  $c_1$  is important for the strongly coupled regime ( $x \ll d$ ) where  $k \approx \frac{k_0}{1+c_1(\frac{x}{d})}$ , and the constant  $c_2$  is important for the weakly coupled regime ( $x \gg d$ ) where  $k \approx \frac{k_0}{c_2(\frac{x}{d})^3}$ .

We define the magnetic coupling of the reference system  $k_{ref}$  by

$$k_{ref} := \frac{0.8}{1 + 10.2 \left(\frac{x}{d}\right) + 16 \left(\frac{x}{d}\right)^3}. \quad (15)$$

The magnetic coupling factor of the reference system was compared to magnetostatic FEA simulations of the scaled baseline coils over a large range of  $x/d$ . The error, shown in Fig. 3, is less than 5% for a large range of  $x/d$ .

### C. Calculation of the New Figure-of-Merit

The new FoM is defined as the ratio of the loss fraction of the reference system and the SUT. The loss fraction of the reference system is derived from  $Q_{ref}$  given by (11) and  $k_{ref}$

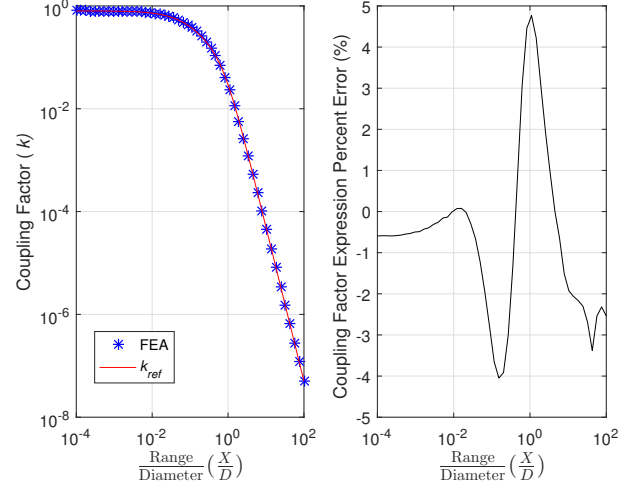


Fig. 3: The coupling factor of the reference coil  $k_{ref}$  is compared to FEA results of the scaled baseline coil over a range of  $x/d$ .  $k_{ref}$  closely approximates the magnetic coupling of the scaled baseline coil, and it has less than 5% error over the range shown.

given by (15). The FoM is defined as

$$\text{FoM} := \frac{\lambda_{ref}}{\lambda_{sut}} = \frac{(1 - \eta_{ref}) \eta_{sut}}{\eta_{ref} (1 - \eta_{sut})}, \quad (16)$$

where

$$\eta_{ref} = \frac{(Q_{ref} k_{ref})^2}{\left(1 + \sqrt{1 + (Q_{ref} k_{ref})^2}\right)^2}, \quad (17)$$

and

$$Q_{ref} k_{ref} = \frac{(3.26 \text{ mm}^{-1}) d}{1 + 10.2 \left(\frac{x}{d}\right) + 16 \left(\frac{x}{d}\right)^3}. \quad (18)$$

## IV. COMPARING WPT SYSTEMS USING THE FOM

The new FoM provides a tool for comparing the performance of WPT systems that is independent of the physical dimensions of the systems. The FoM compares the loss fraction of a SUT to the loss fraction of a reference system with the same overall dimensions. The loss fraction of the reference system can be considered to be an “entitlement”, as it provides the performance of a standard design with the physical dimensions of the SUT. Using the reference system as a standard of performance allows the FoM to assess the performance of WPT coil technology independent of its specific implementation.

It is useful to have a reference system that follows the same scaling laws as the SUT; in this case the FoM is independent of the diameter and transmission distance of the SUT. To assess the degree to which this is achieved in practice, this section compares the scaling laws of generic resonant coils to the scaling laws of the reference system and demonstrates: why the FoM can be used to provide insights into the performance

of WPT independent of size and range, why the FoM can be used to provide a direct comparison between WPT systems with the same  $x/d$ , and when the FoM could be misleading.

For WPT systems that follow the same scaling laws as the reference system, the FoM of an SUT is constant, independent of diameter or transmission distance. Thus, one can accurately predict performance for any size or distance, given the FoM for different scaling. It is sufficient to know the value of the FoM to describe the performance of the SUT.

The general scaling laws of WPT systems are derived in Appendix A. This analysis shows that many well-designed systems have the same scaling with respect to  $d$ , given a constant  $x/d$ . That is,  $Q \propto \epsilon$  and constant  $k$ . A few exceptions are discussed below, but it is reasonable to assume that a typical well designed system at least approximately follows this scaling with respect to  $d$ . However, the scaling behavior with respect to  $x/d$  is less consistent between different system types. Thus, using the FoM for specific quantitative predictions is often more reliable between systems with the same  $x/d$ .

However, the FoM still provides useful information even for systems that do not scale the same way as the reference system. In these cases, the FoM will vary when  $d$  or  $x/d$  is varied. A plot of this varying FoM then indicates not only the overall quality of the SUT, but also the ranges of  $d$  or  $x/d$  where it is most advantageous or where it performs poorly. And despite the possibilities of these variations, a WPT technology that earns a large FoM at a particular  $x/d$  is likely to also perform well if it is designed and operated for different values of  $x/d$ . Therefore, technology that earns a large FoM at any  $x/d$  is a good candidate for use at other  $x/d$  values.

In Appendix A-A, it is shown the  $Q$  of resonant coils scales linearly with  $\epsilon$ , and this is true in most practical scenarios. However, as discussed in [7], there are a few examples of coils that scale differently. For example,  $Q \propto \epsilon^2$  for coils made using: a single-layer winding with a wire diameter less than a skin-depth or a multi-layer winding that successfully utilizes a strategy to eliminate proximity effect such that the ac and dc resistances are approximately equal.

Furthermore, in some designs, the performance of the WPT coils is significantly impacted by loss mechanisms other than winding loss, such as capacitor loss or core loss. If another loss mechanism is dominant, then, once again, the scaling laws of the SUT will not follow the scaling laws of the reference system. For example, consider the impact of capacitor loss. As shown in Appendix A-A, the inductance of a WPT scales with  $\epsilon$ , so in order for the resonant frequency to be constant the capacitance  $C_r$  is proportional to  $1/\epsilon$ . The ESR of a capacitor  $R_c$  is given by  $R_c = \frac{D_d}{C_r \omega}$ , where  $D_d$  is the dissipation factor. Therefore, if the capacitor loss is the dominant loss mechanism, then  $Q$  is not impacted by the scaling factor.

In the scenarios where the SUT does not follow the same scaling laws as the reference system, the FoM still provides insight into the performance of that particular implementation. In this situation, the FoM compares the performance of the SUT to a standard design, so the FoM allows the user to

TABLE II: The FoM is calculated for a sample of WPT systems utilizing various resonant coil technologies in the literature and are catalogued below.

Citation	Frequency	Coil Technology	Max FoM
[3]	7.65 MHz	Solid wire coil	0.11
[4]	15.9 MHz	Solid wire coil	0.61
[8]	6.78 MHz	Multilayer structure	2.43
[10]	10.6 MHz	Solid wire coil	0.97
[11]	3.7 MHz	Surface spiral coil	0.34
[12]	50 kHz	Litz wire coil	0.52

compare other scalable designs to the SUT. However, it is more difficult to assess the performance of the technology as a whole. If the scaling laws of the reference system do not apply, the FoM of the technology may vary for different values of  $d$  or  $x/d$ . Although this does not allow quantitative predictions of the performance for different values of  $x$  and  $x/d$  from those at which the FoM was assessed, the variation of FoM with respect to those parameters can help demonstrate where the particular technology is most advantageous.

## V. COMPARISON OF EXISTING WIRELESS POWER TRANSFER SYSTEMS

The FoM provides a convenient tool for comparing the performance of WPT systems. To demonstrate this ability, FoM values for various WPT systems using different resonant coil technologies are reported in Table II, and plotted as a function of  $x/d$  in Fig. 4.

The proposed FoM requires commonly reported experimental parameters of  $d$ ,  $x$ , and  $\eta$ , and does not require  $Q$  or  $k$ . This allows the FoM to be used on most WPT manuscripts in the literature. In this section, we compare the performance of a few of the existing WPT systems in order to demonstrate the capabilities of the FoM; however, this is not an exhaustive investigation of the literature. Instead, this comparison is intended to demonstrate how to use the FoM to compare WPT systems.

Four types of resonant coils are included in this comparison: solid wire coils, litz wire coils, multilayer self-resonant structures, and surface spiral coils. The best performing solid wire coil included in this comparison is presented in [10]. The design in this work is similar to the reference system, but has a smaller skin-depth and uses a thinner diameter conductor; and therefore achieves slightly worse performance than the reference system. A surface spiral coil design is presented in [11]. Despite the complexity of the structure, the performance is similar to or slightly worse than many of the solid wire designs. A low-frequency litz wire design was presented in [12], and its FoM is slightly worse than the solid wire coils and slightly better than the surface spiral coil. A multilayer self-resonant coil presented in [8] has the largest FoM of the systems included in this comparison. This structure has advantages similar to litz wire, but at multi-MHz frequencies where effective litz wire is not commercially available. This structure significantly outperformed the reference system as it achieved a FoM as high as 3.4. It should be noted that winding

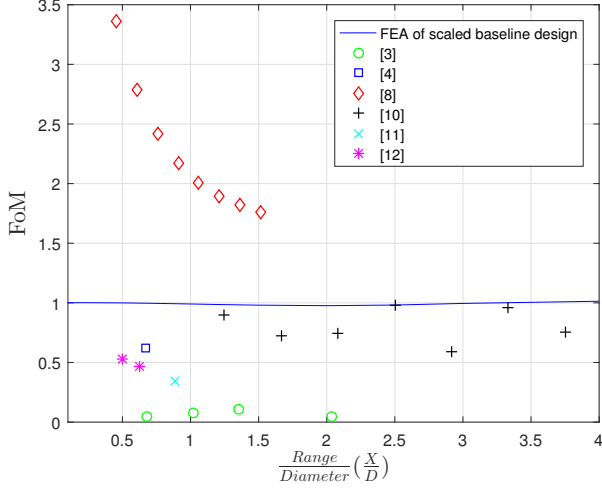


Fig. 4: The FoM is shown as a function of transmission range to resonant coil diameter ( $x/d$ ) for various WPT systems presented in the literature. In addition, the FoM is plotted for the scaled baseline design using FEA to demonstrate that the FoM approximates the behavior of this system.

loss is not the dominant loss mechanism in this coil, and therefore, as discussed in Section IV, the large FoM suggests that this structure would perform well with other dimensions but further data points are required to obtain actual FoM values for other sizes.

The FoM is also plotted for a FEA simulation of the scaled baseline design as the “system-under-test”. As expected, the FoM was nearly one; it had less than 2.5% error across an  $x/d$  range of 0.1 to 4. This suggests that the FoM is based on an accurate approximation of the scaled baseline design.

## VI. CONCLUSION

The conventional FoM used to describe resonant magnetic WPT systems does not consider the diameter and transmission range, and requires parameters ( $Q, k$ ) that are not commonly reported in the literature. These shortcomings make it difficult to compare the effectiveness of various techniques for improving WPT in the literature. This work introduces a new FoM that is in terms of commonly reported parameters ( $x, d, \eta$ ), and considers both the size of the structure and the transmission range of the system-under-test. This new tool can be used to compare the performance of various WPT systems and techniques in the literature, and will help guide engineers and researchers in the development of new and more effective designs.

## APPENDIX A

### GENERAL SCALING LAWS OF RESONANT COILS

This section derives the impact of linearly scaling the physical dimension, by a factor  $\epsilon$ , on the  $Q$  and  $k$  of practical resonant coils. This analysis is similar to the analysis in Section III-A and III-B, but, instead of being focused around a

specific example, it creates general scaling laws that describe most of the resonant coils used today.

#### A. Impact of Scaling on Quality Factor

The  $Q$  of a resonant coil, assuming a constant  $\omega_0$ , is proportional to  $\frac{L}{R_{ac}}$ . Therefore, the influence of  $\epsilon$  on  $Q$  can be understood by investigating the affect of  $\epsilon$  on the inductance  $L$  and resistance  $R_{ac}$  of resonant coils.

The inductance of a coil can be written in terms of the reluctance  $\mathcal{R}$  of the magnetic flux path as  $L = \frac{N^2}{\mathcal{R}}$ , where  $N$  is the number of turns. The reluctance is  $\frac{\ell_e}{\mu_e A_e}$ , where  $\ell_e$  is the effective path length of the magnetic flux,  $A_e$  is the effective area through which the flux passes, and  $\mu_e$  is the effective permeability. In a scaled version of this structure, the path length would be  $\epsilon \ell_e$  and the area would be  $\epsilon^2 A_e$ ; therefore, the inductance of a resonant coil scaled by the linear scaling factor  $\epsilon$  is proportional to the scaling factor,

$$L = \frac{\epsilon^2 A_e N^2 \mu_e}{\epsilon \ell_e} \propto \epsilon. \quad (19)$$

This is a general result that applies to resonant WPT coils of different types and shapes. For example, consider a common resonant coil such as a planar spiral winding. The inductance of such a structure, as modeled by [13], is

$$L \approx \frac{\mu_0 N^2 \epsilon d_{avg}}{2} \left( \ln \left( \frac{2.46}{R_f} \right) + 0.2 R_f^2 \right) \propto \epsilon, \quad (20)$$

where the average diameter  $d_{avg}$  is the average of the inner diameter of the coil  $d_{in}$  and the outer diameter of the coil  $d_{out}$ , and the fill ratio  $R_f = (d_{out} - d_{in}) / (d_{out} + d_{in})$ . As expected, the inductance of this winding is proportional to  $\epsilon$ .

The impact of scaling on winding resistance is considered in [7]. First, the authors consider a single-layer winding in which the thickness of the conductor is larger than a skin-depth. For this scenario, both the length of the winding and the area of the conductor that is utilized scales with  $\epsilon$ , so the resistance is independent of  $\epsilon$ . Next, the authors consider constrained multilayer windings with layers or strands thin compared to the skin depth (e.g. litz wire). There are two constraints that are commonly applicable to resonant coils: the optimal layer thickness given the number of layers, and the optimal number of layers given the layer thickness. The authors show that for both of these constraints the winding resistance is independent of  $\epsilon$ .

For most resonant coils used in WPT applications the inductance of is proportional to the scaling factor ( $L \propto \epsilon$ ) and the ac resistance is independent of the scaling factor; therefore, the  $Q$  of practical resonant coils is

$$Q \propto \epsilon. \quad (21)$$

#### B. Impact of Scaling on Magnetic Coupling Factor

The magnetic coupling of resonant coils can be derived from a general reluctance model of two identical coils separated by an air gap. The reluctance of the leakage path of each coil is



$\mathcal{R}_\ell$  and the reluctance of the mutual path is  $\mathcal{R}_m$ . The resulting inductance matrix is

$$L = \begin{bmatrix} \frac{N^2}{\mathcal{R}_m} + \frac{N^2}{\mathcal{R}_\ell} & \frac{N^2}{\mathcal{R}_m} \\ \frac{N^2}{\mathcal{R}_m} & \frac{N^2}{\mathcal{R}_m} + \frac{N^2}{\mathcal{R}_\ell} \end{bmatrix} \quad (22)$$

Therefore, the magnetic coupling is

$$k = \frac{1}{1 + \frac{\mathcal{R}_m}{\mathcal{R}_\ell}}. \quad (23)$$

If the relative dimensions the WPT system are scaled (i.e.  $x/d$  is constant), then the coupling factor is unaffected by scaling because  $\frac{\mathcal{R}_m}{\mathcal{R}_\ell}$  is constant. The coupling factor of the scaled baseline design provides an example of this relationship because the expression is a function  $x/d$ .

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